Electromagnetic principle

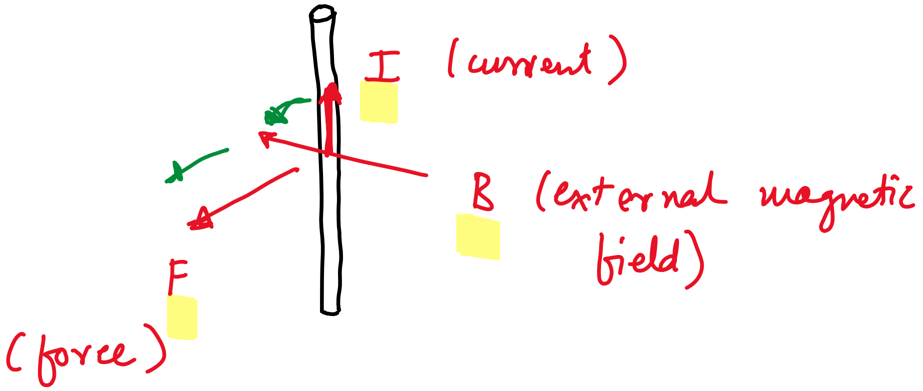
A current carry wire/conductor when subject to an external magnetic field will experience a force

$$\vec{F} = \vec{I} \times \vec{B}$$ [Lorentz Force law]

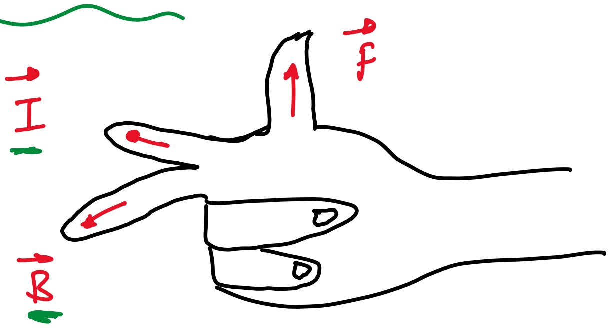
force / unit length

current

external magnetic field



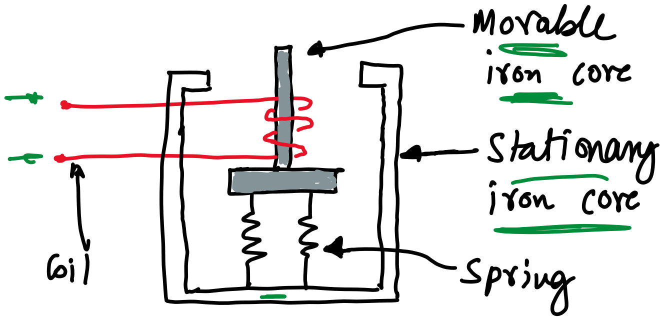
You can remember lorentz effect using Fleming's Right Hand Rule



- → Point index finger of right hand along the current
- → Point middle finger of right hand along the magnetic field
- → Then the thumb of the right hand gives the direction of the force

Solenoids and Relays

Solenoid



Principle

① Current is passed through the coil
② This magnetizes the movable iron core
③ Movable core attracted to stationary iron core
④ spring provides a restoring force to move the movable core back to its normal position

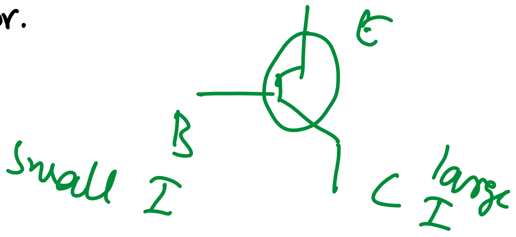
Application : ON-OFF actuators such as latching, locking, & triggering
① Washing machine valves
② Automobile doors and starter solenoids
③ Pin ball machines in plungers and bumpers.

Relay : This is a solenoid, but used to make and break mechanical contact between electrical leads.

