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MECHANICS AND CONTROL OF PUMPING A PLAYGROUND SWING AND ROBOTIC IMPLEMENTATION

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ABSTRACT

We explored the mechanics of a simple pendulum to find the best method to increase the amplitude the fastest way possible. In our experiment, we used MATLAB to simulate the equations of motion and test different approaches to pumping a pendulum. In addition, we used LEGO MINDSTORMS Education EV3 Core Sets to demonstrate the theoretical equations with a physical pumping swing robot. Three different pumping methods were utilized and analyzed by the MATLAB simulations and one was used in the hardware section. The goal was to determine which method increased the amplitude of the swing after a specific amount of oscillations. Upon completion of the experiment, we confirmed that pumping a swing in a pattern-based motion did increase the amplitude of a pendulum. Our results showed that the combined pumping method increased the pendulum amplitude the fastest.

NOMENCLATURE

$\boldsymbol{\theta}$ - Angle between the pendulum string and a line perpendicular to the fixed x-axis
$\boldsymbol{\theta}^+$ - Angle after the pump
$\boldsymbol{\theta}^-$ - Angle before the pump
$\boldsymbol{\ell}$ - Length between the fixed point of rotation and the robot's center of mass
$\boldsymbol{\phi}$ - Angle between $\boldsymbol{\ell}$ and $\mathbf{a}$
$\mathbf{a}$ - Length between the ends of the robot and the center of mass
$\mathbf{T}_{\mathbf{crouch}}$ - Period for stage 1 of standing pump cycle
$\mathbf{T}_{\mathbf{stand}}$ - Period for stage 2 of standing pump cycle
$\mathbf{g}$ - Gravitational acceleration
$\boldsymbol{\omega}$ - Angular velocity , $\frac{d}{dt}(\theta)$
$\boldsymbol{\omega}^+$ - Angular velocity after the pump
$\boldsymbol{\omega}^-$ - Angular velocity before the pump
$\boldsymbol{\alpha}$ - Angular acceleration, $\frac{d^2\theta}{dt^2}$

INTRODUCTION

A simple pendulum is one of the most well known mechanisms of classical physics. A body of mass hangs from a rope that is fixed about a point above. This simple mechanism has led us to the following question: What is the fastest method to increase the amplitude of a simple pendulum? This question has guided us to experiment with different pumping methods: standing, sitting, and combined pumping methods. Utilizing two large LEGO motors and a chassis to move a body of mass will aid in obtaining the three pumping methods physically. The following paragraphs will give a detailed explanation of each pumping method. Understanding how each pumping method works will allow us to accurately compare each method and lead us to the method that increases the amplitude the fastest.

The standing pump method increases the amplitude of the pendulum by changing the length between the fixed point and the robot's center of mass. For this method, one cycle is defined with the following steps: The robot begins in a crouching position at a small angle, $0° < |\boldsymbol{\theta}| < 10°$. As it passes through the lowest point ($\boldsymbol{\theta} = 0$), the robot stands up, decreasing $\boldsymbol{\ell}$ instantaneously. When it reaches the local maximum, the robot increases $\boldsymbol{\ell}$ instantaneously by switching to the crouching position. Again, as it passes through the lowest point in the opposite direction, the robot stands up, decreasing $\boldsymbol{\ell}$ instantaneously until it reaches its local maximum on the opposite end of the pendulum. When the robot reaches the new local maximum, it increases ℓ instantaneously by switching to the crouching position. This concludes one cycle. In theory, according to [1], repeating these steps will result in the robot obtaining an increase in amplitude after each cycle. A step-by-step diagram of the standing pump method is shown in figure 1.

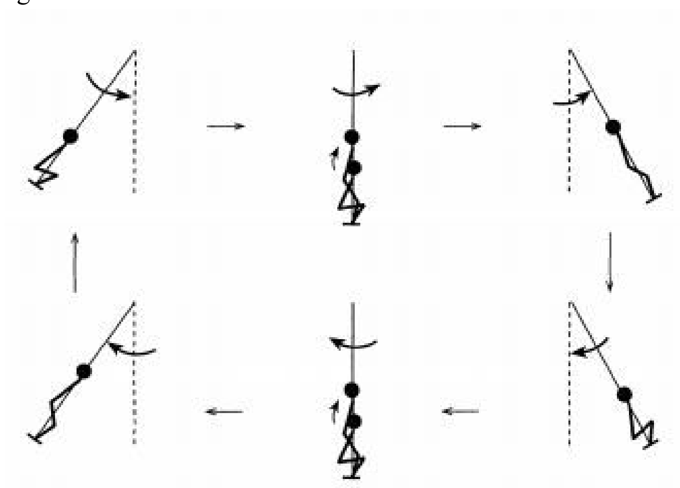

Figure 1. Standing Pump Cycle [1]

