Robotics with MuJoCo, HW 4
Homework due on 02-27-2022, Topics: Inverse kinematics and hybrid systems
Email solutions to pranav@uic.edu.

1. **Drawing an astroid with a three link manipulator:**
   Consider a three-link manipulator with link lengths of $\ell_1 = 1$ m, $\ell_2 = \ell_3 = 0.5$ m and joint angles θ_1, θ_2, and θ_3 as shown in Fig. 1 (left side). Your goal is to get the tip of the three-link manipulator to draw an astroid, an example is shown in Fig. 1 (right side). **While drawing the astroid, the link QR should be vertical or along the y-axis.**

   The equation of an astroid in parametric form is given by

$$
\begin{aligned}
x &= x_0 + a\cos^3\theta \\
y &= y_0 + a\sin^3\theta
\end{aligned} \tag{1}
$$

   where x_0 and y_0 is the center of the astroid and a determines the size of the astroid. Feel free to choose appropriate values for these parameters.

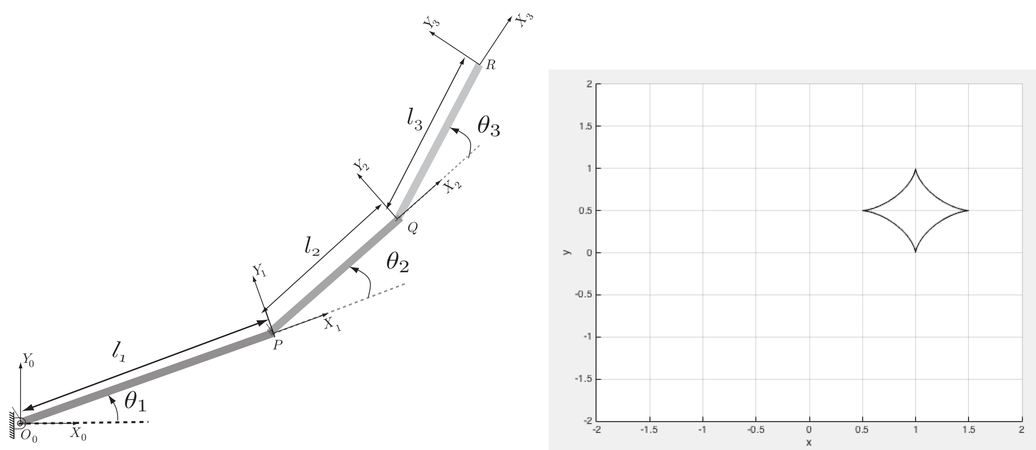

Figure 1: Manipulator and an astroid

   HINT: Here you want to regulate three things: the x-position and y-position of the end-effector, and the z-orientation of the end-effector with respect to the world frame. You would need the Jacobian related to the x, and y position (1st and 2nd row in jacp in *mj_jac*) and z-orientation (third row in jacr in *mj_jac*). The Jacobian you populate will be of dimension 3 × 3. If the end-effector moves in the x-z plane these rows will change to first and third row of of jacp and second row of jacr. Click this link for more info. about *mj_jac*: https://mujoco.readthedocs.io/en/latest/APIreference.html#mj-jac

   Continued on the next page

2. **Balance control of a rotated cart pendulum** Your goal is to balance a pendulum on a cart placed which is placed on an incline as shown in Fig. 2. The only control input to the system is a force applied to the cart. The pendulum should be balanced such that $\theta_1 = 0$, where θ_1 is the angle between the pendulum and the vertical. You do not need to model a cart with wheels. You can model the cart with a mass and sliding joint. You can assume suitable model parameters, but take the slope of the inclined plane as $\gamma = 30°$. Develop an LQR controller for the cart-pendulum and demonstrate that the system is able to balance when pushed by gaussian (normal distribution) forces.

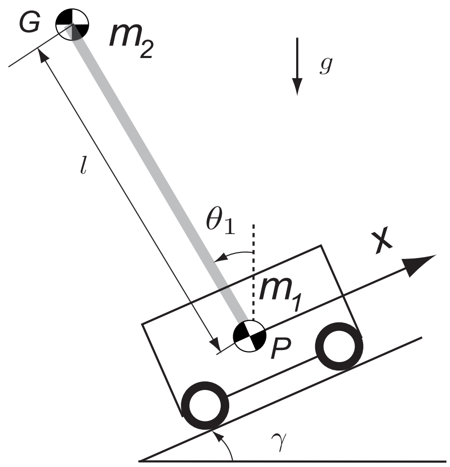

Figure 2: A cart-pendulum

Hint: Develop code to do balance control for $\gamma = 0$. Then turn gravity vector by $\gamma = 30°$ to the vertical. You will have to recompute linearization and evaluate the gains. The alternate to turning gravity is to turn the sliding joint axis to be at $\gamma = 30°$ to the horizontal.