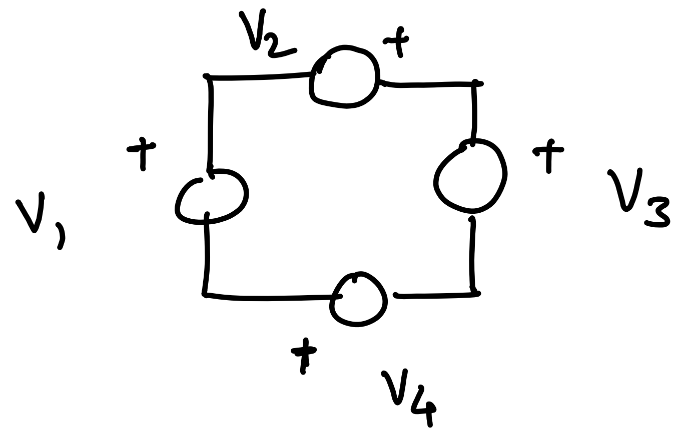
2.3 Kirchhoff's laws

(a) Voltage law

Sum of voltages in a loop adds to zero

$$\sum_{i=1}^{n} V_i = 0$$



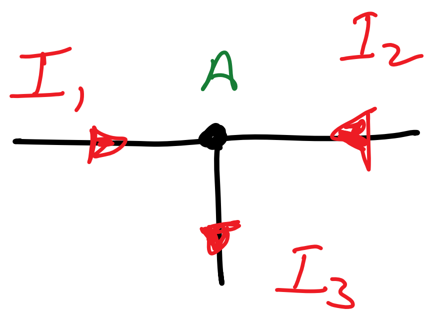
KVL : $$\sum_{i=1}^{4} V_i = 0$$

$$V_1 + V_2 - V_3 + V_4 = 0$$

(b) Current law:

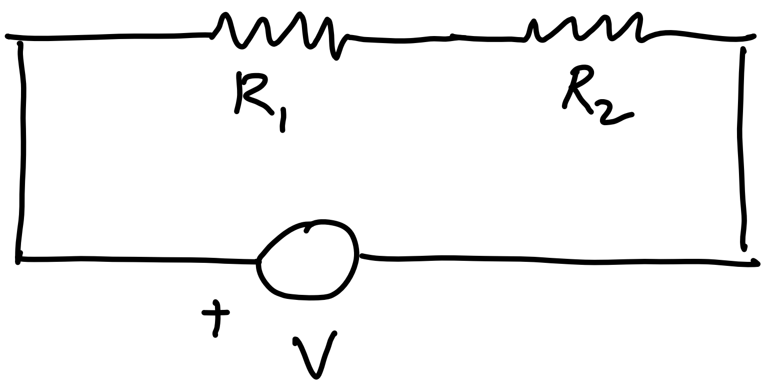
Sum of currents at a node add to zero.