The sitting pump method increases the amplitude of the pendulum only through rotational motion about the center of mass. Using the model provided by [1], the change in ℓ is ignored. For the sitting pump method, one cycle is defined with the following steps: The robot begins in a perpendicular position (ϕ = 90°) at a small initial angle, $0° < |\theta| < 10°$. As the robot swings through the lowest point and reaches its local maximum, the robot's orientation ϕ instantaneously changes to ϕ = 0°. As the robot swings back in the opposite direction, passes through the lowest point, and arrives at the new local maximum, the robot's orientation ϕ instantaneously changes back to ϕ = 90°. This concludes one cycle. Figure 2 shows a diagram of the sitting pump method.

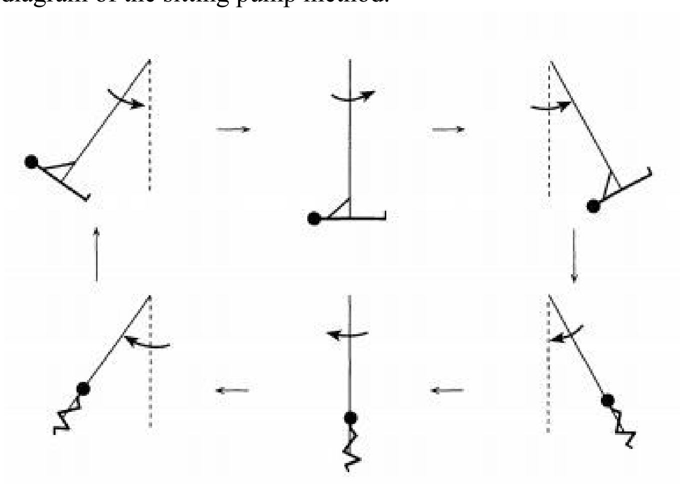

Figure 2. Sitting Pump Cycle [1]

The combined pumping method increases the amplitude of the pendulum by both the change in ℓ and the rotational motion provided by the sitting pump method. In the combined pump method, one cycle is defined with the following steps: The robot begins in a perpendicular position (ϕ = 90°) at a small initial angle, $0° < |\theta| < 10°$. As the robot reaches the lowest point, ϕ instantaneously changes to ϕ = 0° and ℓ decreases. When it reaches the local maximum, ϕ instantaneously changes to ϕ = 90° and ℓ increases. As the robot passes through the lowest point on the return swing, again, ϕ instantaneously changes to ϕ = 0° and ℓ decreases. Finally, when it reaches the new local maximum, ϕ instantaneously changes to ϕ = 90° and ℓ increases. In theory, according to [1], repeating these steps will result in the robot obtaining an increase in amplitude after each cycle. Figure 3 shows the step-by-step process for the combined pump method.

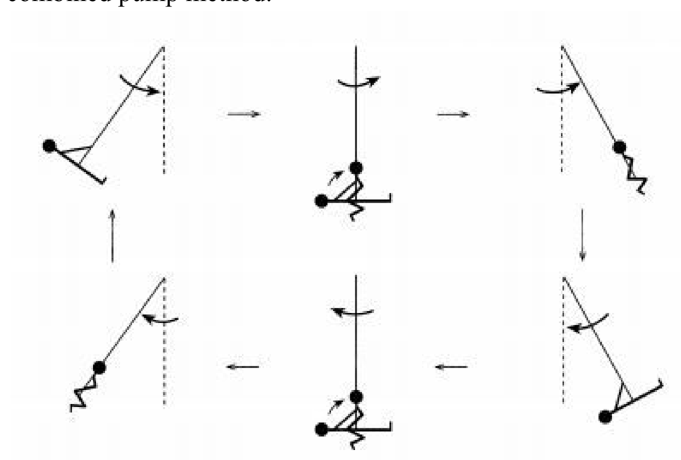

Figure 3. Combined Pump Cycle [1]

SIMULATION

The project was divided into a simulation section and a hardware section. We began the simulation portion of the project by downloading a MATLAB script file of a simple pendulum simulation found on MathWorks [4]. We heavily edited it so we could demonstrate each pumping method.

