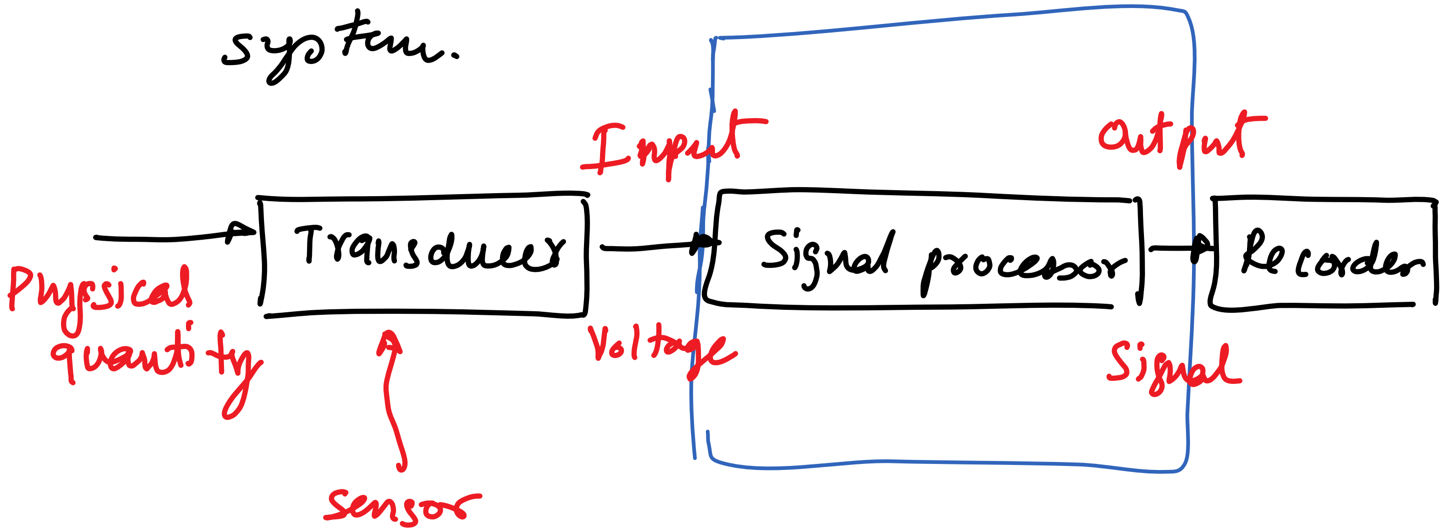
System response

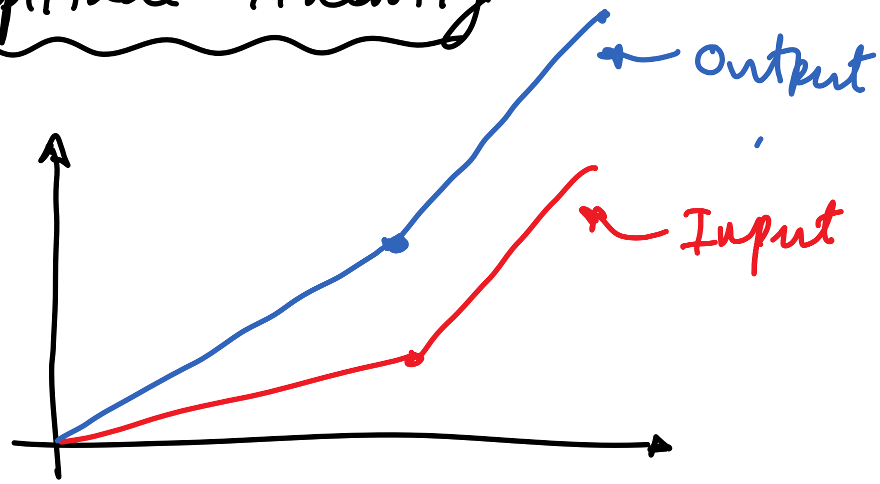
- characteristics of a good measurement system.