$$\sum_{i=1}^{N} I_i = 0$$

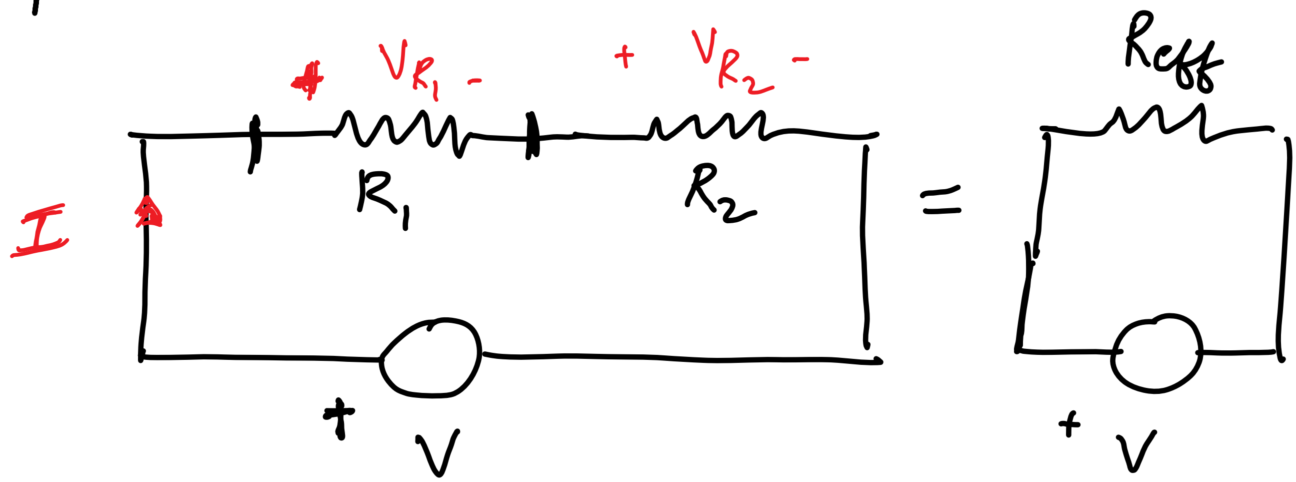


At node A: $+I_1 + I_2 - I_3 = 0$

Series circuit



Compute the current in the circuit



KVL: $\quad \Sigma V_i = 0$

$$V - V_{R_1} - V_{R_2} = 0$$

$$V - IR_1 - IR_2 = 0$$

$$I = \frac{V}{R_1 + R_2} = \frac{V}{R_{eff}}$$

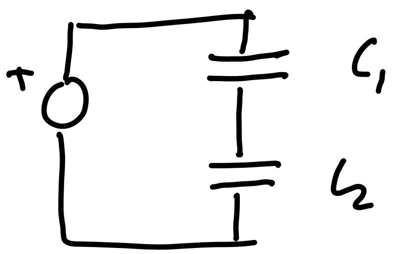
$$R_{eff} = R_1 + R_2$$

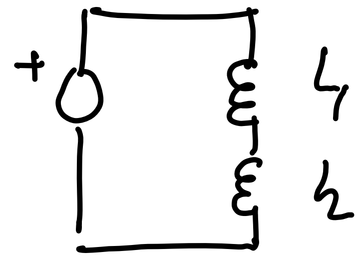
$$V_{R_1} = I R_1 = \left(\frac{V}{R_1+R_2}\right) R_1 = \frac{R_1}{(R_1+R_2)} V$$