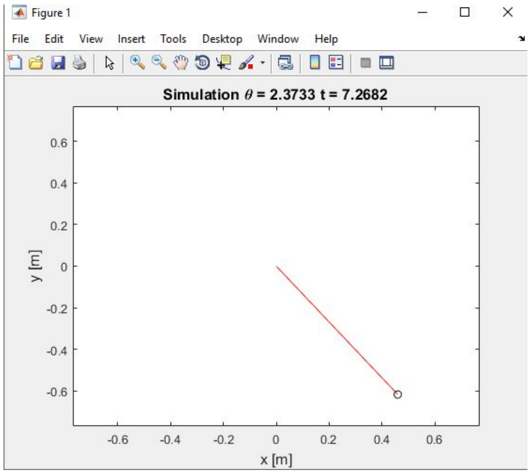

Figure 4. MATLAB Simulation Screenshot

Standing pump simulation

One cycle was defined to be a full swing. The four stages of the standing pump method described in the introduction were used in the simulation. The robot gains energy when it stands up at the bottom of the swing, increasing the potential and kinetic energies. The robot loses potential energy when it squats down at the ends of the swing, but the amount is insignificant compared to the energy gained from the pump. The differential equation solver, ode45, was used to integrate the equation of motion for each stage to get the angular velocities and angles.

$$\frac{d^2\theta}{dt^2} = -g\sin(\theta)/\ell \qquad (1)$$

Equation of Motion for a Simple Pendulum

The location of the fixed point of rotation, the initial angle and velocity, and the length of the pendulum were all predetermined at the beginning of the simulation. We defined the robot to be standing when the pendulum length was 77 cm and crouched when the pendulum length 65 cm. Every odd stage began by defining the length to be 77 cm while every even stage began by defining the length to be 65 cm. The ode45 generated a vector consisting of angles and angular velocities. Using these values, the coordinates of the robot for each instant is determined with the following equations.

$$\text{x-coordinate} = \boldsymbol{\ell} * \sin(\theta) \qquad (2)$$
$$\text{y-coordinate} = -\boldsymbol{\ell} * \sin(\theta) \qquad (3)$$

Equations to determine location of robot

The increased velocity due to the pump is determined by multiplying the velocity before the pump with a ratio of the crouch length over the standing length, squared.

$$\boldsymbol{\omega}^+ = \boldsymbol{\omega}^- * (\boldsymbol{\ell}_{\mathbf{crouch}}/\boldsymbol{\ell}_{\mathbf{stand}})^2 \qquad (4)$$

Equation to determine increased velocity after pump

The increased velocity is added to the initial velocity of the next stage. This allows the simulation to increase its amplitude.

Sitting pump simulation

In the standing pump simulation, one cycle consisted of two stages. Unlike the standing pump, the length of the pendulum was consistent for each stage. This is because the change in $\boldsymbol{\ell}$ is ignored for this model. The length used was 77 cm. The coordinates for the robot's location for each instant is determined using the same equations for the standing pump method. The robot puts energy into the system when it rotates to change its orientation, increasing the kinetic energy.

$$\boldsymbol{\theta}^+ = -(a^2)*(\boldsymbol{\phi}^+ - \boldsymbol{\phi}^-)/(\boldsymbol{\ell}^2 + a^2) + \boldsymbol{\theta}^- \qquad (5)$$

Equation to determine new position after pump

The new position is added to the initial position value of the next stage. This allows the simulation to increase its amplitude.

Combined pump simulation

In the combined pump simulation, one cycle consisted of four stages. This model combined the rate of changes in the standing pump as well as the body orientations in the standing pump. The coordinates for the robot's location for each instant is determined using the same equations for the standing pump method. The equations (4) and (5) are used in the combined model to determine the positions and velocities for each stage.

