

Foot Placement and Ankle Push-off Control for the Orbital Stabilization of Bipedal Robots

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Abstract—The main motivation of this paper is to understand the role of foot placement and ankle push-off control in stabilizing bipedal gaits. We modify the simplest walker (heavy torso, light legs) by incorporating a hip spring, a hip actuator, and a telescopic linear actuator. We consider two stability criteria: one-step dead-beat stabilization for full correction of disturbance in a single step and exponential orbital stabilization using discrete control Lyapunov function. Our findings are as follows: (1) Both control strategies have almost similar robustness as measured by the number of steps walked on stochastic terrain before failure, but both strategies are more robust for changing terrain with step down and less robust for step up. (2) One step dead-beat stabilization is more energy-efficient than exponential stabilization. (3) Control strategy for step up is to decrease foot placement or maintain push-off and for step down is to increase foot placement or decrease the push-off. However, it is most energy-efficient to use foot placement control for step up and push-off control for step down.

I. INTRODUCTION

Dynamic walking robots rely mainly on foot placement control (stepping in the direction of fall) for orbital or step-to-step stability to achieve natural-looking locomotion. However, push-off control (regulating the ankle motion before or after foot-strike) has also emerged as a promising alternative for balance control of dynamic locomotion. Understanding the role of these two strategies for gait stabilization is expected to help in the development of robust dynamic walking robots. In this paper, we investigate the role of foot placement control and ankle push-off control in gait stabilization of a dynamic walking model by considering uneven terrain and two notions of stability — one-step dead-beat stabilization and exponential orbital stabilization.

II. BACKGROUND AND RELATED WORK

Foot placement control was used as early as the 1980's by Marc Raibert to regulate the forward speed of his dynamically balancing hopping robots [1]. He defined the horizontal travel distance of the robot as the CG print. The normal, symmetric gait is created by placing the foot in the middle of the CG print. To increase robot speed, it needs to place its foot slightly before the middle of the CG print and vice versa to decrease its speed. More recently, Pratt et. al. [2] formalized foot placement control by defining the capture point. Capture point is the location that the robot needs to place its foot in order to come to a complete stop when

pushed. It is straightforward to extend this idea to generate and control steady locomotion.

Push-off control was first shown to explain the energetics of human locomotion by Kuo [3]. He showed that ankle push-off just before foot-strike is four times energy-efficient than push-off after foot-strike or hip actuation. Consequently, several dynamically balanced legged robots have used push-off control as means to create energy-efficient walking motions[4], [5]. Hobbelen and Wisse [6] have shown that ankle push-off is also able to improve the robustness while maintaining the energy-efficiency.

Goswami et al. [7] and Spong [8] have investigated the role of ankle torque and hip torque individually in stabilizing the compass gait model. The key idea is to use either actuator to track a reference energy level obtained from a passive dynamic limit cycle [9], but they have not compared the two control strategies. Byl and Tedrake [10] compared the performance of foot placement with push-off. They used the value iteration algorithm to find one-step control policy that maximizes probability of not falling on rough but 'known' terrain. They found that constant push-off control and adjustable foot placement leads to the most robust controller. Our study differs from theirs in that we assume the terrain profile is unknown.

More recently, a study by Kim and Collins [11] investigated the role of foot placement and ankle push-off using a three-dimensional model of human walking with single and double support phase. They found that ankle push-off control is twice as effective as foot placement control to recover from disturbances for changing terrain with step up and step down. Our study investigates the role of two strategies with a simple sagittal model and using two different stability criteria: one-step stabilization and exponential stabilization. Our motivation is to check if Kim and Collins' conclusions hold true for different stability measures and for different perturbations, namely step up and step down considered separately. In doing so, we are able to find which strategy works best for a given terrain disturbance and determine how the stability criteria influence the robustness and energy usage. These principles should aid in making design choices and controller design for bipedal robots and artificial devices.

