MULTIMEDIA EXTENSION
Design, modeling and control of a differential drive rimless wheel that can move straight and turn

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1 Notation

1.1 Robot constant parameters

Symbol	Value	Parameter description
m_1	2.46 kg	Mass of wheel.
m_2	4.45 kg	Mass of torso.
J_1	0	Inertia of wheel in the planar direction (fore-aft).
J_2	0	Inertia of torso in the planar direction (fore-aft).
J_d	0.052 $kg - m^2$	Inertia of wheel in the vertical direction (top-down).
ℓ_0	0.26 m	Leg length when spring is not compressed.
c	0.06 cm	Distance of torso COM along its axis.
k	1315 N/m	Spring constant.
c_v	10^5 Ns/m	Viscous friction in the spring.
$2b$	0.31 m	Distance between the two rimless wheels.
n	10	Number of spokes.
g	9.81 m/s^2	Gravitational constant.
G	5.4	Gear ratio.
K_t	0.034 Nm/A	Motor torque constant.

1.2 Simulation variables

Symbol	Variable description
θ_l	angle between the leg and vertically downward direction for left wheel in the 3D model.
θ_r	angle between the leg and vertically downward direction for right wheel in the 3D model.
θ	angle between the leg and vertically downward direction for the 2D model ($= 0.5(\theta_l + \theta_r)$).
θ_d	difference between left and right wheel stance angle ($= 0.5(\theta_l - \theta_r)$).
ϕ	angle between the torso and vertically downward direction.

ℓ	length of the virtual leg in contact with the ground in the 2D model.
β	heading angle of the rimless wheel.
I_i	motor current in $i = l, r$ left and right motor.
T_i	motor torque in $i = l, r$ left and right motor.
T_a	average of the left and right motor torque.
I_d	differential current for turning.
T_d	differential torque for turning.
(x_c^0, y_c^0)	coordinates of the point midway between the wheels in world frame
(x_c^1, y_c^1)	coordinates of the point midway between the wheels in robot frame
v_P	velocity of point P as so on.
a_P	acceleration of point P as so on.
v_P^-, ϕ^-	velocity of point P, angle *before* support transfer with superscript $-$.
v_P^+, ϕ^+	velocity of point P, angle *after* support transfer with superscript $+$.
$\hat{e}_r$, $\hat{e}_\theta$	unit vectors in r and θ direction.
$\vec{H}^+_{/P}$ $\vec{H}^-_{/P}$	Angular momentum about point P after and before transition.
$\dot{\vec{H}}_{/P}$	Rate of change of angular momentum about point P and so on.
$\vec{M}_{/P}$	External angular moment about point P and so on.

2 Robot model

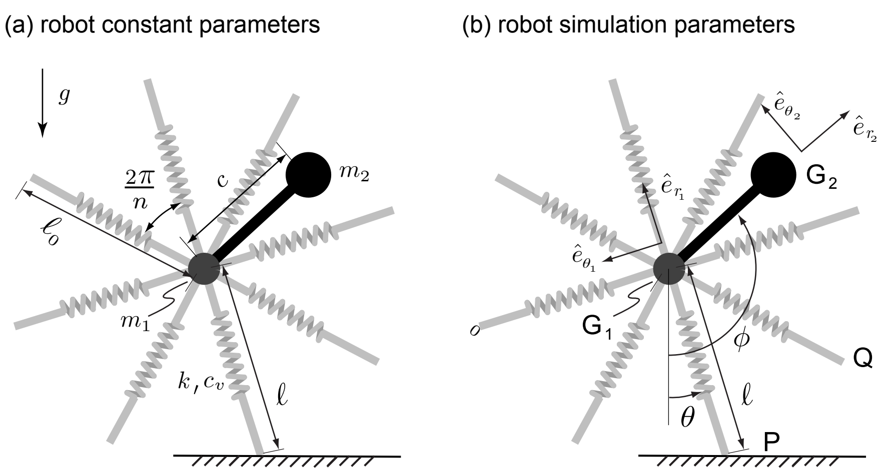

Figure 1: **The 2D sagittal plane model:** The robot is progressing from the left to the right direction. (a) robot constant parameters given in Sec. 1.1 and (b) some of the robot simulation parameters given in Sec. 1.2. Although the robot has 10 legs on each side, we have shown only 8 spokes.

2.1 2D sagittal plane model

2.1.1 Degrees of freedom

We use three independent degrees of freedom to describe the motion of the model in the sagittal plane. These are the length of the spoke that is touching the ground, ℓ, the angle made by the spoke touching the ground with the vertical, θ, and the angle made by the torso with the vertical, ϕ.