HARDWARE

Using the MATLAB simulation and equations provided by [1], the next challenge was to create a standing pump cycle using hardware. The swinging robot was built using 4 LEGO MINDSTORMS Education EV3 Core Sets. Using these sets allowed us to create an apparatus with ease by editing the robot through addition and subtraction of small LEGO pieces. Also, the sets allowed access to sensors and motors that could be programmed with computer software provided by LEGO. The following figures show the robot we used to create a physical standing pump cycle.

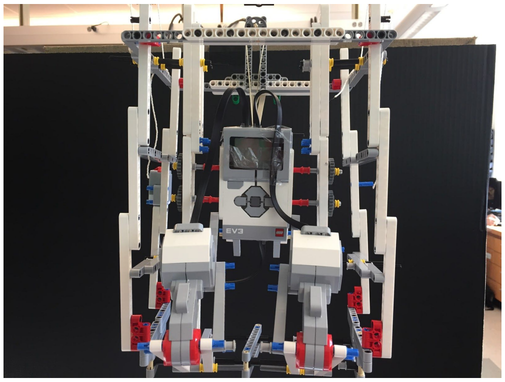

Figure 5. Front View of Robot

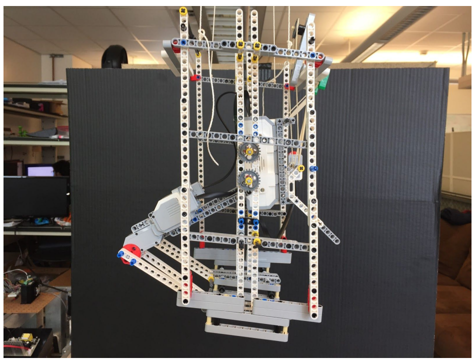

Figure 6. Side View of Robot

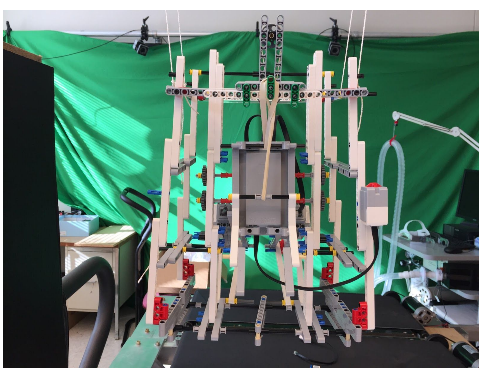

Figure 7. Back View of Robot

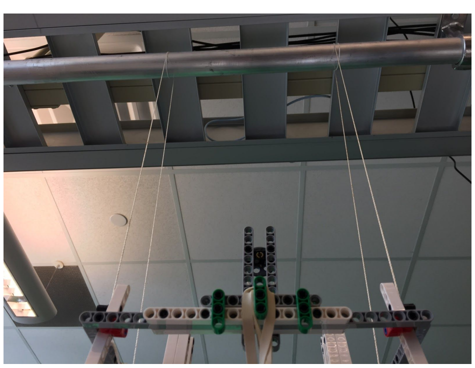

Figure 8. Suspension Setup of Robot

With the setup of this robot, it was found that using 2 large LEGO motors, along with the tension provided by 2 rubber bands, was effective in moving the robot's center of mass completely vertical. Also, it aided in obtaining a sufficient ratio between the crouching position ($\boldsymbol{\ell}_{\mathbf{crouch}}$) and standing position ($\boldsymbol{\ell}_{\mathbf{stand}}$). Furthermore, the LEGO MINDSTORMS Software helped control the 2 motors with sufficient accuracy and precision. With the software, the robot was able to produce a standing pump method by utilizing the period of a simple pendulum and dividing it into 4 stages.

The period of a simple pendulum was utilized in calculating the theoretical time the robot took from being in a crouched position to the lowest point and in the standing position from the lowest point to a local maximum. By dividing the entire period by 4 sub-periods, the robot could recreate a standing pump motion by programming the 2 motors to alternate counterclockwise and clockwise at the given sub-periods. Since the entire period is defined by starting from one position and ending in the same position, $\mathbf{T}_{\mathbf{crouch}}$ and $\mathbf{T}_{\mathbf{stand}}$ were defined as a fourth of the entire period and each utilized twice for one complete period.