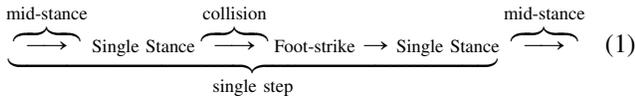
III. MODEL

A. Model description

Fig. 1 shows a model of the simplest walker. The model has a mass M at the hip and point mass m at each of the feet. Each leg has length ℓ . Gravity g points downwards. The leg in contact with the ground is called the stance leg while

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the other leg is called the swing leg. The angle made by the stance leg with the normal to the ground is θ and the angle made by the swing leg with the stance leg is ϕ . The hip torque is T . There is a torsional spring with spring constant K between the two legs (not shown). The rest length of the spring is zero and corresponds to the position when both legs are parallel. There is a linear actuator which is used to apply an impulsive push-off, P , just before foot-strike. A complete walking step of the model is given as follows:



A single step consists of two phases; a single stance phase where the swing leg rotates about the pin joint connecting to the stance leg and a foot-strike phase where there is support transfer and the legs change their roles. These phases are connected through two events; a mid-stance event where the stance leg is along gravity direction and a collision phase where the leading leg touches the ground.

B. Equations of motion

1) *Single stance phase (continuous dynamics)*: In this phase of motion, the stance leg pivots and rotates about the stationary foot; while the swing leg pivots and rotates about the hinge connecting the two legs. We assume that the stance leg does not slip, there is no hip hinge friction, and we ignore foot scuffing during leg swing. We obtain Eqns. 2 and 3 defined below by taking moments about stance foot contact point and hip hinge respectively, and non-dimensionalizing time with $\sqrt{\ell/g}$ and applying the limit, $m/M \rightarrow 0$. In Eqn. 3, τ is the non-dimensional torque obtained by dividing the torque, T , by $Mg\ell$. The non-dimensional spring constant is k and is obtained by dividing K by $Mg\ell$. The equations are;

$$\ddot{\theta} = \sin(\theta), \quad (2)$$

$$\ddot{\phi} = \sin(\theta) + \{\dot{\theta}^2 - \cos(\theta)\} \sin(\phi) - k\phi + \tau. \quad (3)$$

2) *Foot-strike phase (discontinuous dynamics)*: In this phase of motion, the legs exchange their roles, that is, the current swing leg becomes the new stance leg and the current stance leg becomes the new swing leg. There is an instantaneous plastic collision (no slip and no bounce) of the swing leg. The swapping of legs is expressed by Eqns. 4 and 5. The angular rates of the legs after support exchange are given by Eqns. 6 and 7 and are obtained by

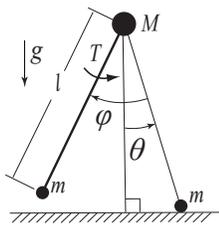


Fig. 1. Model: The simplest walking model on level ground [3].

applying conservation of angular momentum about stance foot contact point and hip hinge respectively, followed by non-dimensionalizing time with $\sqrt{\ell/g}$ and applying the limit, $m/M \rightarrow 0$. In the equations below, the state variables before and after collision are denoted using the superscript $-$ and $+$ respectively.

$$\theta^+ = -\theta^-, \quad (4)$$

$$\phi^+ = -\phi^-, \quad (5)$$

$$\dot{\theta}^+ = \dot{\theta}^- \cos \phi^- + P \sin \phi^-, \quad (6)$$

$$\dot{\phi}^+ = (1 - \cos \phi^-)(P \sin \phi^- + \dot{\theta}^- \cos \phi^-). \quad (7)$$

3) *Mid-stance event*: The mid-stance event occurs when the stance leg is normal to the ground and is given by $\theta = 0$.

4) *Collision event*: The collision event occurs when the foot of the leading leg makes contact with the ground. We introduce a step down of height h at the collision. This is taken to be zero, except for testing the robustness of the control approach. The collision event is given by, $\cos(\phi^- - \theta^-) - \cos(\theta^-) = h$.

5) *Failure modes*: There are two failure modes for the simplest walker and lead to two conditions on the state of the system and are checked at each integration step. Violation of any of these conditions is interpreted as system failure.

1) *Falling Backwards*: Falling backwards is detected when the angular velocity of the stance leg is positive (note that forward velocity is indicated by a negative angular velocity). The condition for failure is: $\dot{\theta} \geq 0$.

