Alternating voltage and current

So far $V =$ constant (e.g. battery)

This class $V(t) = V_m \sin(\omega t + \phi)$

time

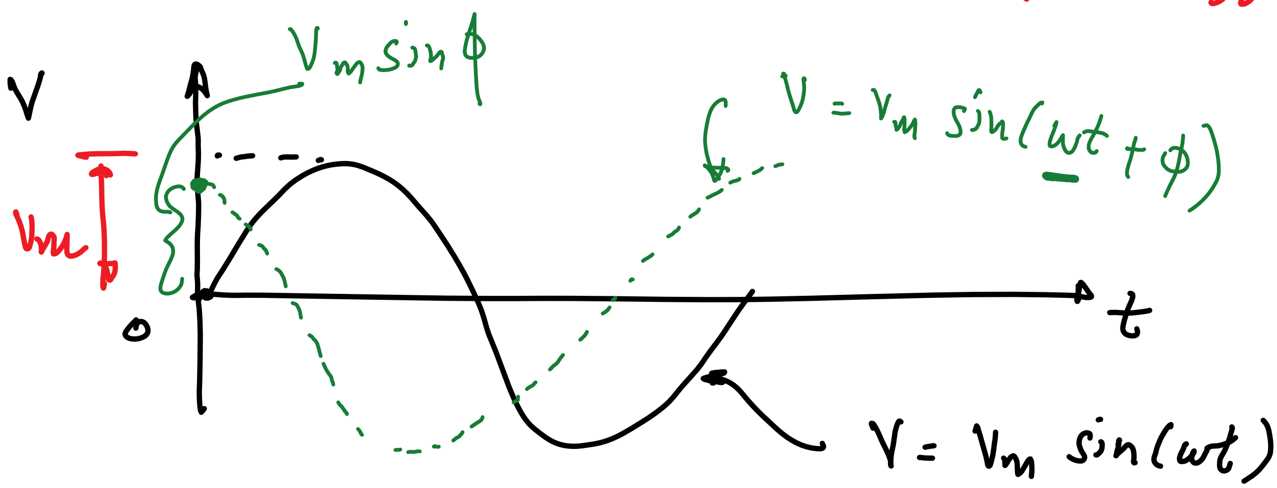
amplitude

frequency

phase

$\phi > 0$ leading

$\phi <$ lagging



$\omega \rightarrow \frac{1}{s} = s^{-1}$

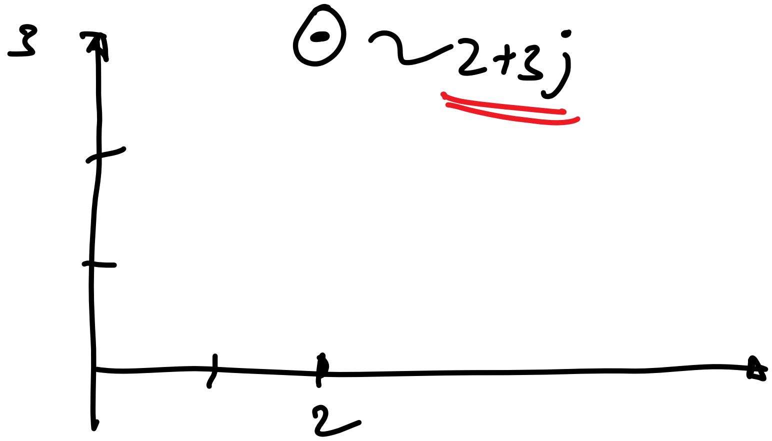
$f = \frac{\omega}{2\pi}$

Complex numbers e.g. $2 + 3j$

2 ← real (x-axis)

$3j$ ← complex (y-axis)

$j = \sqrt{-1}$



$$e^{j(\omega t+\phi)} = \cos(\omega t+\phi) + j\sin(\omega t+\phi)$$

$$V = V_m e^{j(\omega t+\phi)} = \underbrace{V_m\left[\cos(\omega t+\phi) + j\sin(\omega t+\phi)\right]}_{\text{rectangular form}}$$

$$= \underbrace{V_m \angle \phi}_{\text{polar form}}$$

$$V = V_x + jV_y$$

$$V_m = \sqrt{V_x^2 + V_y^2} \qquad \phi = \tan^{-1}\left(V_y / V_x\right)$$

Idea: $\dfrac{V}{I} = Z$ (impedance, can be complex number)