2.1.2 Kinematics in stance phase

The acceleration of the center of mass of the rimless wheel $\vec{a}_{G_1}$ and torso $\vec{a}_{G_2}$ are obtained from Fig. 1 (a)

$$\vec{a}_{G_1} = (\ddot{\ell} - \ell\dot{\theta}^2)\hat{e}_{r_1} + (2\dot{\ell}\dot{\theta} + \ell\ddot{\theta})\hat{e}_{\theta_1} \tag{1}$$

$$\vec{a}_{G_2} = \vec{a}_{G_1} - c\dot{\phi}^2\hat{e}_{r_2} + c\ddot{\phi}\hat{e}_{\theta_2} \tag{2}$$

where $\hat{e}_{r_1}$ and $\hat{e}_{\theta_1}$ are coordinate axis attached to the rimless wheel and $\hat{e}_{r_2}$ and $\hat{e}_{\theta_2}$ are coordinate axis attached to the torso as shown.

2.1.3 Equations of motion in the stance phase

The equation of the stance phase are derived using the Fig. 1 (b). We need three equations to describe the acceleration of the three degrees of freedom $(\ddot{\ell}, \ddot{\theta}, \ddot{\phi})$. Two equations are obtained from the principle of angular momentum and one equation is obtained from principle of linear momentum balance as follows.

$$\dot{\vec{H}}_{/P} = \vec{M}_{/P}$$

$$\vec{r}_{G_1/P} \times m_1 \vec{a}_{G_1} + \vec{r}_{G_2/P} \times m_2 \vec{a}_{G_2} + J_1 \ddot{\vec{\theta}} + J_2 \ddot{\vec{\phi}} = \vec{r}_{G_1/P} \times m_1 \vec{g} + \vec{r}_{G_2/P} \times m_2 \vec{g} \tag{3}$$

$$\dot{\vec{H}}_{/G_1} = \vec{M}_{/G_1}$$

$$\vec{r}_{G_2/G_1} \times m_2 \vec{a}_{G_2} + J_2 \ddot{\vec{\phi}} = \vec{T}_a + \vec{r}_{G_2/G_1} \times m_2 \vec{g} \tag{4}$$

$$(m_1 \vec{a}_{G_1} + m_2 \vec{a}_{G_2}) \cdot \hat{e}_{r_1} = (\vec{F} - m_1 \vec{g} - m_2 \vec{g}) \cdot \hat{e}_{r_1}$$

$$m_1 \vec{a}_{G_1} \cdot \hat{e}_{r_1} + m_2 \vec{a}_{G_2} \cdot \hat{e}_{r_1} = F_r - m_1 \vec{g} \cdot \hat{e}_{r_1} - m_2 \vec{g} \cdot \hat{e}_{r_1} \tag{5}$$

where $F_r = -k(\ell - \ell_0) - c_v\dot{\ell}$, k is the spring constant, c_v is the damping in the leg, ℓ_0 is the spring free length, and average torque $T_a = 0.5(T_l + T_r)$.

From Eqns. 3, 4, and 5 we can write

$$\mathbf{A}_{ss}\ddot{\mathbf{X}}_{ss} = \mathbf{b}_{ss}, \tag{6}$$

where $\mathbf{X}_{ss} = \{\theta,\ \phi,\ \ell\}$, $\mathbf{A}_{ss}$ is a 3x3 matrix, and $\mathbf{b}_{ss}$ is a 3x1 vector. If $A_{i,j}$ is the element on the ith row and jth of $\mathbf{A}_{ss}$, and if b_i is the ith row of $\mathbf{b}_{ss}$ then

$$A_{1,1} = c\,\ell \mathrm{m}_2 \cos(\theta - \phi) - \ell^2, \mathrm{m}_1 - \ell^2, \mathrm{m}_2 - \mathrm{I}_1$$

$$A_{1,2} = -\mathrm{m}_2\, c^2 + \ell \mathrm{m}_2\, \cos(\theta - \phi)\; c - \mathrm{I}_2$$

$$A_{1,3} = c\, \mathrm{m}_2\, \sin(\theta - \phi)$$

$$A_{2,1} = c\, \ell \mathrm{m}_2\, \cos(\theta - \phi)$$

$$A_{2,2} = -\mathrm{m}_2\, c^2 - \mathrm{I}_2$$

$$A_{2,3} = c\, \mathrm{m}_2\, \sin(\theta - \phi)$$

$$A_{3,1} = 0$$

$$A_{3,2} = c\, \mathrm{m}_2\, \sin(\theta - \phi)$$

$$A_{3,3} = -\mathrm{m}_1 - \mathrm{m}_2$$

$$\begin{aligned} b_1 = {} & 2\, \ell \dot{\ell} \mathrm{m}_1\, \dot{\theta} + 2\, \ell \dot{\ell} \mathrm{m}_2\, \dot{\theta} + c\, g\, \mathrm{m}_2\, \sin(\phi) - g\, \ell \mathrm{m}_1\, \sin(\theta) - g\, \ell \mathrm{m}_2\, \sin(\theta) \\ & - 2\, c\, \dot{\ell} \mathrm{m}_2\, \dot{\theta}\, \cos(\theta - \phi) + c\, \ell \mathrm{m}_2\, \dot{\theta}^2\, \sin(\theta - \phi) - c\, \ell \mathrm{m}_2\, \dot{\phi}^2\, \sin(\theta - \phi) \end{aligned}$$