$$V_{R_2} = \qquad = \frac{R_2}{R_1+R_2} V$$

Voltage divider

HW

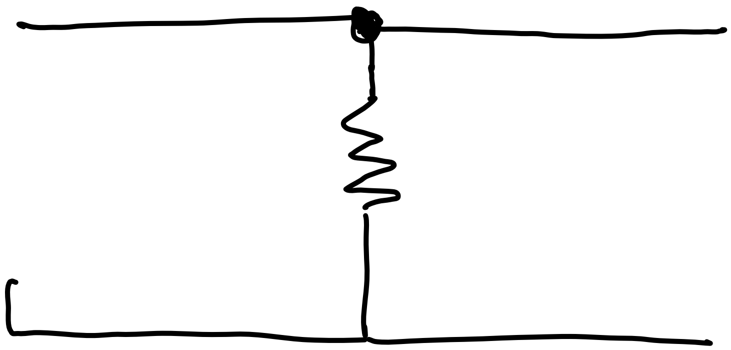




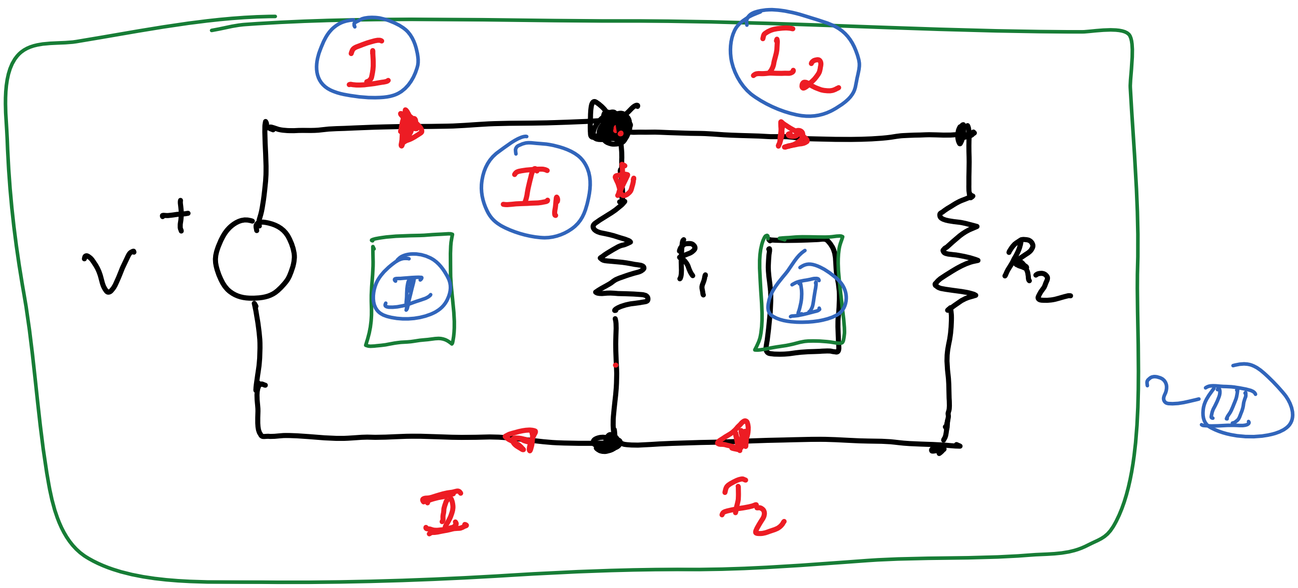
Compute effective capacitance & inductance

$$C_{eff} = \frac{C_1 C_2}{C_1 + C_2} \qquad L_{eff} = L_1 + L_2$$

(b) Parallel circuit



Compute the current and effective resistance.



KVL: $\Sigma V = 0$

(I) $V - I_1 R_1 = 0$; $V - I_2 R_2 = 0$ (II)

$I_1 R_1 - I_2 R_2 = 0$ ~ (III)

KCL: $I - I_1 - I_2 = 0$

$$I = I_1 + I_2$$

Summary

$$I_1 R_1 = V \Rightarrow I_1 = V/R_1$$

$$I_2 R_2 = V \Rightarrow I_2 = V/R_2$$

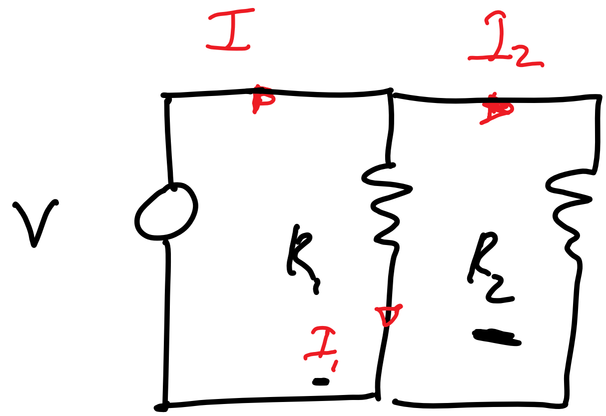
$$I_1 + I_2 - I = 0 \Rightarrow I = V/R_1 + V/R_2$$

$$I = V\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

$$\frac{V}{I} = R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = \left[\left(\frac{R_1 R_2}{R_1 + R_2}\right) I\right] \frac{1}{R_1}$$