$$T = 2\pi\sqrt{\frac{\ell}{g}} \tag{6}$$

Period of a simple pendulum equation

Using this simple equation, T was divided into 2 equations. Each equation describes the approximate time the robot endures in both the crouching and standing position. Considering that the standing pump method changes $\boldsymbol{\ell}$ at the lowest point of the pendulum path ($\boldsymbol{\theta}$ = 0°), $\mathbf{T}_{\mathbf{crouch}}$ was applied during which the robot moved from the local maximum displacement on both sides where $\boldsymbol{\omega}$ = 0°/sec towards the lowest point of the pendulum. Furthermore, $\mathbf{T}_{\mathbf{stand}}$ was utilized during which the robot moved from the lowest point of the pendulum path to the local maximum on both sides of the pendulum.

$$T crouch = \frac{\pi}{2}\sqrt{\frac{\ell crouch}{g}} \tag{7}$$

$$T stand = \frac{\pi}{2}\sqrt{\frac{\ell stand}{g}} \tag{8}$$

Period of a Standing Pump Pendulum Equations

With the setup of the robot, we decided that the best way to test the theoretical equations provided by [1] and the simulation created in MATLAB was to divide the experiment into 3 parts. First, testing the robot with a random pump pattern with no initial angle. It was agreed upon to use a random pump to show that randomly changing $\boldsymbol{\ell}$ would not conserve the angular momentum of the swinging robot entirely. Secondly, the robot was to perform a standing pump method with no initial angle ($\boldsymbol{\theta} = 0°$). Performing the standing pump method would allow to compare a pattern-based pump method to a random pump method to show if there was an increase in amplitude in both systems over the same amount of time (30 seconds). Lastly, the robot was setup to perform a standing pump method with a starting angle ($0° < |\boldsymbol{\theta}| < 10°$). This last part would show that starting with an initial angle would increase the amplitude in a shorter amount of time than a standing pump cycle with $\boldsymbol{\theta} = 0°$ initially. A video of the robot doing a standing pump is in the reference [5].

RESULTS

By subtracting the amplitude of the pendulum after one oscillation from the initial angle, we can determine the amplitude increase produced by each pumping method. After one oscillation with an initial angle of $\pi/60$, the standing pump method achieved an amplitude increase of 0.2036, the sitting pump method reached an amplitude increase of 0.0483, and the combined pump method achieved an amplitude increase of 0.2941. After one oscillation with an initial angle of $\pi/40$, the standing pump method achieved an amplitude increase of 0.3066, the sitting pump method achieved an amplitude increase of 0.0483, and the combined pump method reached an amplitude increase of 0.3938.

The swinging robot that we built using LEGO MINDSTORMS showed that the methods used for pumping a simple pendulum heavily depended on the position of the center of mass at a given stage during one complete period of the swing. This dependency was determined experimentally using the MATLAB simulation and the conservation of angular momentum given by equation (9).

The random pumping approach (with no initial angle, $0° < |\boldsymbol{\theta}| < 10°$) to increase the amplitude was ineffective. As $\boldsymbol{\ell}$ randomly changed due to the motion of the center of mass, the pendulum as a whole showed no significant increase in amplitude over 30 seconds. Initially, the robot increased $|\boldsymbol{\theta}|$ by approximately 2-5° by pumping since $\boldsymbol{\theta} = 0°$. The maximum amplitudes are shown in figures 9 and 10.

Figure 9. Maximum Amplitude for Standing Random Pump Method (Right side)

Figure 10. Maximum Amplitude for Standing Random Pump Method (left side)

With $\boldsymbol{\theta} = 0°$ initially, the angular velocity $\boldsymbol{\omega}$ increased a small amount according to the equation below provided by [1]:

$$\boldsymbol{\ell}^2_{\mathbf{stand}} * \boldsymbol{\omega}_{\mathbf{stand}} - \boldsymbol{\ell}^2_{\mathbf{crouch}} * \boldsymbol{\omega}_{\mathbf{crouch}} = -\int_{t0}^{t0+\Delta t} g\ell sin(\theta)dt \qquad (9)$$

Integrating Equation (1) over [t_0 and $t_0+\Delta t$]

The momentum in the system is perfectly conserved if the integral goes to 0. In the robot's case, $\boldsymbol{\theta} \rightarrow 0°$ will nullify the entire integral because $\sin(\boldsymbol{\theta})$ is not a function in time in the equation. In other words, the robot's amplitude will only increase when the impulse integral is equal to 0, making the entire equation conservative.

