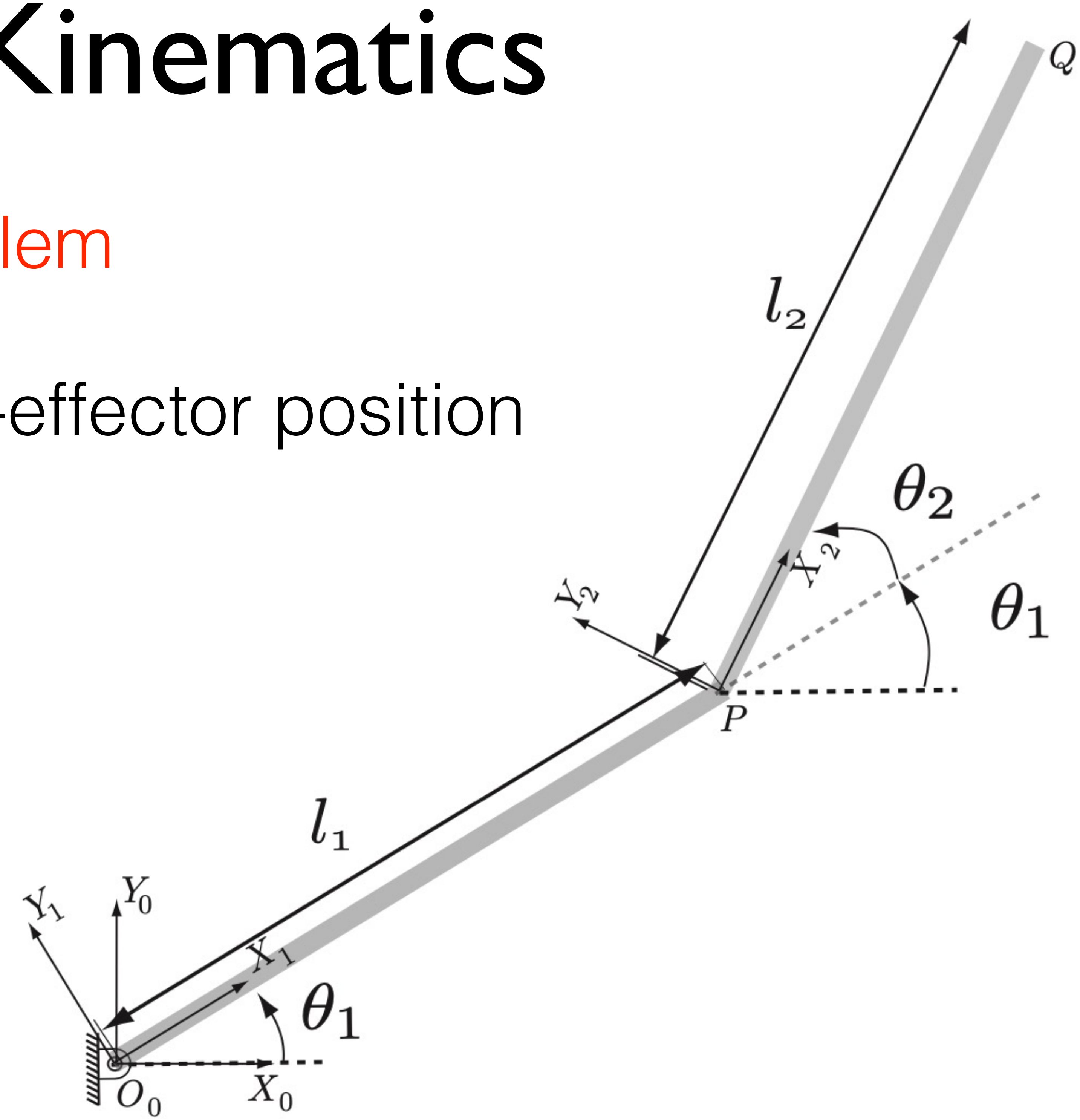
Inverse Kinematics

Inverse kinematics problem

For a given desired end-effector position

$$X_{\mathrm{ref}} = \{x_{ref}, y_{ref}\}$$

Compute: $q = \{\theta_1, \theta_2\}$



We know

$$r_Q = X_{\mathrm{ref}} = f(q)$$

Compute

$$q = f^{-1}(X_{\mathrm{ref}})$$ f is nonlinear

Inverse Kinematics using Jacobian

$$\mathbf{f} = \begin{bmatrix} f_1(\mathbf{q}), & f_2(\mathbf{q}), & f_3(\mathbf{q}), & \ldots & f_m(\mathbf{q}) \end{bmatrix}$$

size = m

$$\mathbf{q} = \begin{bmatrix} x_1, & x_2, & \ldots & x_n \end{bmatrix}$$

size = n

$$\mathbf{J} = \frac{\partial \mathbf{f}}{\partial \mathbf{q}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \ldots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \ldots & \frac{\partial f_2}{\partial x_n} \\ \ldots & \ldots & \ldots. & \ldots & \ldots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \frac{\partial f_m}{\partial x_3} & \ldots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

size = mxn

Jacobian

Inverse Kinematics (Theory)