$$b_2 = c\, \ell \mathrm{m}_2\, \sin(\theta - \phi)\; \dot{\theta}^2 - 2\, c\, \dot{\ell} \mathrm{m}_2\, \cos(\theta - \phi)\; \dot{\theta} - \mathrm{T}_2 + c\, g\, \mathrm{m}_2\, \sin(\phi)$$

$$b_3 = g\, \mathrm{m}_1\, \cos(\theta) - F_r + g\, \mathrm{m}_2\, \cos(\theta) - \ell \mathrm{m}_1\, \dot{\theta}^2 - \ell \mathrm{m}_2\, \dot{\theta}^2 + c\, \mathrm{m}_2\, \dot{\phi}^2\, \cos(\theta - \phi)$$

2.1.4 Kinematics in support transfer

We denote the linear and angular position and velocities at the instance before support transfer using the super-script $(-)$ and instance after support transfer using the super-script $(+)$ as shown in Fig. 2.

$$\vec{v}^{\,+}_{G_1} = \dot{\ell}^{+} \hat{e}^{+}_{r_1} + \ell_0 \dot{\theta}^{+} \hat{e}^{+}_{\theta_1}, \tag{7}$$
$$\vec{v}^{\,+}_{G_2} = \vec{v}^{\,+}_{G_1} + c\dot{\phi}^{+} \hat{e}^{+}_{\theta_2}, \tag{8}$$
$$\vec{v}^{\,-}_{G_1} = \dot{\ell}^{-} \hat{e}^{-}_{r_1} + \ell^{-} \dot{\theta}^{-} \hat{e}^{-}_{\theta_1}, \tag{9}$$
$$\vec{v}^{\,-}_{G_2} = \vec{v}^{\,-}_{G_1} + c\dot{\phi}^{-} \hat{e}^{-}_{\theta_2}. \tag{10}$$

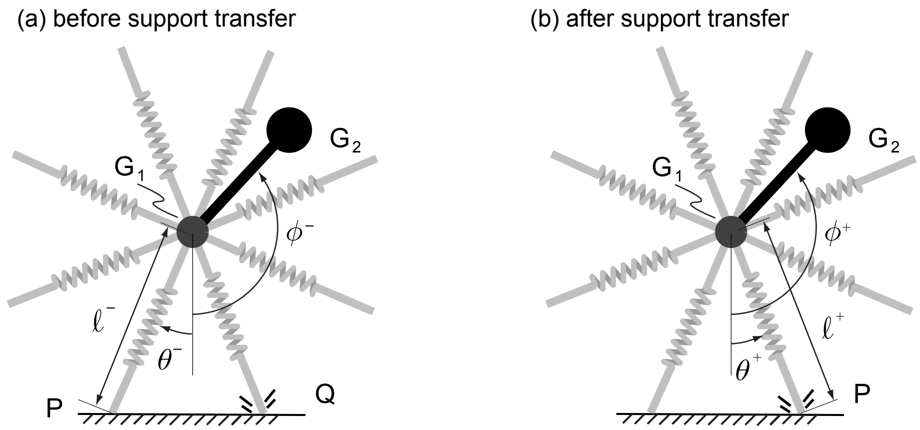

Figure 2: **Support transfer:** The robot is progressing from the left to the right direction. (a) Instance just before support transfer uses superscript (-) and (b) instance just after support transfer uses superscript (+).

2.1.5 Equations of motion for support transfer

The equation relating the three degrees of freedom after support transfer to that before support transfer are obtained by comparing the configuration of the robot before and after support transfer as shown in Fig. 2. These are

$$\ell^+ = \ell_0 \tag{11}$$

$$\theta^+ = \theta^- + \frac{2\pi}{n} \tag{12}$$

$$\phi^+ = \phi^- \tag{13}$$

where the first equation is based on the fact the new spoke that will contact the ground is not stretched.

The equations relating the rate of change of the three degrees of freedom after support transfer are derived using the notation in Fig. 2. The first two equations are derived by applying conservation of angular momentum and the last one is derived by applying conservation of linear momentum along the axial direction.

$$\overrightarrow{H}_P^{+} = \overrightarrow{H}_Q^{-}$$

$$\overrightarrow{r}_{G_1/P} \times m_1 \overrightarrow{v}_{G_1}^{+} + \overrightarrow{r}_{G_2/P} \times m_2 \overrightarrow{v}_{G_2}^{+} + J_1 \dot{\overrightarrow{\theta}}^{+} + J_2 \dot{\overrightarrow{\phi}}^{+} = \overrightarrow{r}_{G_1/Q} \times m_1 \overrightarrow{v}_{G_1}^{-} + \overrightarrow{r}_{G_2/Q} \times m_2 \overrightarrow{v}_{G_2}^{-} + J_1 \dot{\overrightarrow{\theta}}^{-} + J_2 \dot{\overrightarrow{\phi}}^{-} \tag{14}$$

