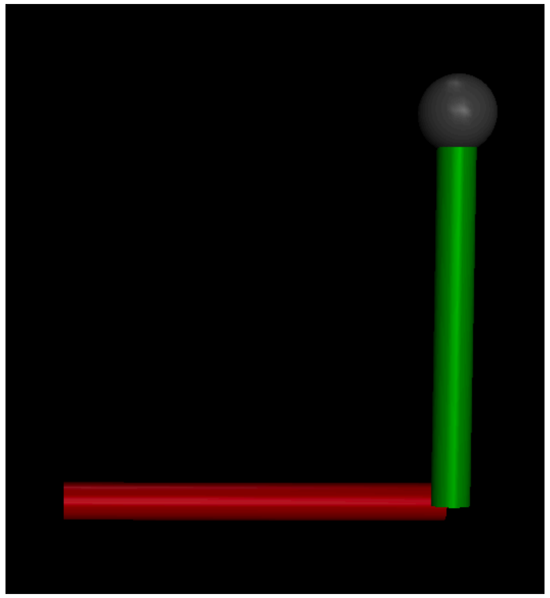
Trajectory tracking control

Trajectory generation and tracking

Start



End

Red: q0 = -pi/2

Green: q1 = 0

Red: q0 = pi/2

Green: q1 = pi/2

Trajectory generation

Generate a trajectory q(t) and track the trajectory

Cubic Trajectory

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

a0, a1, a2, a3 are constants

Boundary conditions

$$q(t=0) = q_0, \quad q(t=t_f) = q_f, \quad \dot{q}(t=0) = 0, \quad \dot{q}(t=t_f) = 0$$

Solving for a's

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{(t_f - t_0)^3} \begin{bmatrix} q_f t_0^2 (3t_f - t_0) + q_0 t_f^2 (t_f - 3t_0) \\ 6 t_0 t_f (q_0 - q_f) \\ 3(t_0 + t_f)(q_f - q_0) \\ 2(q_0 - q_f) \end{bmatrix}$$

Trajectory generation

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Cubic Trajectory

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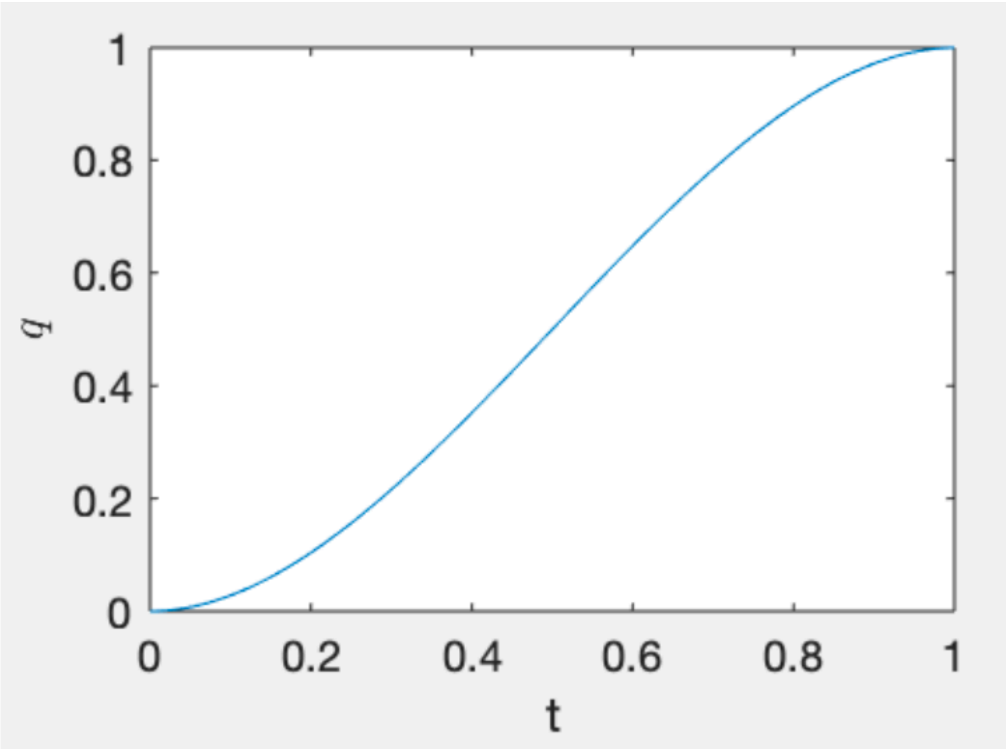