2) *Flight phase*: Flight phase is detected when the vertical ground reaction force, $R_y = \cos(\theta) - \dot{\theta}^2 \leq 0$. Thus the condition for failure is: $\dot{\theta}^2 - \cos(\theta) \geq 0$

IV. METHODS

A. Overview of control technique

From the single stance phase Eqns. 2 and 3, we see that: (1) the hip torque can be used to control the swing leg and (2) the motion of the swing leg does not affect the motion of the stance leg. Thus, by the controllability definition [12], although the swing leg motion, $\phi(t)$, is fully controllable, the motion of the stance leg, $\theta(t)$, is not controllable *within a step*. However, we can find a function, F , that maps the mid-stance between consecutive steps and is indexed by step number, k . Thus

$$\dot{\theta}_{k+1} = F(\dot{\theta}_k, P_k, \phi_k^-) \quad (8)$$

where ϕ_k^- is swing leg angle at foot-strike at step k and is related to the step length, and P_k is impulsive push-off at step k . Given the measurement at step k , $\dot{\theta}_k$, the control variables, ϕ_k^- and P_k , can be used to modulate $\dot{\theta}_{k+1}$. Thus, the stance leg is fully controllable *between steps*. Based on these observations, we use a hierarchical control approach: *the stance leg velocity is controlled between steps using foot placement and push-off control, while the swing leg is controlled in the step using trajectory tracking controller based on the foot placement angle*. Note that it is not possible to find closed form solution for F . The map, F , is obtained by numerically integrating the equations of motion.

B. Mid-stance to mid-stance map for stance leg:

In this section we present equations that can be used to numerically solve for the mid-stance to mid-stance map, F , given by Eqn. 8.

We do an energy balance for the stance leg between the mid-stance and the instant just before foot-strike and then again from just after foot-strike to the next mid-stance to get Eqns. 9 and 10 respectively

$$\frac{(\dot{\theta}_k)^2}{2} + 1 = \frac{(\dot{\theta}^-)^2}{2} + \cos\left(\frac{\phi^-}{2}\right), \quad (9)$$

$$\begin{aligned} \frac{(\dot{\theta}_{k+1})^2}{2} + 1 &= \frac{(\dot{\theta}^+)^2}{2} + \cos\left(\frac{\phi^+}{2}\right), \quad (10) \\ &= \frac{(\dot{\theta}^- \cos \phi^- + P \sin \phi^-)^2}{2} + \cos\left(\frac{\phi^-}{2}\right). \end{aligned}$$

To get the Eqn. 10 we have used the foot-strike conditions given by Eqns. 5 and 6. We have also assumed that controller is not going to be aware of the step up/down disturbance, so we set $h = 0$.

C. Discrete control Lyapunov function (DCLF) and one-step dead-beat control

We use a discrete control Lyapunov function (DCLF) to create an exponential stabilizing controller. The key idea is to create a control Lyapunov function at an event (e.g., mid-stance) and use it to regulate the velocity of the stance leg at a predefined convergence rate. More details are in [13].

First, we need a nominal limit cycle so that we are able to construct a DCLF to stabilize it. A nominal limit cycle is specified by setting $\dot{\theta}_{k+1} = \dot{\theta}_k = \dot{\theta}_0$, $P_k = P_0$, and $\phi_k^- = \phi_0^-$ in Eqn. 8 to get

$$\dot{\theta}_0 = F(\dot{\theta}_0, P_0, \phi_0^-) \quad (11)$$

Second, we define the DCLF as follows

$$V(\Delta\dot{\theta}_k) = \Delta\dot{\theta}_k^2 = (\dot{\theta}_k - \dot{\theta}_0)^2, \quad (12)$$

where $V(0) = 0$ and $V(\Delta\dot{\theta}_k) > 0$ at the mid-stance event, $\theta = 0$. For the system to be exponentially stable, we have the following condition

$$V(\Delta\dot{\theta}_{k+1}) - V(\Delta\dot{\theta}_k) = -cV(\Delta\dot{\theta}_k), \quad (13)$$

where c is a user chosen positive constant such that, $0 < c < 1$ for exponential stabilization and $c = 1$ for one-step dead-beat stabilization. Using Eqn. 12 in Eqn. 13 yields

$$(\dot{\theta}_{k+1} - \dot{\theta}_0)^2 - (1 - c)(\dot{\theta}_k - \dot{\theta}_0)^2 = 0. \quad (14)$$