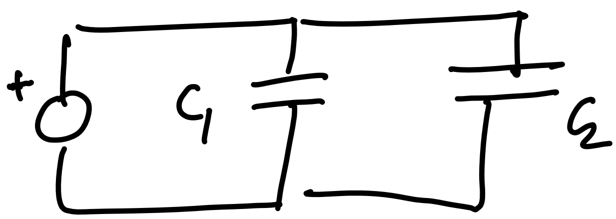
$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I$$

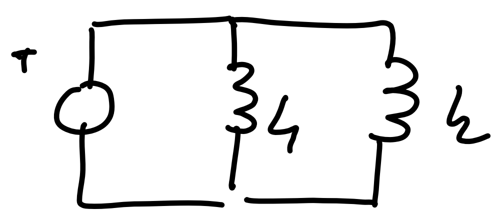


$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I$$

Current Divider Circuit

HW

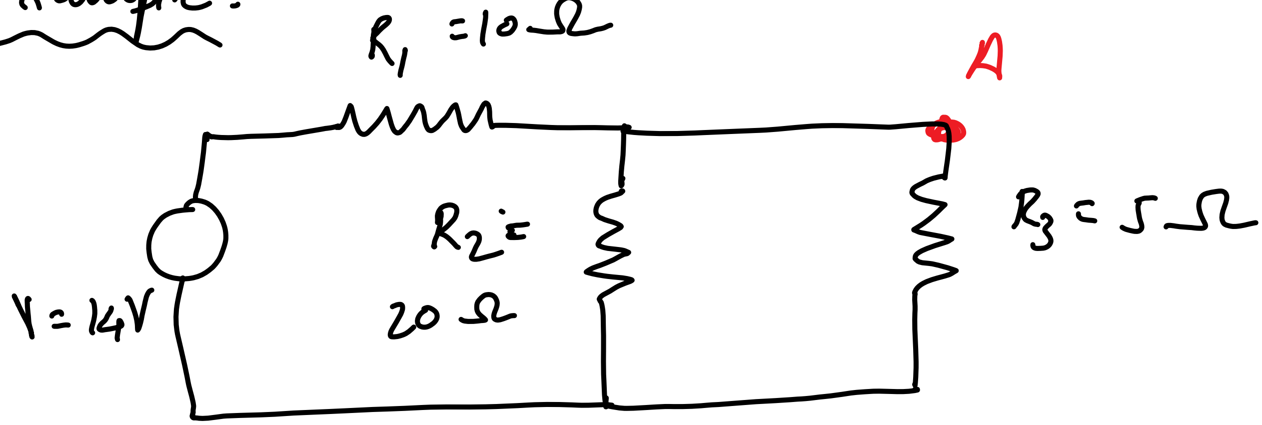




$$C_{eff} = C_1 + C_2$$

$$L_{eff} = \frac{L_1 L_2}{L_1 + L_2}$$

Example:

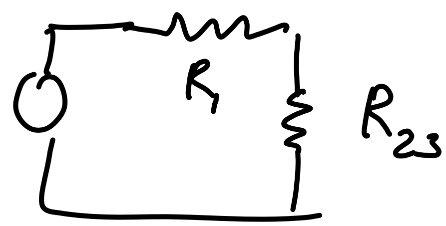


(i) Compute equivalent resistance

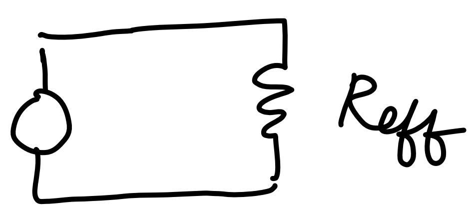
(ii) Compute the currents through R_1, R_2, R_3

(iii) Compute voltage at node A.