$$\overrightarrow{H}_{G_1}^{+} = \overrightarrow{H}_{G_1}^{-}$$

$$\overrightarrow{r}_{G_2/G_1} \times m_2 \overrightarrow{v}_{G_2}^{+} + J_2 \dot{\overrightarrow{\phi}}^{+} = \overrightarrow{r}_{G_2/G_1} \times m_2 \overrightarrow{v}_{G_2}^{-} + J_2 \dot{\overrightarrow{\phi}}^{-} \tag{15}$$

$$\overrightarrow{L}^{+} \cdot \hat{e}_{r_1}^{+} = \overrightarrow{L}^{-} \cdot \hat{e}_{r_1}^{-}$$

$$m_1 \overrightarrow{v}_{G_1}^{+} \cdot \hat{e}_{r_1}^{+} + m_2 \overrightarrow{v}_{G_2}^{+} \cdot \hat{e}_{r_1}^{+} = m_1 \overrightarrow{v}_{G_1}^{-} \cdot \hat{e}_{r_1}^{-} + m_2 \overrightarrow{v}_{G_2}^{-} \cdot \hat{e}_{r_1}^{-} \tag{16}$$

In the Eqn. 16 we assume that there is no impulsive force along the stance leg direction. This is because the spring force acts along the radial direction and can be considered to provide negligible impulse force in the short support transfer duration.

We can combine Eqn. 14, 15, and 16 to get

$$\mathbf{A}_{hs}\dot{\mathbf{X}}_{hs}^{+} = \mathbf{b}_{hs}, \tag{17}$$

where $\mathbf{X}_{hs} = \{\theta,\ \phi,\ \ell\}$, $\mathbf{A}_{hs}$ is a 3x3 matrix, and $\mathbf{b}_{hs}$ is a 3x1 vector. If $A_{i,j}$ is the element on the ith row and jth of $\mathbf{A}_{hs}$, and if b_i is the ith row of $\mathbf{b}_{hs}$ then

$$A_{1,1} = -c\,\ell^{+}\mathrm{m}_2\,\cos\left(\theta^{+} - \phi^{+}\right)$$

$$A_{1,2} = \mathrm{m}_2\,c^2 + \mathrm{I}_2$$

$$A_{1,3} = -c\,\mathrm{m}_2\,\sin\left(\theta^{+} - \phi^{+}\right)$$

$$A_{2,1} = \mathrm{I}_1 + (\ell^{+})^2\mathrm{m}_1 + (\ell^{+})^2\mathrm{m}_2 - c\,\ell^{+}\mathrm{m}_2\,\cos\left(\theta^{+} - \phi^{+}\right)$$

$$A_{2,2} = \mathrm{m}_2\,c^2 - \ell^{+}\mathrm{m}_2\,\cos\left(\theta^{+} - \phi^{+}\right)\,c + \mathrm{I}_2$$

$$A_{2,3} = -c\,\mathrm{m}_2\,\sin\left(\theta^{+} - \phi^{+}\right)$$

$$A_{3,1} = 0$$

$$A_{3,2} = -c\,\mathrm{m}_2\,\sin\left(\theta^{+} - \phi^{+}\right)$$

$$A_{3,3} = \mathrm{m}_1 + \mathrm{m}_2$$

$$b_1 = \mathrm{I}_2\,\dot{\phi}^- - \mathrm{m}_2\,\Big(c\,\cos(\phi^-)\,\Big(\dot{\ell}^-\,\sin(\theta^-) - c\,\dot{\phi}^-\,\cos(\phi^-) + \ell^-\dot{\theta}^-\,\cos(\theta^-)\Big) - c\,\sin(\phi^-)\,\Big(\dot{\ell}^-\,\cos(\theta^-) + c\,\dot{\phi}^-\,\sin(\phi^-) - \ell^-\dot{\theta}^-\,\sin(\theta^-)\Big)\Big)$$

$$\begin{aligned} b_2 = {} & \mathrm{I}_1\,\dot{\theta}^- + \mathrm{I}_2\,\dot{\phi}^- \\ & - \mathrm{m}_2\,\Bigg(\Bigg(\ell_0\,\sin\Bigg(\theta^- + \frac{2\,\pi}{n}\Bigg) - c\,\sin(\phi^-)\Bigg)\,\Big(\dot{\ell}^-\,\cos(\theta^-) + c\,\dot{\phi}^-\,\sin(\phi^-) - \ell^-\dot{\theta}^-\,\sin(\theta^-)\Big) \\ & \qquad - \Bigg(\ell_0\,\cos\Bigg(\theta^- + \frac{2\,\pi}{n}\Bigg) - c\,\cos(\phi^-)\Bigg)\,\Big(\dot{\ell}^-\,\sin(\theta^-) - c\,\dot{\phi}^-\,\cos(\phi^-) + \ell^-\dot{\theta}^-\,\cos(\theta^-)\Big)\Bigg) \\ & + \mathrm{m}_1\,\Bigg(\ell_0\,\cos\Bigg(\theta^- + \frac{2\,\pi}{n}\Bigg)\,\Big(\dot{\ell}^-\,\sin(\theta^-) + \ell^-\dot{\theta}^-\,\cos(\theta^-)\Big) \\ & \qquad - \ell_0\,\sin\Bigg(\theta^- + \frac{2\,\pi}{n}\Bigg)\,\Big(\dot{\ell}^-\,\cos(\theta^-) - \ell^-\dot{\theta}^-\,\sin(\theta^-)\Big)\Bigg) \end{aligned}$$

$$b_3 = \dot{\ell}^-\mathrm{m}_1 + \dot{\ell}^-\mathrm{m}_2 - c\,\mathrm{m}_2\,\dot{\phi}^-\,\sin(\theta^- - \phi^-)$$