Characteristics of a good measurement system

1. Amplitude linearity
2. Adequate Band width
3. Phase Linearity.

① Amplitude linearity



$$\frac{\text{Output}}{\text{Input}} = \text{constant}$$

However, in real systems

① ratio is constant only for a range of input amplitude

② ratio is constant only for a certain range of input frequency.

bandwidth

Fourier series representation of signals

any periodic waveform can be represented as an infinite series of sines and cosines

Mathematically, $f(t)$ is periodic

$$f(t) = C_0 + \sum_{n=1}^{\infty} \left(A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \right)$$

infinite series

C_0, A_n, B_n are constants

ω_0 — fundamental frequency or first harmonic or the lowest frequency in the periodic waveform

$$\omega_0 = \frac{2\pi}{T} \qquad T = \text{time period}$$

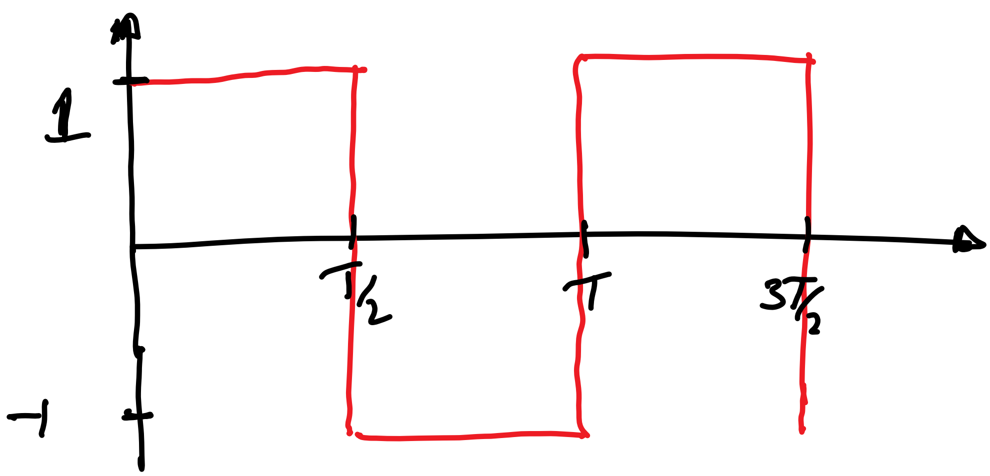
$$C_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{A_0}{2}$$

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

EXAMPLE

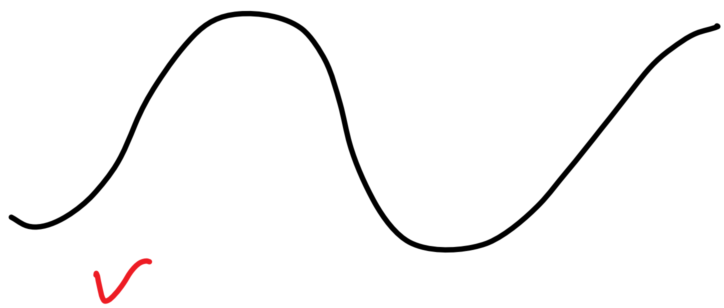
$$f(t) = \begin{cases} 1 & 0 \le t \le T/2 \\ -1 & T/2 \le t \le T \end{cases}$$