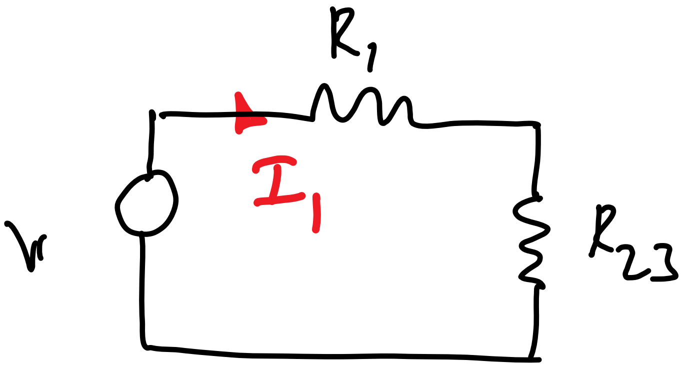
(i) $R_{23} = \dfrac{R_2 R_3}{R_2 + R_3}$ $\Rightarrow$



$R_{eff} = R_1 + R_{23}$

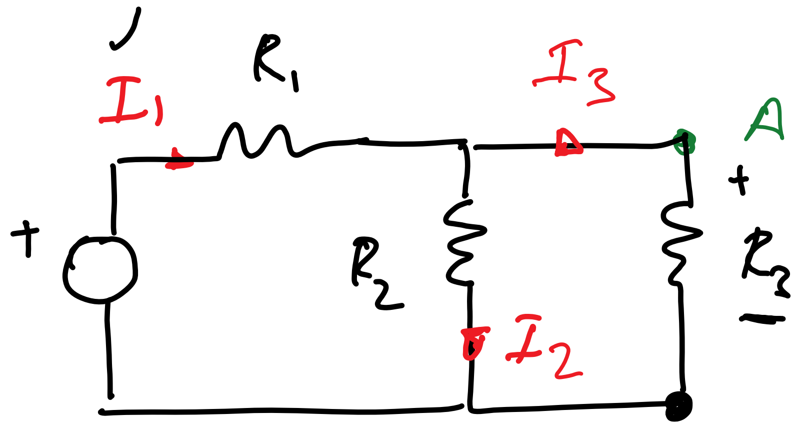


$$R_{eff} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 14\ \Omega$$



$$I_1 = \frac{V}{R_{eff}} = \frac{14}{14}$$

$$I_1 = 1 \text{ A}$$



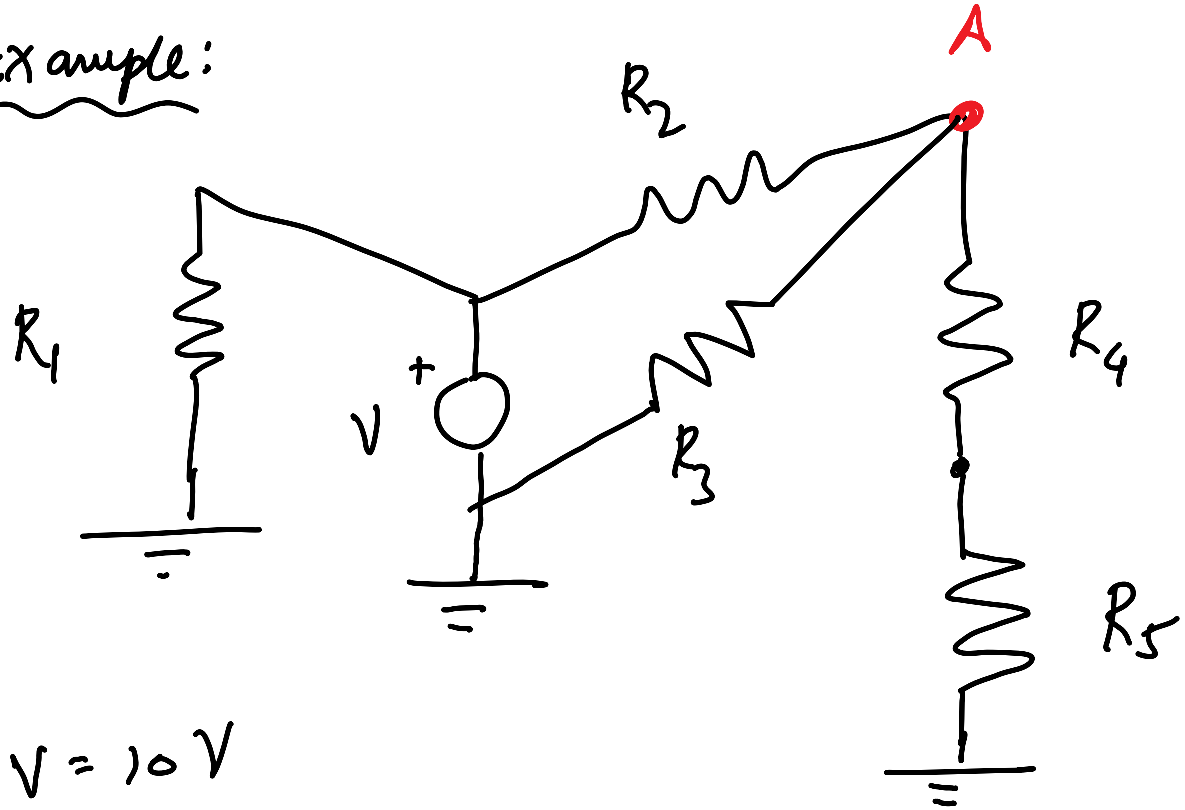