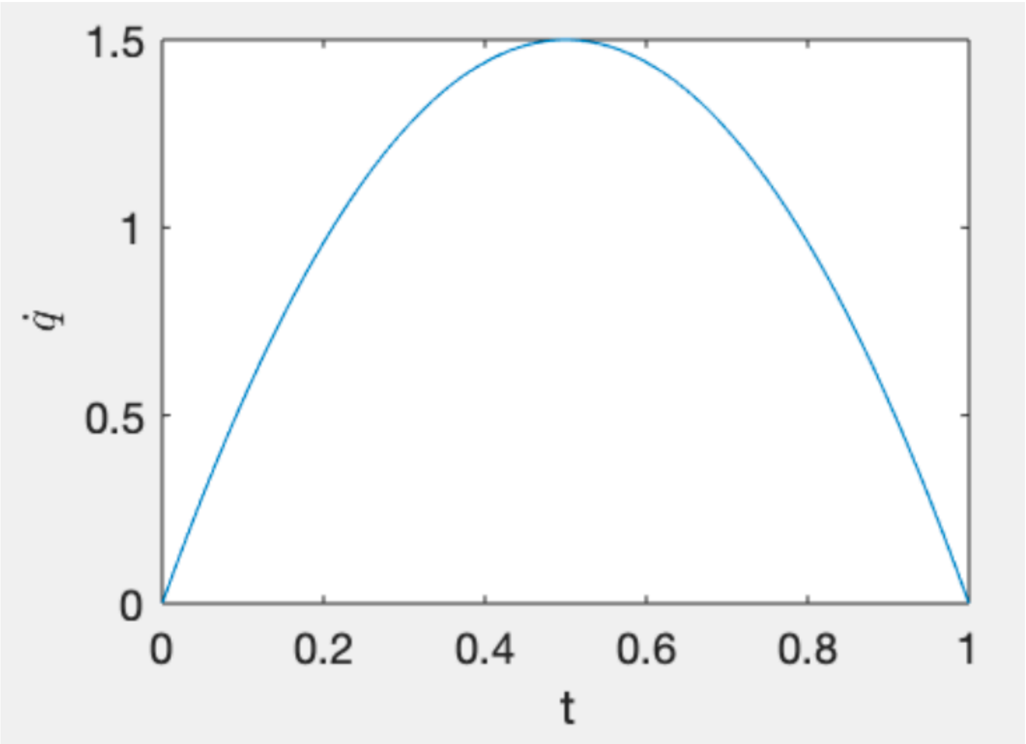

Equations of manipulator

Equations of motion: M qddot + C + G = tau

Equations of motion (MuJoCo notation):

M qacc + qfrc_bias = qfrc_applied + ctrl

- M is the mass matrix
- qddot = qacc = acceleration of degrees of freedom
- C + G (gravity + Coriolis) = qfrc_bias
- tau (torque) = qfrc_applied OR ctrl
- qfrc_applied is always available (generalized force)
- ctrl is available only if an actuator is defined

Tracking Control

Equations: M qddot + f = tau where f = C + G

3) Controllers

i) Proportional-Derivative control

tau = -Kp*(q-q_ref) - Kd*(qdot-qdot_ref)

ii) (gravity + coriolis forces) + PD control

tau = f -Kp*(q-q_ref) - Kd*(qdot-qdot_ref)

iii) Feedback linearization

tau = M(-Kp*(q-q_ref) - Kd*(qdot-qdot_ref)) + f