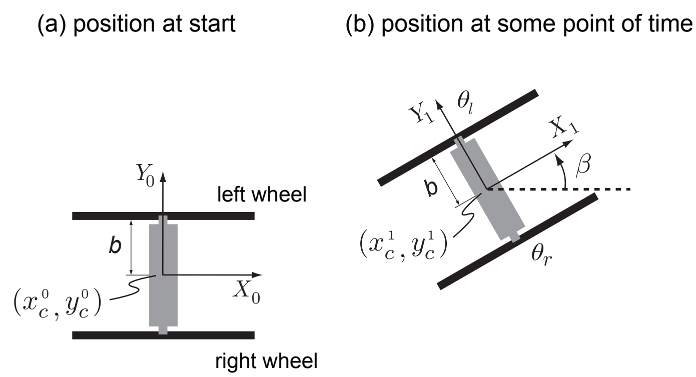

Figure 3: **The steering model:** The robot is progressing from the left to the right direction. (a) The world or fixed frame is $X_0 - Y_0$ (b) The local frame attached to the robot $X_1 - Y_1$. The angle between the spoke in contact with the ground and vertical is θ_i and the corresponding angular rate is $\dot{\theta}_i$, where $i = r$ for the right wheel and $i = l$ for the left wheel. We keep a track of the center point between two wheels in local frame, $\{x_c^1, y_c^1\}$, the world frame, $\{x_c^0, y_c^0\}$, and angle made by the perpendicular to the torso and world frame, β, also known as the heading angle.

2.2 3D model: combining steering with sagittal model

Earlier we have derived the equations of motion in the 2D sagittal plane, namely the pitching motion of the robot. Here we use differential drive kinematics formulation borrowed from differential drive wheel robots [2] to describe motion in the heading or steering direction. We combine the sagittal motion with the steering model to build the complete 3D model. Note that we ignore rolling in our analysis.

2.2.1 Differential drive kinematics

We use the Fig. 3 to derive an expression for the kinematics of the robot for the heading motion. We use two frames, $X_0 - Y_0$ is the fixed or world frame and $X_1 - Y_1$ is the local frame attached to the torso and moves as the torso moves. In this exposition, we are interested in keeping track of the mid-point on the torso (C), (x_c, y_c), and the heading angle β.

The velocity vector for C in frame $X_1 - Y_1$ is $\dot{c}^1 = \{\dot{x_c}^1, \dot{y_c}^1\}$ then [2]

$$\dot{x_c}^1 = 0.5\ell(\dot{\theta}_r + \dot{\theta}_l)$$
$$\dot{y_c}^1 = 0$$

If the velocity vector for C in frame $X_0 - Y_0$ is $\dot{c}^0$ then the relation between the two velocity vectors is

$$\dot{c}^0 = \mathbf{R}_1^0 \dot{c}^1 \quad \text{where } \mathbf{R}_1^0 = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \tag{18}$$

Using $\dot{c}^0 = \{\dot{x}_c^0, \dot{y}_c^0\}$, we simplify the expressions

$$\dot{x}_c^0 = 0.5\ell(\dot{\theta}_r + \dot{\theta}_l)\cos(\beta) \tag{19}$$
$$\dot{y}_c^0 = 0.5\ell(\dot{\theta}_r + \dot{\theta}_l)\sin(\beta) \tag{20}$$

The angular velocity for heading $\dot{\beta}$ may be obtained arguing about the change in heading as the speed of the two sides of the rimless wheel changes.

$$\dot{\beta} = 0.5\frac{\ell}{b}(\dot{\theta}_r - \dot{\theta}_l) \tag{21}$$

For given $\dot{\theta}_r$, $\dot{\theta}_l$, the Eqns. 19, 20, and 21 have to be integrated to find the movement of the center point C in the world frame, x_c^0, y_c^0, β.

2.2.2 Equations for stance phase in 3D

There are a total of 7 equations that describe the motion of the robot in 3D. These are given by $\dot{x}_c^0, \dot{y}_c^0, \dot{\beta}, \ddot{\phi}, \ddot{\ell}, \ddot{\theta}_l, \ddot{\theta}_r$. All these variables except the last two been defined, so we define them first and summarize all the variables toward the end of the section.

