Robotics with MuJoCo, HW 2
Topics: Modeling and controlling a coupled pendulum
Email solutions to pranav@uic.edu.

1. **Coupled pendulum:** Figure **??** shows two single link pendulums with link length ℓ, mass m, with negligible inertia. The rotation of the each link is described by angles θ_1 and θ_2 as shown. The pendulums are coupled through a spring at point C_1 and C_2 at a distance c from the hinge point for both links. The spring constant is k. the spring is unstretched when $\theta_1 = \theta_2$. Assume suitable values for the model parameters.
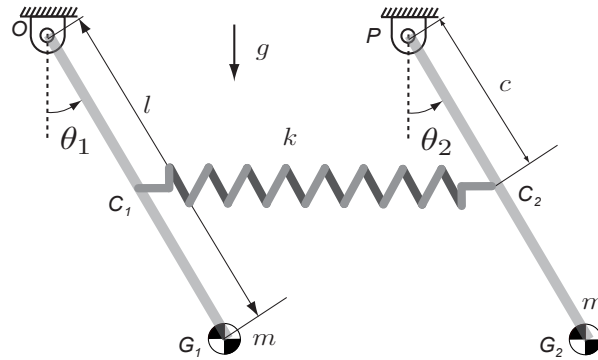

Figure 1: Coupled pendulum

(a) Model the two pendulums in xml. Then write code in C to simulate the spring. HINT: You should be able to use *xfrc_applied* to simulate the spring force.

(b) Set the following initial condition $z_0 = \begin{bmatrix} \pi/10, & 0, & \pi/10, & 0 \end{bmatrix}$. Observe the animation and see if it looks similar to the uncoupled oscillation in this video: https://youtu.be/CguKKl9mX2s?t=137 (starts at 2 min 17 sec).

(c) Now change the initial condition to $z_0 = \begin{bmatrix} \pi/10, & 0, & -\pi/10, & 0 \end{bmatrix}$. Observe the animation and see if it looks similar to the uncoupled oscillation in this video: https://youtu.be/CguKKl9mX2s?t=156 (starts at 2 min 36 sec).

(d) Now change the initial condition to $z_0 = \begin{bmatrix} \pi/10, & 0, & 0, & 0 \end{bmatrix}$. Observe the animation and see if it looks similar to the mixed modes observed in this video: https://youtu.be/CguKKl9mX2s?t=171 (starts at 2 min 51 sec). Generate a plot for θ_1, θ_2, $\theta_1 + \theta_2$, and $\theta_1 - \theta_2$. You should see that the sum and difference to be periodic while the individual values are not. The two angle interfere with each other to create a phenomenon known as beats https://en.wikipedia.org/wiki/Beat_(acoustics). The sum and difference have a dominant frequency that one can find using Fast Fourier Transform: https://en.wikipedia.org/wiki/Fast_Fourier_transform.

Submit a folder containing all files needed to run the simulation. Set the C file to initial conditions for the third (last) case.