A small voltage input to the relay controls a large current through the relay contact.

Applications : Power switches

Relay is similar to transistor : both act like a switch ; small voltage in relay / small current in base causes large voltage in contact / large current in collector.



Advantages of relays over transistors

① Relays Control potentially much larger currents than transistors

② Relays can switch AC and DC, transistors only DC.

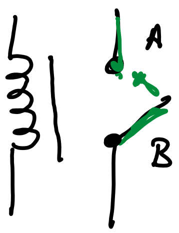
③ Relays are electricaly isolated from output hence virtualy noise free. Transistors share common ground with output (collector), hence can noise and ground faults can impact transistors.

Disadvantages of relays over transistors

① Relays switch slower because the mechanical element has inertia

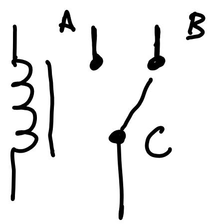
② Relays wear out because they use mechanical elements

Different types of relays



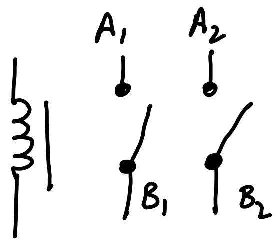
Single pole
Single throw
(SPST)
4 leads

①



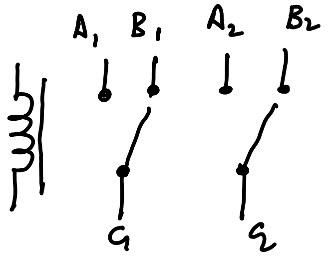
Single pole
Double throw
(SP DT)
5 leads

②



Double pole
Single throw
(DPST)
6 leads

③



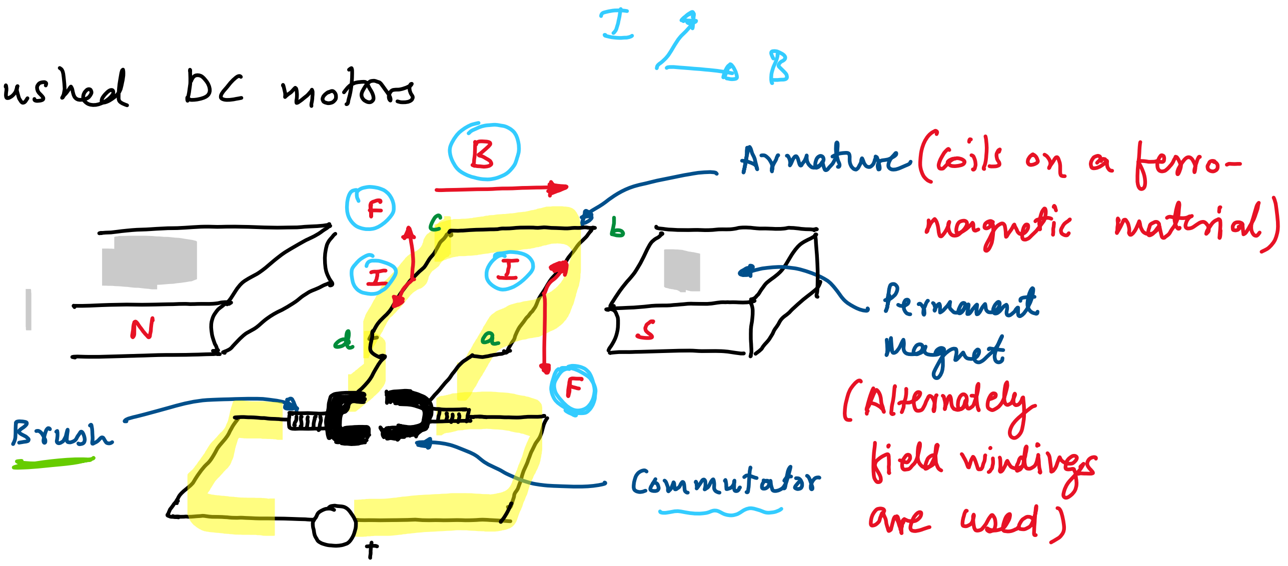
Double pole
Double throw
(DPDT)
8 leads

④

Electric motors

We will cover ① Brushed DC motors
② Brushless DC motors.