D. Two control techniques

1) *Foot-placement control with fixed push-off:* We use the foot-placement to control the stance leg velocity between consecutive steps by fixing the impulsive push-off at P_0 . Thus Eqn. 8 can be written as

$$\dot{\theta}_{k+1} = F(\dot{\theta}_k, P_0, \phi_k^-) \quad (15)$$

Substituting Eqn. 15 for $\dot{\theta}_{k+1}$ in Eqn. 14 leads to the condition for exponential stability

$$\left(F(\dot{\theta}_k, P_0, \phi_k^-) - \dot{\theta}_0\right)^2 - (1 - c)(\dot{\theta}_k - \dot{\theta}_0)^2 = 0. \quad (16)$$

The stabilization problem for this case can be stated as follows: For the limit cycle parametrized by the stance leg nominal velocity at mid-stance, $\dot{\theta}_0$, a fixed rate of convergence specified by c , the stance leg velocity measured at mid-stance $\dot{\theta}_k$, and impulsive push-off maintained at its nominal value, P_0 , find the foot-placement angle ϕ_k^- so that the Eqn. 16 is met.

2) *Push-off control with fixed foot placement:* We use the impulsive push-off to control the stance leg velocity between consecutive steps by fixing the foot-placement at ϕ_0^- . Thus Eqn. 8 can be written as

$$\dot{\theta}_{k+1} = F(\dot{\theta}_k, P_k, \phi_0^-) \quad (17)$$

Substituting Eqn. 17 for $\dot{\theta}_{k+1}$ in Eqn. 14 leads to the condition for exponential stability as

$$\left(F(\dot{\theta}_k, P_k, \phi_0^-) - \dot{\theta}_0\right)^2 - (1 - c)(\dot{\theta}_k - \dot{\theta}_0)^2 = 0. \quad (18)$$

The stabilization problem for this case can be stated as follows: For the limit cycle parametrized by the stance leg nominal velocity at mid-stance, $\dot{\theta}_0$, a fixed rate of convergence specified by c , the stance leg velocity measured at mid-stance $\dot{\theta}_k$, and foot-placement angle maintained at its nominal value ϕ_0^- , find the control action P_k so that the Eqn. 18 is met.

E. Hip torque control for foot placement

We need to define a controller for the hip torque based on the computed swing leg angle at foot-strike. We use 2 third-order, time-based trajectories for the swing leg; one for the instant from mid-stance to foot-strike and another one from instant after foot-strike to the next mid-stance. Each of the third order polynomials has four coefficients that are computed based on the initial and final values of the position and velocity which are completely known. One also needs the time from mid-stance to instant just before foot-strike ($T_{\text{mid-fs}}$) and from instant after foot-strike to mid-stance ($T_{\text{fs-mid}}$). These can be computed as follows

$$\begin{aligned} T_{\text{mid-fs}} &= \int_{\theta=\frac{\phi^-}{2}}^{\theta=0} \frac{d\theta}{\sqrt{(\dot{\theta}_k)^2 + 2(1 - \cos(\theta))}} \\ T_{\text{fs-mid}} &= \int_0^{\theta=\frac{\phi^-}{2}} \frac{d\theta}{\sqrt{(\dot{\theta}^+)^2 + 2(\cos(\frac{\phi^-}{2}) - \cos(\theta))}} \end{aligned}$$

The time-based trajectory is supplemented with a proportional derivative controller to ensure good tracking performance.

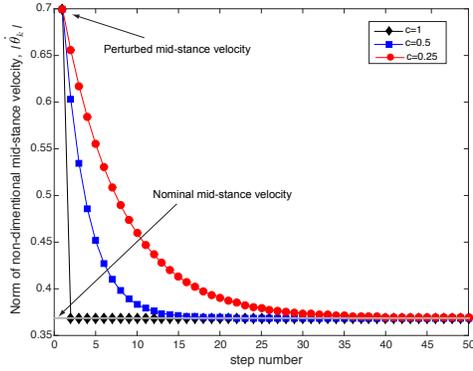


Fig. 2. Rate of stabilization: The convergence is exponential for $0 < c < 1$ and “one-step dead-beat” for $c = 1$.