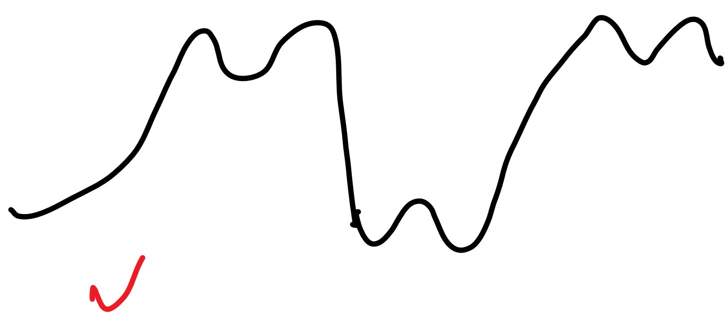
$$A_n = 0 = C_0 \qquad B_n = \frac{2}{n\pi}\left(1 - \cos(n\pi)\right)$$

$$f(t) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin\left[(2n-1)\omega_0 t\right]$$

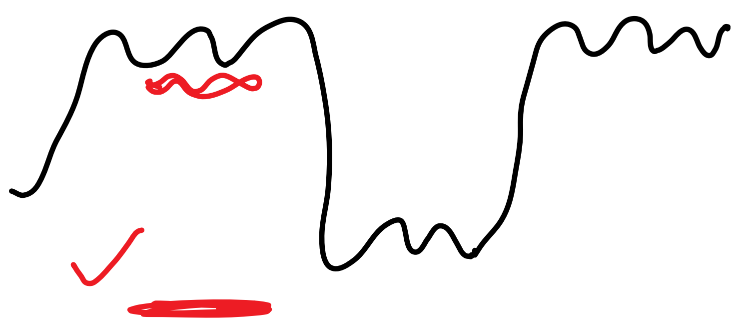
Only first term $n=1$



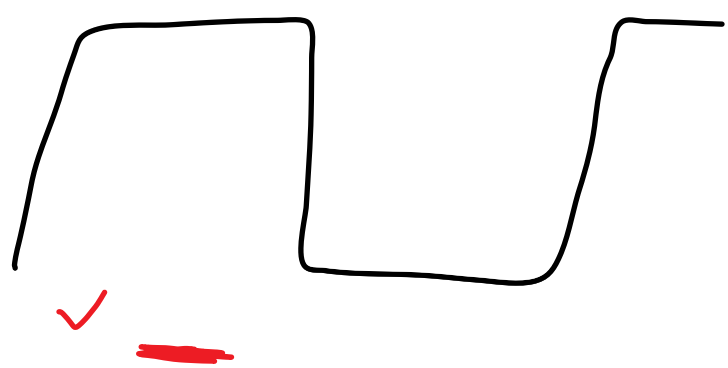