The robot, for the first 3-5 seconds of random pumping, obtained a conservative system in momentum since the value of $\boldsymbol{\theta}$ was small enough to neglect. However, it was observed that the robot's small amplitude initiated by the first 3-5 seconds damped over time due to the erratic motion of the center of mass of the pendulum's cycle.

Next, the robot experimented with the standing pump method with 2 cases: (1) Starting with no initial angle $\boldsymbol{\theta} = 0°$ and (2) starting with an initial angle $0° < |\boldsymbol{\theta}| < 10°$. It was recorded that in both cases, the robot did increase its amplitude over a duration of 30 seconds. In case 1, the robot took a longer amount of time to gain a maximum amplitude than it did when starting with an initial angle. Using these 2 cases and comparing them to the random pump cycle showed that the standing pump method was a more efficient and effective way to increase the amplitude of a swing. On the other hand, when comparing case (1) with case (2), the experiment showed that starting with an initial angle in the crouched position ($\boldsymbol{\ell} = \boldsymbol{\ell}_{\mathbf{crouch}}$), was the most preferred of the standing pump methods to maximize amplitude in a small amount of time. The figures show the maximum amplitude obtained using the standing pump method with an initial angle $\boldsymbol{\theta}$.

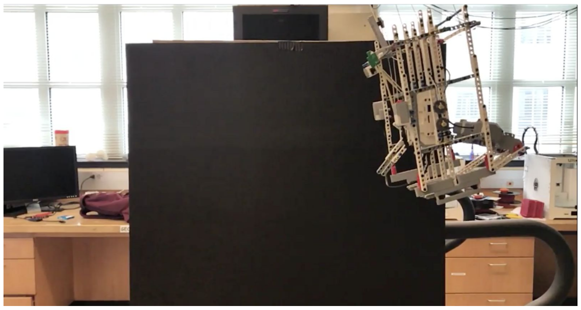

Figure 11. Maximum Amplitude for Standing Pump Method (right side)

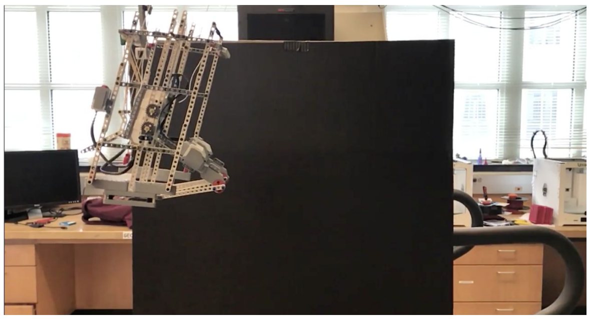

Figure 12. Maximum Amplitude for Standing Pump Method (left side)

Lastly, it was recorded that the standing pump method did indeed increase the amplitude, however, once the robot reached a maximum amplitude, the angular momentum of the system began to become non-conservative. The dampening of the pendulum's amplitude was caused by energy absorption by small twisting of the robot's suspension setup, large values for $\boldsymbol{\theta}$ and primarily due to the non-instantaneous change in $\boldsymbol{\ell}$ as the robot passes through the lowest point of the cycle and local maximum when $\boldsymbol{\omega}$ is equal to 0.

DISCUSSION

Simulation

The goal of this project was to determine which strategy produced the largest amplitude after one oscillation. The results showed that the combined pump method produced the largest amplitude after one oscillation, the standing pump produced the second largest amplitude after one oscillation, and the sitting pump produced the smallest amplitude after one oscillation.

To better understand why the combined pumping method increased its amplitude the fastest, we analyzed the equations each method used. For the standing pump method, the equation to determine the new velocity after the pump is equal to the velocity before the pump times a ratio of the crouched length over the standing length squared, making amplitude increase geometrically. For the sitting pump method, the equation to determine the new position after the pump is equal to the old position plus a constant, making the amplitude increase arithmetically. It is clear that the combined pumping method increases the amplitude the fastest because its amplitude uses equations from the standing and sitting methods, increasing the amplitude geometrically and arithmetically.

After running the simulation several times at different initial angles, we determined the initial angles that demonstrated our methods well were between $\pi/60$ and $\pi/40$. Angles that were larger sent the robot into a perpetual 360° swing, which did not demonstrate our concept very well. The angles that were smaller showed an increase in amplitude, but it took several cycles and the increase was not clearly visible.