$$I_2 = \frac{R_3}{R_2 + R_3} I_1 \quad ; \quad I_3 = \frac{R_2}{R_2 + R_3} I_1$$

$$I_2 = 0.2 \text{ A} \qquad I_3 = 0.8 \text{ A}$$

(c) $V_A = V_{R_3} = I_3 R_3 = 0.8(5) = 4 \text{ V}$

$$V_A = 4 \text{ V}$$

Example:



$V = 10\,V$

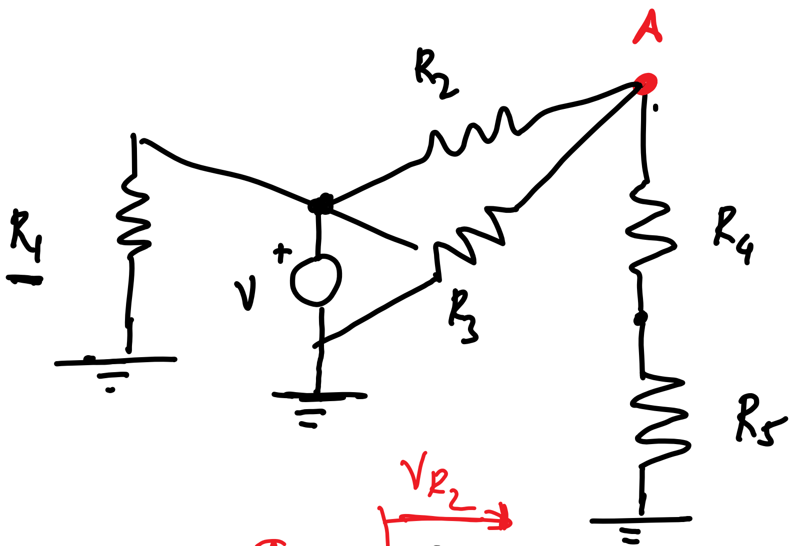
$R_1 = 1\,k\Omega$, $R_2 = 2\,k\Omega$, $R_3 = 3\,k\Omega$