V. RESULTS

A. Nominal limit cycle

We choose average human walking kinematics to create the nominal limit cycle. The non-dimensional average human speed is 0.41, step length is 0.73, and consequently the step time is 1.78 [14]. The spring constant k is chosen to be 2.11 in order to correspond to average human swing frequency. Next, we find the impulsive push-off and fixed point that give a passive leg swing. The nominal limit cycle values are: mid-stance robot state, $\{\theta_0, \dot{\theta}_0, \phi_0, \dot{\phi}_0\} = \{0, -0.3686, 0.0063, -1.2495\}$, impulsive push-off, $P_0 = 0.22$, and foot placement angle, $\phi_0^- = -0.7802$.

B. Rate of convergence

To demonstrate the rate of convergence we perturb the mid-stance speed, $\dot{\theta}_k = -0.7$. Then we run the one-step dead-beat controller, $c = 1$, and DCLF controller for $c = 0.25$ and $c = 0.5$. Figure 2 shows a plot of the mid-stance velocity as a function of the steps. As expected, the one-step dead-beat controller is able to regulate the velocity in a single step while the DCLF controller shows exponential convergence to the nominal limit cycle.

C. Robustness

1) *Simulation details:* In order to assess robustness of our controller, we compute the average number of steps that the robot can withstand using either controllers, foot-placement control with fixed push-off and push-off control with fixed foot placement, without falling down on uneven terrain chosen from a random distribution with a deviation of σ . Note that σ can be interpreted as a maximum step down ($+\sigma$) or step up ($-\sigma$) normalized by the leg length. We create 10 terrains with 400 steps, selected from a random distribution with standard deviation of σ . For each terrain, we do forward simulations of the system for three values of c . We evaluate the average number of steps to failure; control inputs ϕ^- and P ; and average Cost of Transport (COT) defined as the energy used per unit weight per unit distance travelled.

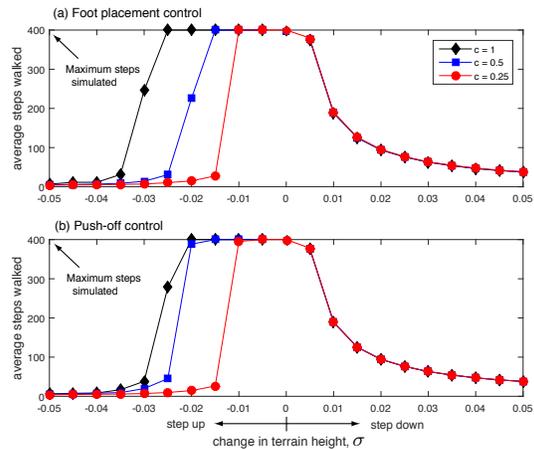


Fig. 3. Robustness to step-down and step-up disturbance: (a) Average steps to failure using foot placement control with push-off held constant, and (b) Average steps to failure using push-off control with foot placement held constant.

2) *Average number of steps to failure:* Figure 3 shows the average number of steps walked before failure as a function of deviation in terrain height σ for three values of c . The top plot corresponds to foot placement control, ϕ^- , with constant push-off, P_0 (see Sec. IV-D.1), while the bottom plot corresponds to push-off control, P , with foot placement held constant, ϕ_0^- (see Sec. IV-D.2). Since we limited the steps to 400 in the simulation, walking for 400 steps corresponds to maximum robustness. As σ increases from 0 to 0.05 (step down) or decreases from 0 to -0.05 (step up), the average steps to failure decreases as expected. The plots indicate the robustness is same for different c values for step down but marginally better for higher c values for step up. The similar robustness for different c values is because we did not enforce actuator limits. We expect that when actuator limits are enforced, more aggressive control (higher c values) will show lower robustness. The average steps profile is not symmetric about the y-axis; the drop in average steps walked is more gradual for step-down and more dramatic for step-up. This is due to the asymmetry in the hip controller: during step down the model has enough time to complete the step as planned but during step up the robot takes a shorter step because the robot takes a premature step increasing the chance of failure.

3) *Control strategy:* Figure 4 depicts the control strategy used for the robustness test shown in Fig. 3. The top plot shows the swing-leg angle at foot-strike for foot placement control strategy with push-off held constant to its nominal value while the bottom plot shows the impulsive push-off for push-off control strategy with swing-leg at foot-strike held constant at its nominal value.