Hardware

After observing the robot perform a random pumping motion, it was determined that the motion was not an effective way to pump a swing. Considering that the robot did not repeat a pattern based pumping method, it was clear that angular momentum was not conserved since the location of the center of mass relative to the fixed point of rotation did not change at either of the critical point of the pendulum: The lowest point of the path and local maximum on left and right side of the pendulum. Instead, the mass body changed in between these critical points, resulting in either no increase or dampening in amplitude.

As the random pumping did not produce a significant result in terms of increasing of amplitude, the standing pump with no initial angle did. After the first 3-5 seconds of pumping, the robot started to show a slow increase in amplitude. Although the robot acquired a sufficient amplitude after 30 seconds of pumping, it was evident that if the robot had started with an initial angle $\boldsymbol{\theta}$, the robot would start with a greater value of $\boldsymbol{\omega}$, hence, increasing the amplitude in a shorter amount of time than starting with no initial angle.

Starting with an initial angle gave the pendulum system an initial gravitational potential energy that would transform to kinetic energy as the mass body is released. When the center of mass passes through the lowest point of the path and change $\boldsymbol{\ell}$, it would produce an initial "boost" as [1] states. Initializing the standing pump method as such would in turn increase the amplitude faster. Although the amplitude of the robot increased the fastest out of all three methods tested, the system began to dampen the amplitude as previously stated in the results section due to non-instantaneous change in $\boldsymbol{\ell}$ and large values for $\boldsymbol{\theta}$. As the robotic implementation experiment was successful in showing how standing pump can effectively increase the amplitude of a pendulum, there were faults in the experiment as well.

Using the LEGO MINDSTORMS EV3 software was convenient to use for this experiment since it allowed ease of programming. The software was interactive and aided in reducing the time and money constraints to adjust the motion of the robot if needed. However, the 2 large motors came at a disadvantage when trying to increase the speed of the motors. Since the motors had a capped running torque of 20 N/cm, the robot could change ℓ as fast as the MATLAB simulation. If the robot were to use servo motors with a higher torque and speed, the robot could have possibly gained a greater amplitude by changing ℓ faster, closely replicating an instantaneous change to conserve angular momentum. Furthermore, if the robot were to use a gyro sensor that could accurately measure the change in $\boldsymbol{\theta}$, the lab could have recorded tangible values for $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ at each time instance. If this experiment were to be repeated in implementing a robotic swing, using stronger, faster and more accurate motors and possibly using a motion capture system and/or gyro sensor could further show the differences and similarities in each pumping method through accurate and precise analyzation for both motion of the robot and numerical values for each parameter presented throughout this paper.

CONCLUSION AND FUTURE WORK

The simulations demonstrated the three pumping methods well, and the robotic swing that was built to perform the MATLAB simulated pumping methods was overall a success. The robot showed that a pattern-based pumping method is ideal in order to increase the amplitude of a pendulum. In addition, the robot revealed that starting with an initial angle, $\boldsymbol{\theta}$, did increase the amplitude much faster than starting at the lowest point of the pendulum's path ($\boldsymbol{\theta} = 0°$). On the other hand, utilizing the relatively low torque/speed motors was a major draw back. If the robot had changed ℓ faster, it could have possibly gained a greater amplitude with higher values of $\boldsymbol{\theta}$.

The current simulations show a point mass rotating about a fixed point, but future work could incorporate a simulation using a two-dimensional representation of the robot we built with LEGO MINDSTORMS. Future researchers could also explore creating simulations that demonstrate methods that dampen amplitudes the fastest.

To extend the robotic implementation section of this experiment, the robot should perform not only the standing pump method, but also the sitting and combined pump method. In doing so could aid in accurately drawing a conclusion to the question: Which pumping method increases the amplitude of a swing the fastest? Lastly, to improve the entirety of the robot experiment, applying a gyroscopic sensor to measure $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ at given time intervals and/or capturing the motion of the robot during pumping methods could result in obtaining raw visual and numerical data. Acquiring graphical and numerical data could strengthen the answer(s) to this and further research experiments regarding the motion of a simple pendulum.

ACKNOWLEDGMENTS

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