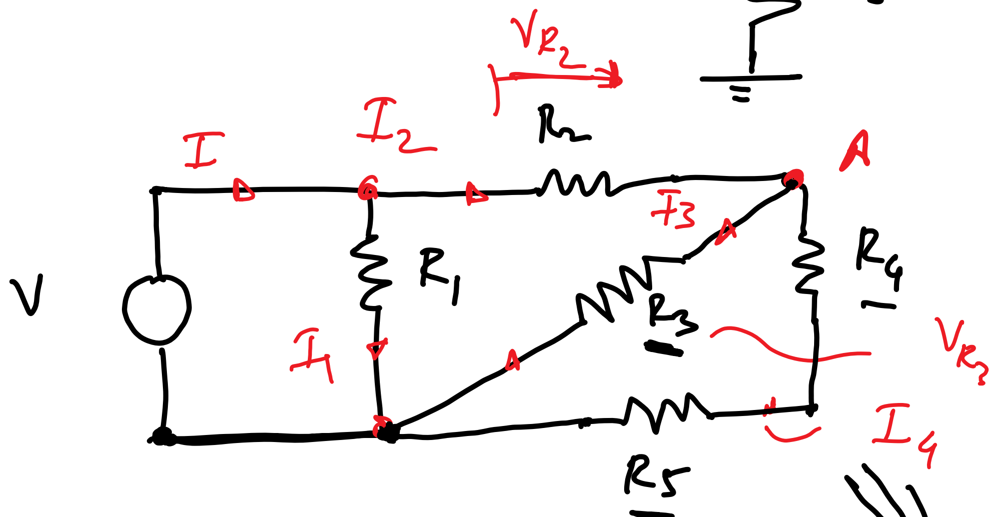
$R_4 = 4\,k\Omega$; $R_5 = 1\,k\Omega$

Compute (i) Equivalent resistance

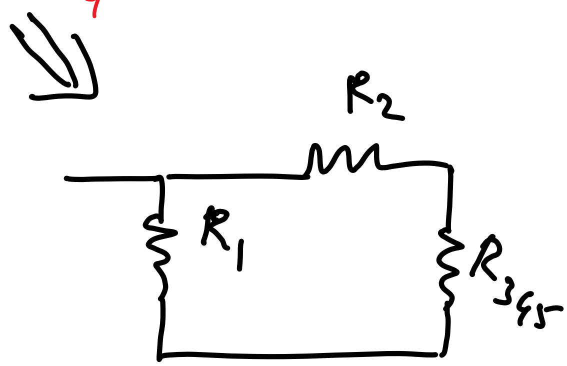
(ii) Voltage at node A

(iii) Current through resistor R_5





(i) $R_{45} = R_4 + R_5$



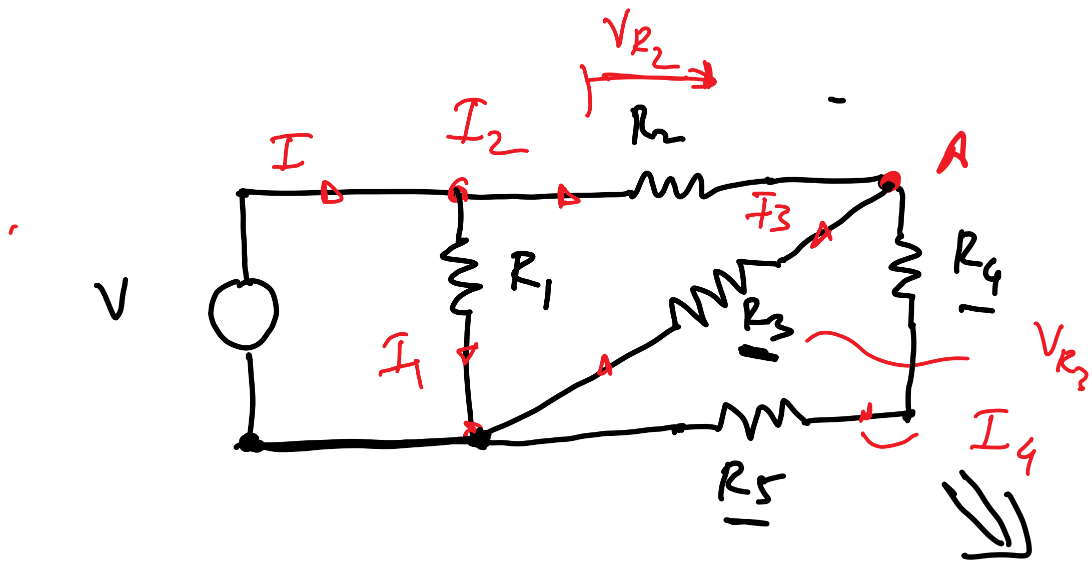
$$R_{345} = \frac{R_3 R_{45}}{R_3 + R_{45}}$$