$\sim \dfrac{V}{I} = R$

Resistor

$$V = RI$$

$$V = Z_R I \qquad \boxed{(Z_R = R)}$$

Inductor

$$V = L \frac{dI}{dt} \rightsquigarrow V = Z_L I$$

Let $I = I_m e^{j(\omega t + \phi)}$

$$\frac{dI}{dt} = I_m e^{j(\omega t+\phi)} \frac{d}{dt}\left[j(\omega t+\phi)\right]$$

$$= \underline{I_m e^{j(\omega t+\phi)} (j\omega)}$$

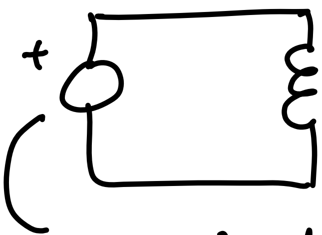
$$V = L \, \underbrace{I_m \, e^{j(\omega t + \phi)}}_{I} \, (j\omega)$$

$$V = (j\omega L) \, I$$

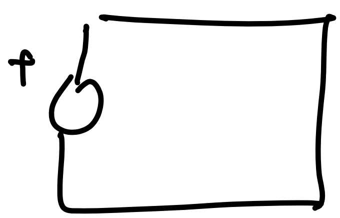
$$= z_L \, I$$

$$\boxed{z_L = j\omega L}$$

If $\omega = 0$ DC source ; $z_L = 0$
short circuit
(0 resistance)



$V =$ constant $(\omega = 0)$



short circuit
0 resistance

Capacitor

$$q = CV \rightsquigarrow V = Z_C I$$

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

$$I = C\frac{dV}{dt}$$

$$V = V_m e^{j(\omega t+\phi)}$$

$$\frac{dV}{dt} = V_m e^{j(\omega t+\phi)}(j\omega)$$

$$I = \underline{C}\ \underbrace{V_m e^{j(\omega t+\phi)}}_{V}(\underline{j}\omega)$$

$$I = j\omega C V$$

$$\frac{I}{j\omega C} = V$$

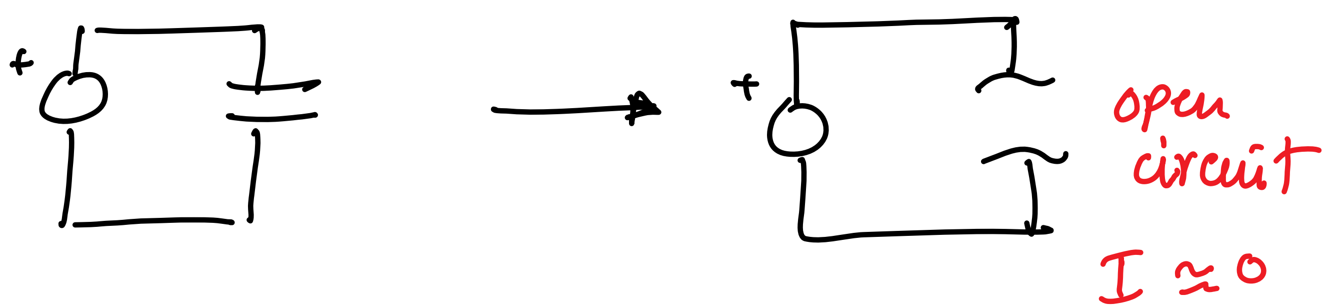
$$V = \frac{I}{j\omega C} = \frac{j}{j^2}\frac{I}{\omega C}$$

$$j = \sqrt{-1} \Rightarrow j^2 = -1$$

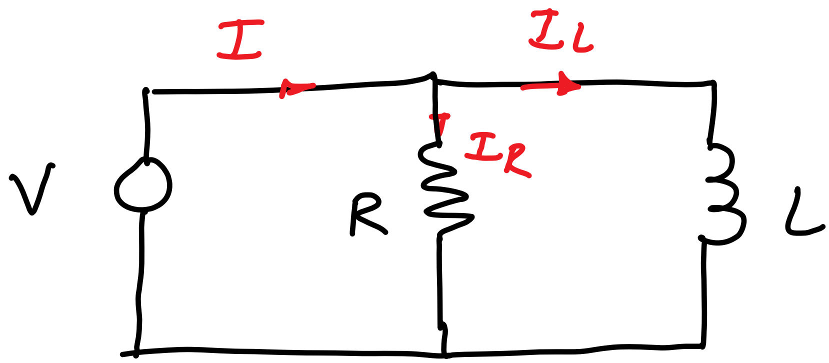
$$V = -\frac{j}{\omega C} I \simeq Z_c I$$