We explain these plots by looking at the response for $c = 1$ or one-step dead-beat control (black diamond). For foot placement control, the swing leg angle changes from smaller than nominal to larger than nominal as the terrain height changes from step up to step down. This can be understood as follows. The model’s energy is lower than nominal for

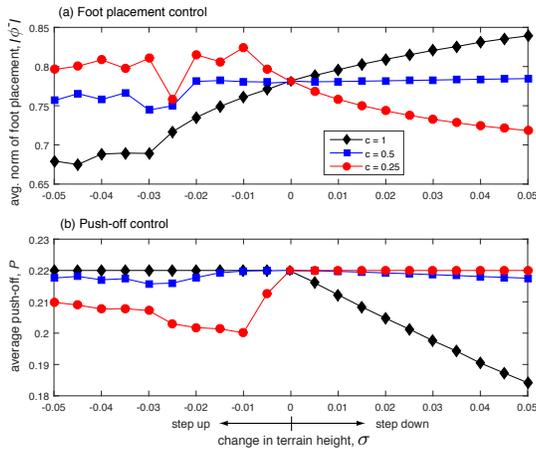


Fig. 4. Control strategy for robustness test: (a) Average norm of swing-leg angle at foot-strike for foot placement control with push-off held constant. (b) Average push-off with swing-leg angle at foot-strike held constant. The average is taken over all the 10 runs and across the complete terrain for given deviation, σ .

step up and higher than nominal for step down. To regulate the energy for a fixed push-off, the swing leg angle needs to monotonically increase. Indeed, energy loss at foot-strike is directly proportional to foot placement angle [15]. However, for push-off control for $c = 1$, the push-off is almost constant for step up and it decreases monotonically for step down. Since push-off work is directly proportional to added energy, a decrease in the push-off allows the robot to maintain its speed while walking down. Thus we would expect the push-off to increase for step up but it remains constant. This is because the step up introduces an unexpected disturbance and the model takes a short step which has the effect of supplying energy to the system (note that foot placement angle is directly proportional to the energy loss).

The plots for $c = 0.25$ and $c = 0.5$ can be understood from the plot for $c = 1$. For example, consider the foot placement control for step down (top-right figure). The swing-leg angle is almost constant for $c = 0.5$ (blue squares) and decreasing for $c = 0.25$ (red circles). These values of c require the robot to dissipate the excess energy gained through step down more gradually than $c = 1$. The result for $c = 0.5$ is that the model needs to just maintain its nominal swing leg angle and for $c = 0.25$ the model needs to decrease the swing leg angle in order to dissipate the energy more slowly to satisfy the rate of convergence given by $c = 0.25$.

4) *Energy usage:* Figure 5 shows the average Cost Of Transport defined as the energy used per unit weight per unit distance travelled. The left side plots show the COT due to hip work while the right side plots show the COT due to push-off work. We used the absolute value of mechanical work for the energies for the hip and ankle COT. Since the hip is massive and legs are light it is not meaningful to sum up the COT of hip and push-off work. The top plots are for foot placement control and the bottom plots are for push-off control.

The COT for the hip work (left side plots) increases as σ

deviates from 0. Note that the COT for hip work for $\sigma = 0$ is zero because the nominal limit cycle has a passive leg swing. The increase in COT with σ on the top plot for foot placement is obvious; the swing-leg angle needs to change for foot placement control, thus torque needs to be supplied and work needs to be done. The increase in COT with σ on the bottom plot for push-off control for constant swing-leg angle is slightly more subtle. When the model is subjected to varying terrain heights, the speed changes but so does the time to foot-strike. The hip torque then needs to do work to move the leg faster or slower as needed even though the final swing-leg angle is the same, thus requiring more work.

The COT for push-off work (right side plots) shows that in general, push-off COT decreases for step down and increases for step up with some exceptions. For $c = 0.25$, step down, foot placement control (top plot, right side, red circles), the COT for push-off increases. As noted earlier, this is to regulate the energy decay at a slower rate than the natural decay rate. A similar trend and reasoning apply for $c = 0.25$, step up, push-off control (bottom plot, right side, red circles).