Sum of first 3 terms
$n=1, 2, 3$



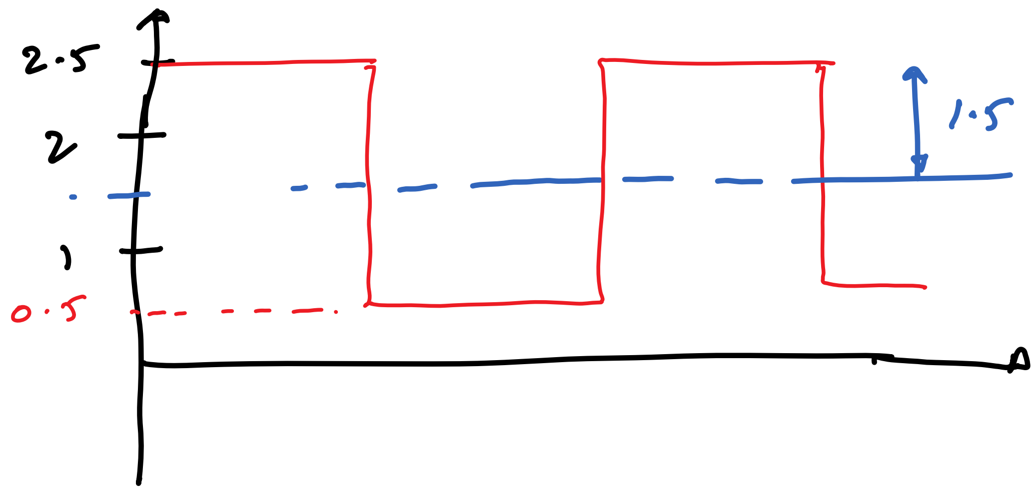
Sum of first 5 terms
$n=1, 2, 3, 4, 5$



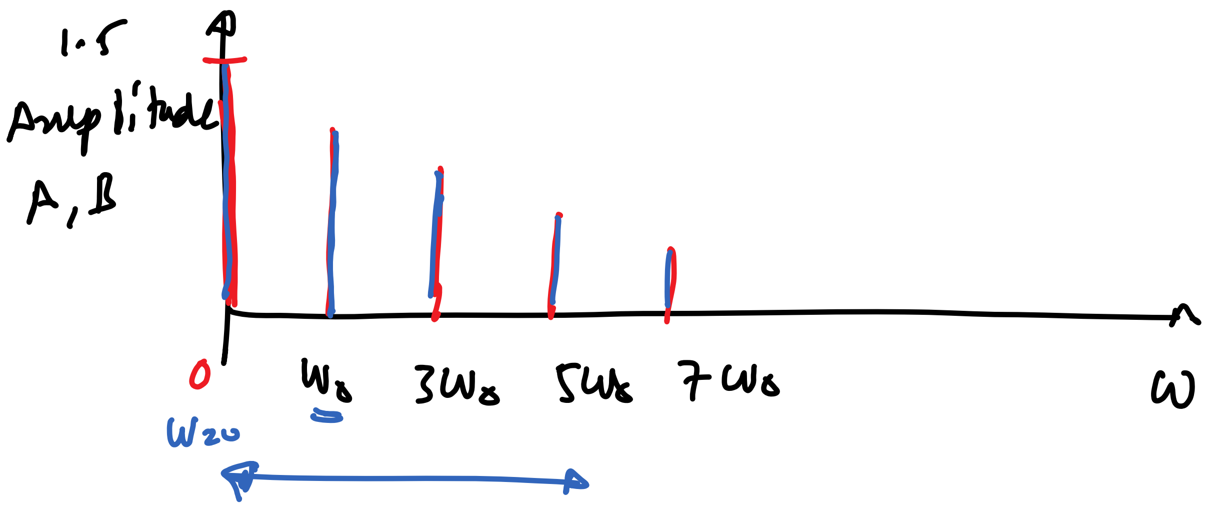
Sum of all terms



Consider the same signal with an offset 1.5



Fourier series

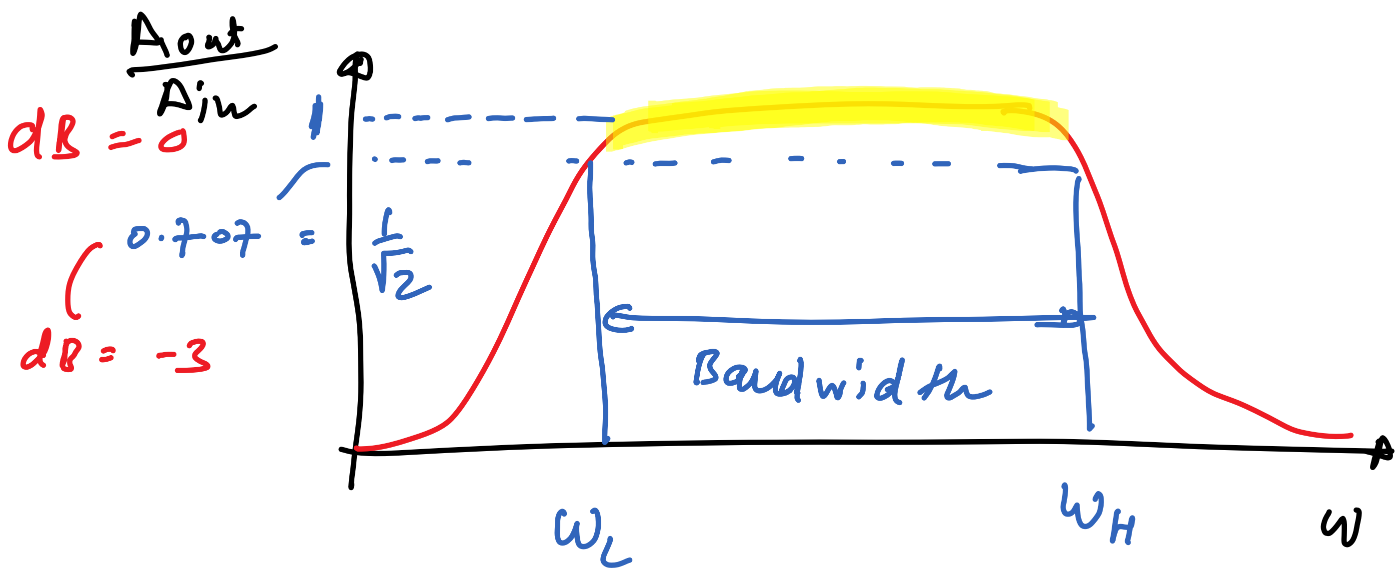