First, we define the following θ and θ_d and their rates

$$\theta = 0.5(\theta_l + \theta_r),$$
$$\dot{\theta} = 0.5(\dot{\theta}_l + \dot{\theta}_r),$$
$$\theta_d = 0.5(\theta_l - \theta_r),$$
$$\dot{\theta}_d = 0.5(\dot{\theta}_l - \dot{\theta}_r).$$

With this notation, we can now find $\ddot{\theta}_l$ and $\ddot{\theta}_r$

$$\begin{aligned}\ddot{\theta}_l &= \ddot{\theta} + \ddot{\theta}_d,\\ \ddot{\theta}_r &= \ddot{\theta} - \ddot{\theta}_d.\end{aligned} \tag{22}$$

Thus, we can replace the pair $(\ddot{\theta}_l, \ddot{\theta}_r)$ with the pair $(\ddot{\theta}, \ddot{\theta}_d)$. Thus our equation set is $\dot{x}_c^0, \dot{y}_c^0, \dot{\beta}, \ddot{\phi}, \ddot{\ell}, \ddot{\theta}, \ddot{\theta}_d$. The first three are given by Eqn. 19, 20, and 21. The next three are given by Eqns. 3, 4, and 5. The last one is given by

$$\ddot{\theta}_d = \frac{T_d}{J_d} \tag{23}$$

where T_d is the net torque in the lateral plane and J_d is the inertia along the vertical axis. Next, we derive expressions for T_d and J_d. If traction forces between the contact spoke in the sagittal plane for the two wheels is $F_l = T_l/\ell_0$ and $F_r = T_r/\ell_0$ (where we assume that the spring compression is negligible $\ell \sim \ell_0$). Thus,

$$\begin{aligned}T_d &= (F_l - F_r)b\\ &= (T_l - T_r)\frac{b}{\ell_0}\end{aligned} \tag{24}$$

To derive an expression for J_d (inertia), we assume that the robot is not translating $\dot{x}_c^0 = \dot{y}_c^0 = 0$ and the two wheels are at rest $\dot{\theta}_l = \dot{\theta}_r = 0$. The net kinetic energy of the system is

$$\begin{aligned}\text{KE} &= 0.5J_b\dot{\beta}^2 + 2 \times (0.5J_w\dot{\beta}^2 + 0.5m_1b^2\dot{\beta}^2)\\ &= (0.5J_b + J_w + m_1b^2)\dot{\beta}^2\\ &= 0.5J_d\dot{\beta}^2.\end{aligned} \tag{25}$$

where J_b is the inertia of the body about the vertical axis and J_w is the inertia of the wheel about the vertical axis. Thus, we have

$$J_d = J_b + 2(J_w + m_1b^2) \tag{26}$$

Note that Eqn. 25 may also be derived from parallel axis theorem.

2.2.3 Collision condition

The collision condition for the left side wheel and right side wheel are given by

$$\begin{aligned}h_l &= \ell^- \cos(\theta_l^-) - \ell_0 \cos\left(\theta_l^- + \frac{2\pi}{n}\right) = 0,\\ h_r &= \ell^- \cos(\theta_r^-) - \ell_0 \cos\left(\theta_r^- + \frac{2\pi}{n}\right) = 0.\end{aligned} \tag{27}$$

Both these conditions are checked during integration. We can have three conditions: (1) spokes on both sides collide simultaneously, thus $h_l = 0$ and $h_r = 0$ are both true simultaneously, (2) only the left side spoke strikes the ground, thus only $h_l = 0$ is true, and (3) only the right side spoke strikes the ground, thus only $h_r = 0$ is true.

2.2.4 Equations for support transfer in 3D

We make the assumption that the net difference of velocity before collision is that same as that after collision. Thus

$$\Delta\dot{\theta}^{+} = \dot{\theta}_l^{+} - \dot{\theta}_r^{+} = \dot{\theta}_l^{-} - \dot{\theta}_r^{-} \tag{28}$$

For the support transfer we need to find the following positions: $\ell^{+}, \phi^{+}, \theta_l^{+}, \theta_r^{+}$ and their corresponding rates: $\dot{\ell}^{+}, \dot{\phi}^{+}, \dot{\theta}_l^{+}, \dot{\theta}_r^{+}$. Note that the steering coordinates after support transfer $\dot{x}_c^{+0}, \dot{y}_c^{+0}, \dot{\beta}^{+}$ may be found using the positions and rates after support transfer using Eqns. 19, 20, and 21. The support transfer may involve any of the three conditions

(1) Collision of left and right side spoke simultaneously

In order to find all angles, we define the angle θ^{-} as follows

$$\theta^{-} = 0.5(\theta_l^{-} + \theta_r^{-}) \tag{29}$$

Next, we can find ℓ^{+}, θ^{+}, ϕ^{+} using Eqns. 11, 12, and 13 respectively. The angles θ_l^{+} and θ_r^{+} are given by

$$\theta_l^{+} = -\theta_l^{-} \tag{30}$$
$$\theta_r^{+} = -\theta_r^{-} \tag{31}$$