Position of Q $r_Q = f(q)$

Velocity of Q

$$V_Q = \frac{\partial f}{\partial q}\dot{q} = J\dot{q}$$

$$\frac{dr_Q}{dt} = J\frac{dq}{dt}$$

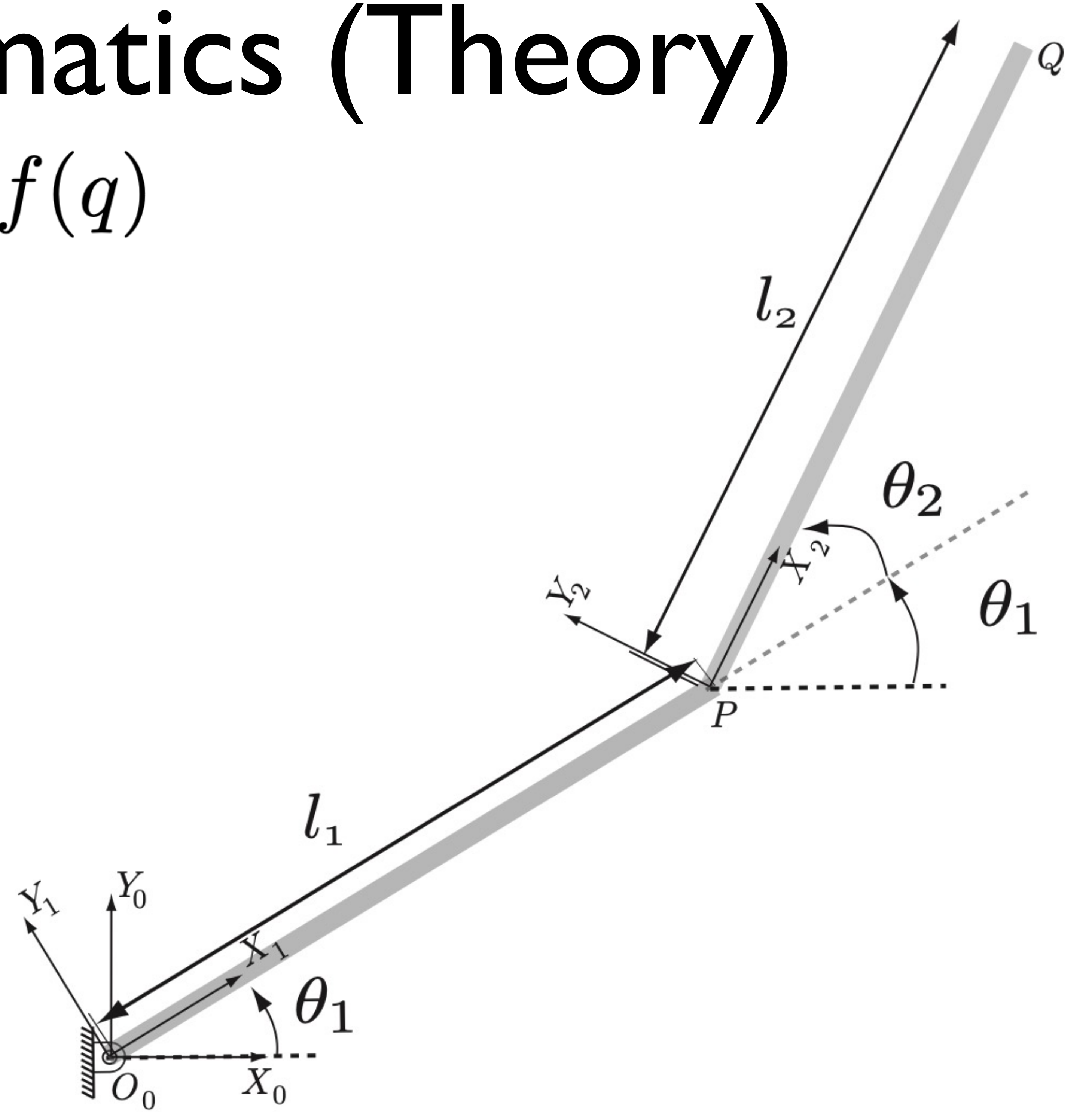
$$\Delta r_Q = J\Delta dq$$

Key equation

$$\Delta dq = J^{-1}\Delta r_Q$$

$$\Delta dq = J^{-1}(X_{\text{ref}} - X)$$

Unknown Known Measured



Inverse kinematics example