Bandwidth and frequency response

$$dB = 20 \log_{10} \left(\frac{A_{out}}{A_{in}} \right)$$

decibels

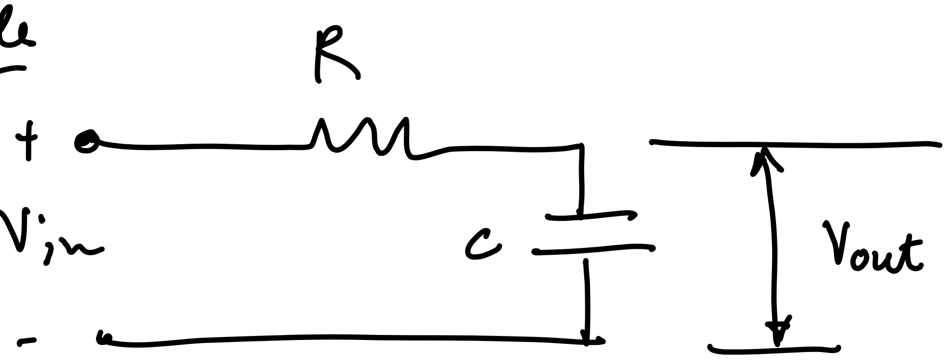
ratio of amplitudes



Bode plot

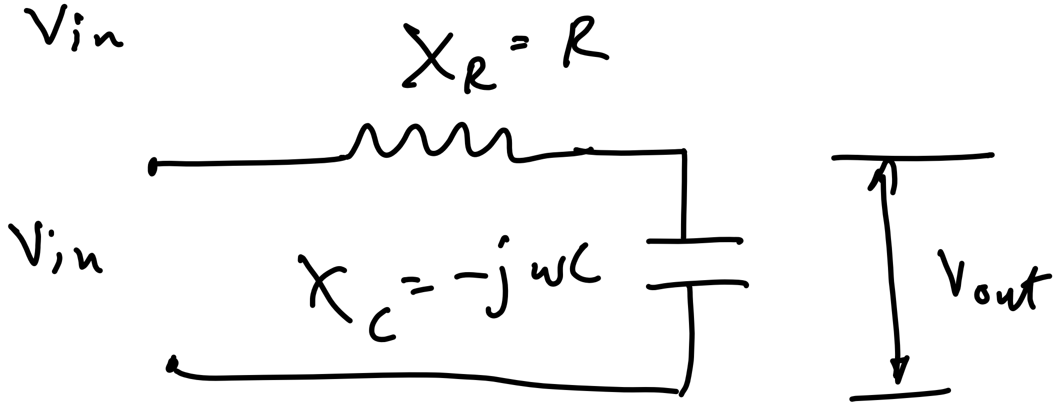
Bandwidth range of frequencies where the input is not attenuated by more than −3 dB