In order to find all rates, we define the rate $\dot{\theta}^{-}$ as follows

$$\dot{\theta}^{-} = 0.5(\dot{\theta}_l^{-} + \dot{\theta}_r^{+}) \tag{32}$$

Next, we find $\dot{\ell}^{+}$, $\dot{\theta}^{+}$, and $\dot{\phi}^{+}$ using Eqn. 17. Finally, we can find the rate $\dot{\theta}_l^{+}$ and $\dot{\theta}_r^{+}$ using

$$\dot{\theta}_l^{+} = \dot{\theta}^{+} + \Delta\dot{\theta}^{+}$$
$$\dot{\theta}_r^{+} = \dot{\theta}^{+} - \Delta\dot{\theta}^{+}$$

(2) Collision of left side spoke only

All conditions are similar to the simultaneous collision except for Eqn. 29, 30, 31, and 32 are replaced in the same order to be

$$\theta^{-} = \theta_l^{-}$$
$$\theta_l^{+} = -\theta_l^{-}$$
$$\theta_r^{+} = \theta_r^{-}$$
$$\dot{\theta}^{-} = \dot{\theta}_l^{-}$$

(3) Collision of right side spoke only

All conditions are similar to the simultaneous collision except for Eqn. 29, 30, 31, and 32 are replaced in the same order to be

$$\theta^- = \theta_r^-$$
$$\theta_l^+ = \theta_l^-$$
$$\theta_r^+ = -\theta_r^-$$
$$\dot{\theta}^- = \dot{\theta}_r^-$$

2.3 Motor torque and power model

The motor torque model relates the current to the torque and is given by

$$\begin{aligned} T_i &= GK_tI_i \\ T_d &= GK_tI_d \end{aligned} \tag{33}$$

The motor power model relates the torque and speed to the power and is given by

$$P_i = T_i\dot{\theta}_i \tag{34}$$

The net power is $P = P_l + P_r$.

3 Software

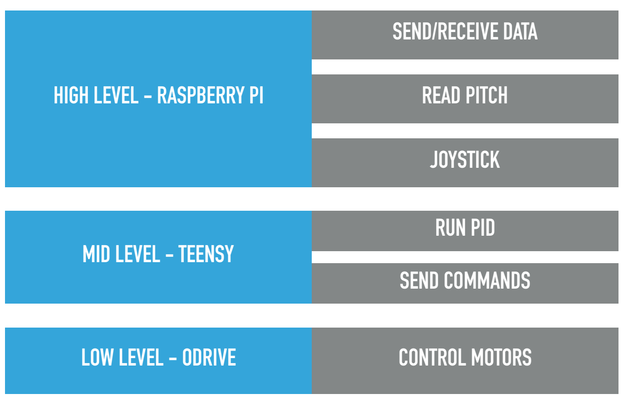

Figure 4: Software hierarchy

The software is categorized into three levels: high level, mid level, and low level (Figure 4). Each corresponds to a different computing device in the robot. At the high level block, the Raspberry Pi is the system's scheduler and data logger. The Pi communicates bidirectionally with both the Teensy microcontroller and the ODrive at 100 Hz (Figure 5). The communication with the micro-controller is serial over USB. For the ODrive, the communication is through a library that directly communicates through USB. The Teensy calculates the PID loop at 1 kHz and communicates with the ODrive uni-directionally through physical serial. Lastly, the ODrive controls the motors at a frequency of around 4 kHz.