$$\boxed{Z_c = \frac{-j}{\underline{\omega} C}}$$

For a DC source $\omega = 0$ $\underline{\underline{Z_c = \infty}}$

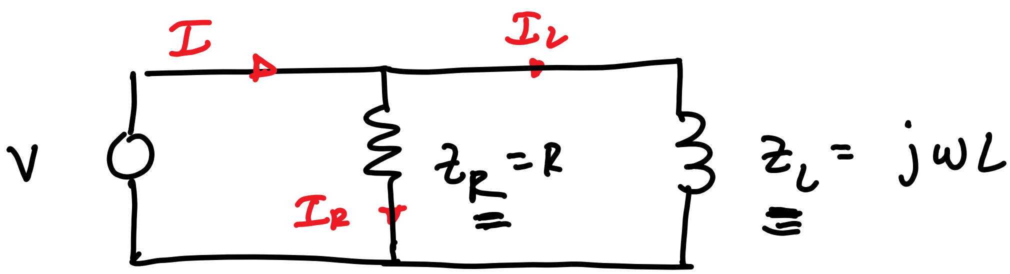


Example

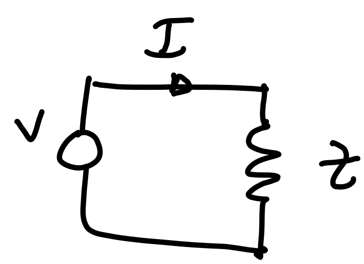


V = AC source

Compute I, I_L, I_R.

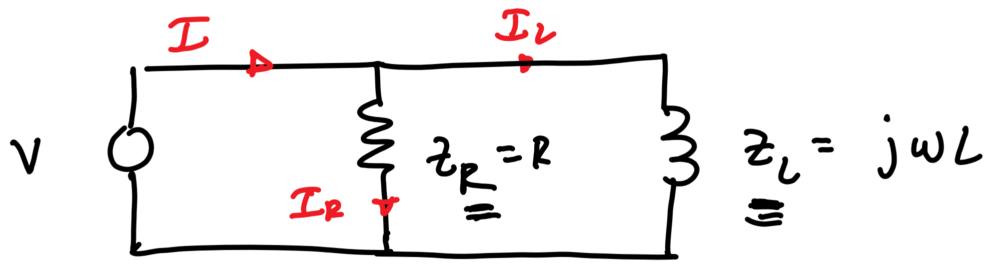


$$z = \frac{z_R z_L}{z_R + z_L}$$ (z_R & z_L are in parallel)

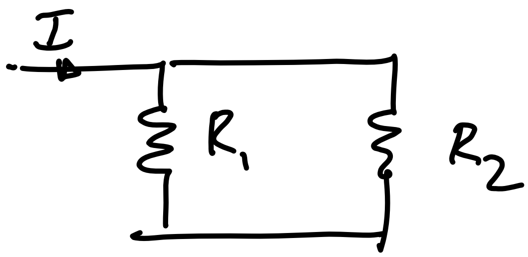


$$I = \frac{V}{z} = \frac{V\,(z_R + z_L)}{z_R z_L}$$

$$\boxed{I = \frac{V\,(R + j\omega L)}{j\omega L R}}$$



I ✓



$$I_1 = \frac{R_2}{R_1+R_2} I$$

current divider

$$I_2 = \frac{R_1}{R_1+R_2} I$$

$$I_R = \frac{Z_L}{Z_R+Z_L} I = \frac{\cancel{Z_L}}{\cancel{Z_R+Z_L}} \cdot \frac{V \cancel{(Z_R+Z_L)}}{Z_R \cancel{Z_L}}$$

$$\boxed{I_R = V/Z_R = V/R}$$

$$I_L = \frac{Z_R}{Z_R+Z_L} I = \frac{\cancel{Z_R}}{\cancel{Z_R+Z_L}} \cdot \frac{V}{\cancel{Z_R} Z_L} \cancel{(Z_R+Z_L)}$$

$$\boxed{I_L = \frac{V}{Z_L} = \frac{V}{j\omega L}}$$