Example



Compute the bandwidth of this circuit.

$$\frac{V_{out}}{V_{in}} = ?$$



$$V_{out} = \frac{X_C}{X_C + X_R} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{-jwC}{-jwC + R} * \frac{(jwC + R)}{(+jwC + R)}$$

$$\frac{V_{out}}{V_{in}} = \frac{-j\omega C}{-j\omega C + R} * \frac{(j\omega C + R)}{(+j\omega C + R)}$$

$$\frac{V_{out}}{V_{in}} = \frac{+\omega^2 C^2 - j\omega CR}{\omega^2 C^2 + R^2}$$

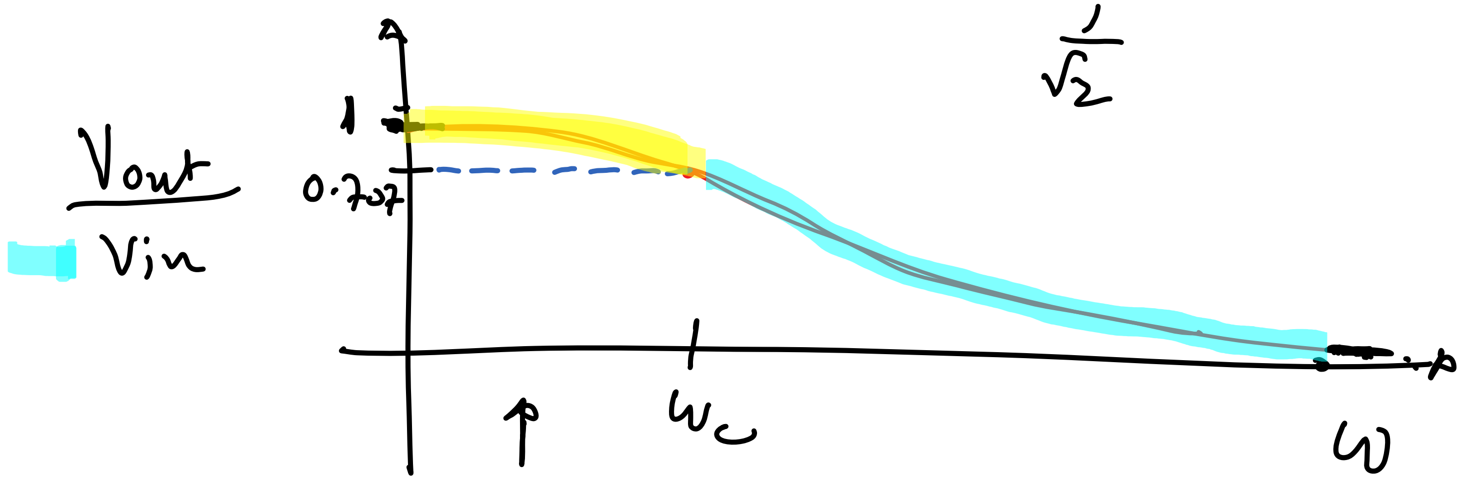
$$= \frac{1 - j\frac{R}{\omega C}}{1 + \frac{R^2}{\omega^2 C^2}} = \frac{1 - j\frac{\omega_c}{\omega}}{1 + \left(\frac{\omega_c}{\omega}\right)^2}$$