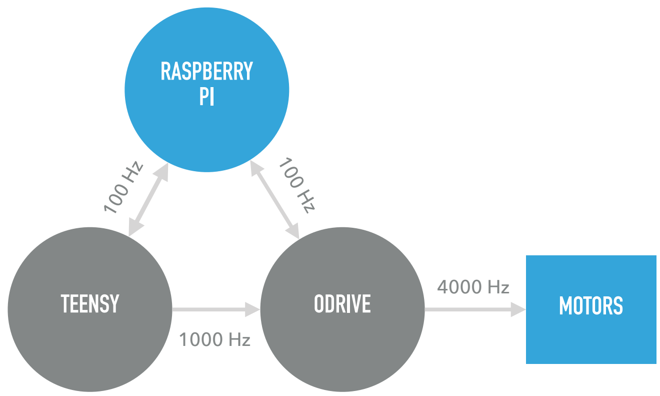

Figure 5: Communication diagram

3.1 Raspberry Pi

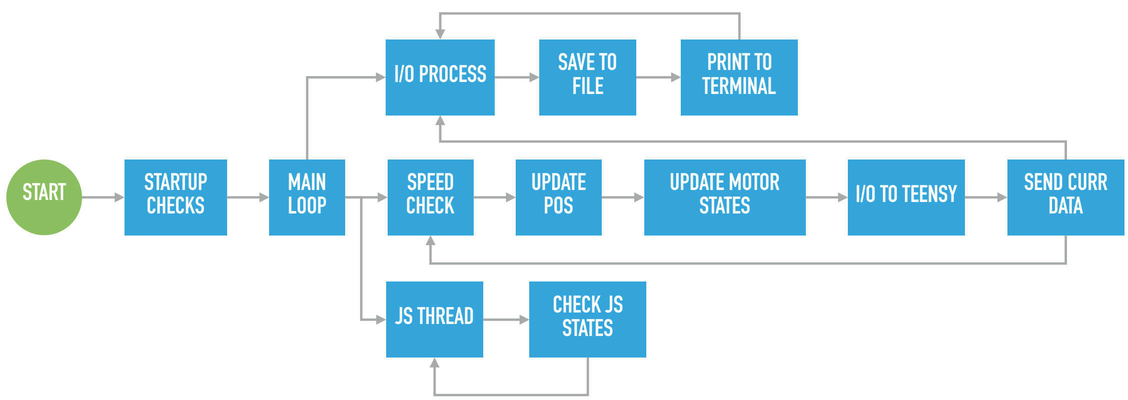

Figure 6: Raspberry Pi software diagram

The Raspberry Pi software is written as a single script using Python 3, and uses both multi-threading and multi-processes to run functions simultaneously (Figure 6). Once the program is started, the Pi performs startup checks by connecting to the orientation sensor, the ODrive, and the Teensy. The main program consists of three loops: a main loop, an I/O process, and a joystick update thread. These are all run simultaneously to avoid the latency from writing acquired data to the disk. The I/O process collects data from the main loop, saves it to the disk, and prints desired information to the console. The data logged data includes: torso pitch, motor position, motor speed, PID output, pitch setpoint, battery voltage, and battery current. These are all saved for later analysis of walking trials for the robot. Data is collected at 100 Hz from the main loop, by passing all of the values in a Python dictionary. This is necessary when using multi-processes, as the processes do not share memory space. A multi-process approach was taken with the I/O process because writing to disk is the slowest portion of the Pi code. The separate process can be run on a different core, alleviating bottlenecking from too many slow instructions in a single process.

The main loop starts a thread that monitors the Dualshock 3 joystick states. This is done using the built-in Linux `linux_js` application programming interface and a script that maps all of the buttons and axes into usable dictionaries. Since the joystick monitoring is event based, e.g. the loop waits until a button or axis value changes, the joystick function was placed in a thread, preventing it from stopping the main loop. The benefit of a thread, however, is that it shared memory space with the main loop, so no deliberate communication must happen between the thread and the main loop. All joystick related commands are placed within this thread. Pitch setpoint changes, motor calibrations, and motor on/off are all done in this thread.

The first action from the main loop is to check the motor speed as a safety precaution. The script queries the motors for their velocity, and if either one is greater than our speed limit, the Pi instructs the motor controller to turn off the motors immediately. Next, the torso pitch read from the position sensor at 100 Hz. The ODrive is then queried for the information from the power system: motor velocities and positions, bus (battery) voltage, and bus current. The current body pitch is then sent to the Teensy at 100 Hz, and, if available, the PID out is received from the Teensy. Lastly, the data is placed into a dictionary and that is sent to the I/O process to be saved and viewed.

3.2 Teensy

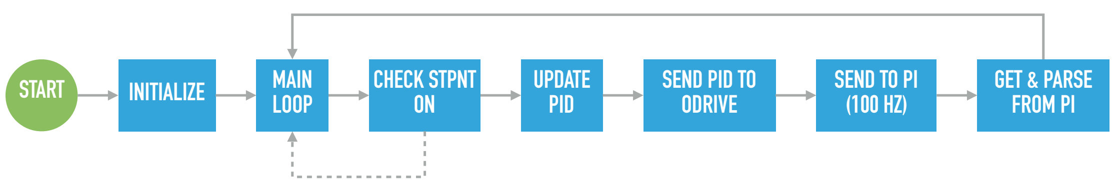

Figure 7: Teensy software diagram

The PID library for Arduino [1] was modified by enabling sub millisecond PID operation frequency, adding an I-term limit, and adding a band limited derivative term. The microncontroller is reset each time the main program on the Raspberry Pi is run. The reset pin is pulled to low using the Pi's I/O pins. Once the Teensy initializes, the main loop starts by checking the value `setpoint_on` (Figure 7). This value is sent from the Pi when the start button is pressed, and if True, the PID controller starts. The PID is then updated with a built in function, completing the calculation. If the PID update function is called faster than the time step for 1 kHz (1 millisecond), the calculation is not performed. If the calculation has been performed, the PID output, in amps, is sent to the ODrive over serial with the correct formatting required by the ODrive. The PID output is then sent to the Pi over serial at 100 Hz. Lastly, the Teensy receives and parses the setpoint, the boolean `setpoint_on`, and the torso pitch. The information from the Pi must be parsed, because the message, encoded in ASCII, begins with a `<` and ends with a `>`. This prevents partial or corrupted messages from irregularities in communication.

References

[1] Brett Beuregard. Arduino pid library.

[2] Roland Siegwart, Illah Reza Nourbakhsh, and Davide Scaramuzza. Introduction to autonomous mobile robots (intelligent robotics and autonomous agents series), 2011.