VI. DISCUSSION, CONCLUSION, AND FUTURE WORK

We have presented an analysis of two disparate control strategies, foot placement and ankle push-off, to regulate robot velocity between steps. We compared one-step dead-beat stabilization with exponential stabilization. The approach was tested by doing a forward simulation with the terrain height changing monotonically, i.e., either step up or step down. Our findings are as follows:

- 1) Average number of steps to failure is the same for both control strategies and stabilization protocols. This is because of lack of actuator limits.
- 2) In general, average number of steps walked for step down disturbance is more than that for step up. This is because in our simulation, step up leads to premature stepping, thus throwing off the planned foot placement location.
- 3) Control strategy for fast convergence to the nominal for step down is to increase the foot placement angle or to decrease ankle push-off, while for step up is to decrease foot placement or maintain same ankle push-off.
- 4) For both control strategies, the mechanical energy usage (as measured by COT) for ankle push-off and foot placement is almost the same, except for $c = 0.25$ where hip COT for push-off control is half of that of foot placement control for step down. However, ankle COT for step down is less than step up while hip COT for step up is less than step down. Thus energy-wise, foot placement control is ideal for step up and ankle push-off for step down.
- 5) Overall, one-step dead-beat stabilization is more energy-efficient in terms of hip work and ankle work than exponential stabilization.

We found that ankle push-off and foot placement have similar robustness as measured by the steps to failure on changing terrain. This is in contrast to the work by Kim

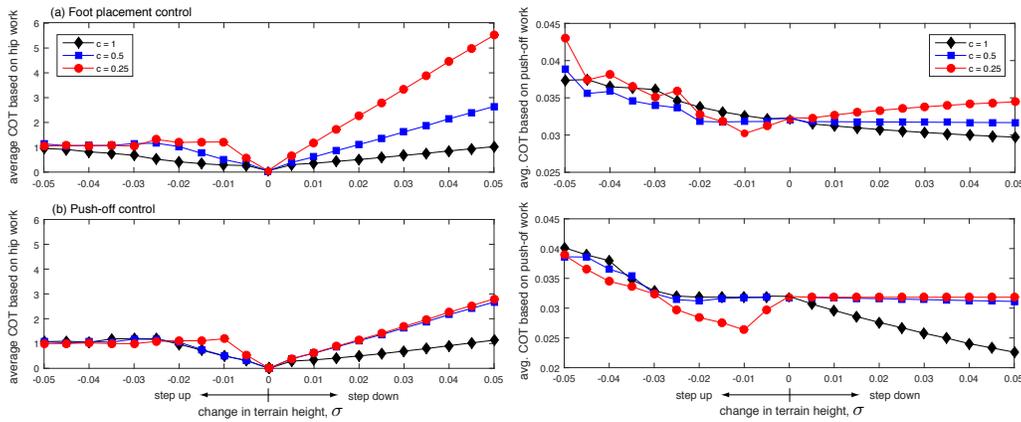


Fig. 5. Average COT for robustness test: (a) Foot placement control (b) Push-off control. The COT based on hip torque work is shown on the left side and the COT based on push-off work is shown on the right side.

and Collins [11] who found that push-off is at least twice as effective as foot placement. We suspect that this is in part due to the fact that we have no actuator limits in our simulation allowing the robot to do drastic corrections to regulate walking. Similarly we found that the energy usage is similar for both control strategies except for slow convergence to the nominal limit cycle. However, it is more economical to use ankle push-off for step down and foot placement for step up.

We also compared full stabilization of disturbances in a single step, also called one-step dead-beat control ($c = 1$), with exponential stabilization of disturbances ($0 < c < 1$), and found that although both have similar robustness as measured by average steps to failure, full correction of disturbances is more energy-efficient. The latter is due to the fact that although fast convergence to the nominal limit cycle is more expensive in the short-term (due to faster control actions), it has long-term advantages of being on the energy-efficient nominal limit cycle.

This work has several limitations which we discuss next. The simulation model has no actuator limits. Adding actuator limits will reduce the robustness of the controller. It will also favor the control strategy which has less stringent limitations. For example, if push-off has stringent limits then foot placement will be more robust. The model does not have many features of human-like walking kinematics and dynamics. For example, the legs are massless, which prevents us from comparing the work done by the hip motor and the ankle motor. A massy leg would make this comparison meaningful and would help in making design choices on how to size actuators. We have not explored the simultaneous use of push-off control and foot placement control. Based on the current work, we hypothesize that combining push-off and foot placement control will be most effective when the terrain consists of both, step up and step down. Our future work will address these limitations as well as looking at combination of the two control strategies, comparing robustness and energy usage for different walking speeds, and looking at terrain

with combination of step up and down.

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