$$\omega_c = 1/RC$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

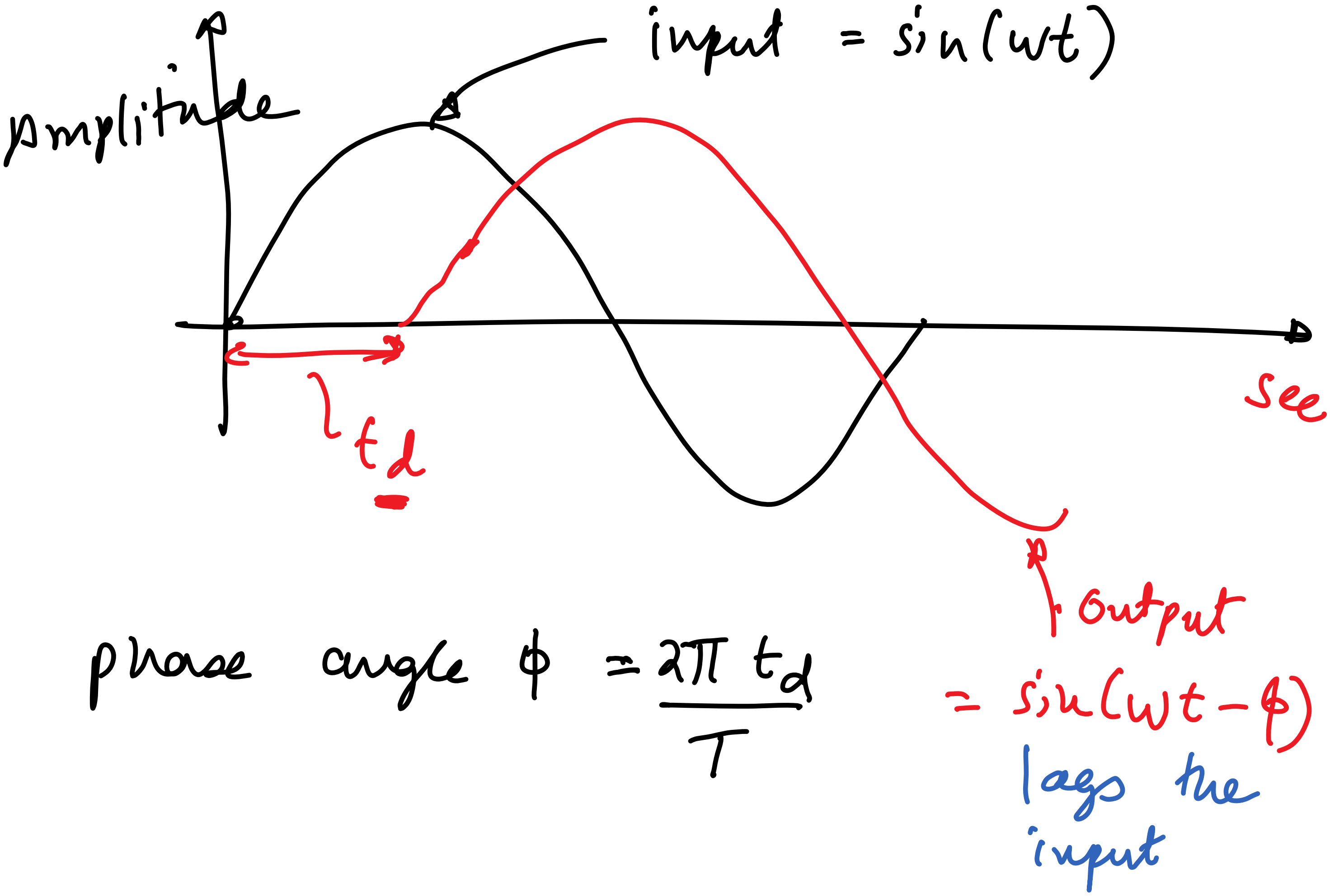
$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$ $\leftarrow \frac{1}{\infty}$

$\frac{1}{\sqrt{2}}$



$\Rightarrow$ Bandwidth 0 to $\omega_c = \frac{1}{RC}$

③ Phase linearity



phase angle $\phi = \dfrac{2\pi t_d}{T}$

$$F = \frac{1}{T}$$

$$\phi = 2\pi F t_d$$

$\phi \propto F$ (Phase linearity)