$$R_{2345} = R_2 + R_{345}$$

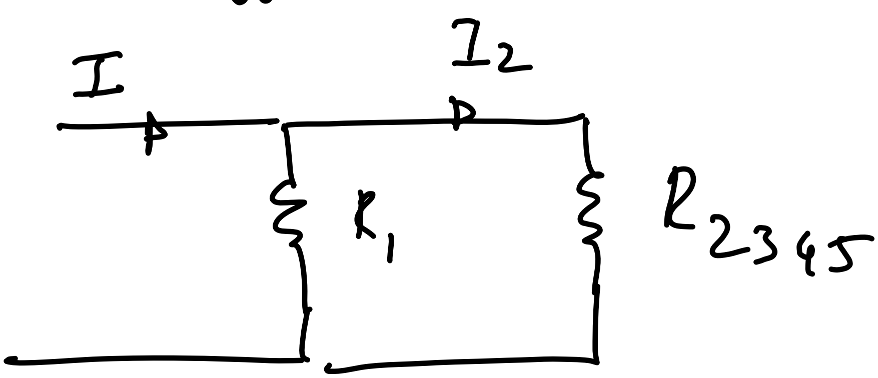
$$R_{eff} = R_1 R_{2345} / (R_1 + R_{2345})$$

$$\boxed{R_{eff} = 0.795 \text{ k}\Omega}$$

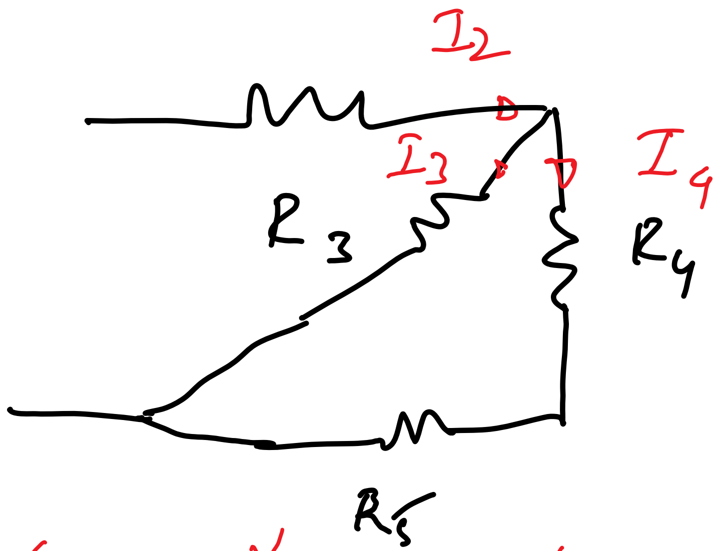
$$V_A = I_2 R_2 + I_3 R_3$$



$$I = \frac{V}{R_{eff}}$$



$$I_2 = \left(\frac{R_1}{R_1 + R_{2345}} \right) I$$



$$I_3 = \frac{R_{45}}{R_3 + R_{45}} I_2$$

$$\boxed{V_A = 4.84\,V}$$

(iii) $$I_4 = \frac{R_3}{R_3 + R_{45}} I_2$$

$$\boxed{I_4 = 0.97\,mA}$$