① Brushed DC motors



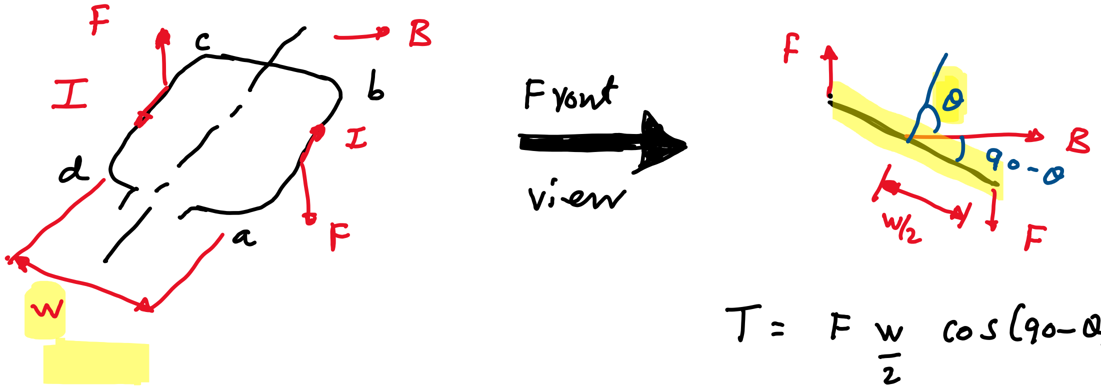
Components

- Two permanent magnets. These do not rotate (stator)
- A coil connected to a voltage source through commutator rings and brushes. These rotate (rotor)

Working

- The coil draws a current I.
- The permanent magnet produces a magnetic field B that is directed from North to South.
- By Lorentz law this will produce the force $\vec{F} = \vec{I} \times \vec{B}$. The force is upward on length c-d and downwards on length a-b as shown
- The commutator (split ring) ensures that the current direction is not reversed after 180° rotation.

- To compute the torque we redraw the coil in a rotated position as shown below

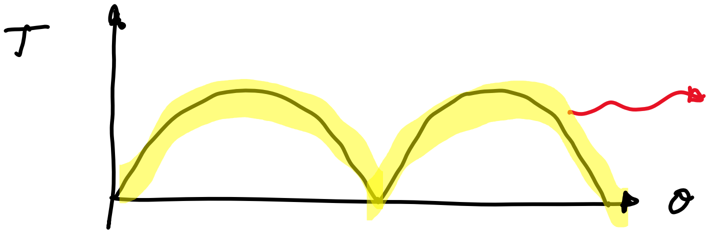


$$T = F \, \frac{w}{2} \, \cos(90-\theta)$$

$$T = (IB)(0.5\,w)\,\sin\theta$$

$$\Rightarrow T \propto \sin\theta$$

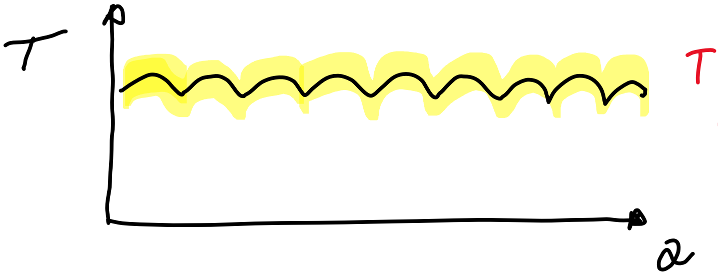
If we draw T as a function of θ



Torque ripple (not good)

This is tackled by have n coils, each with its own commutator ring. n = 50 is common.

For example, with n = 6, we will have



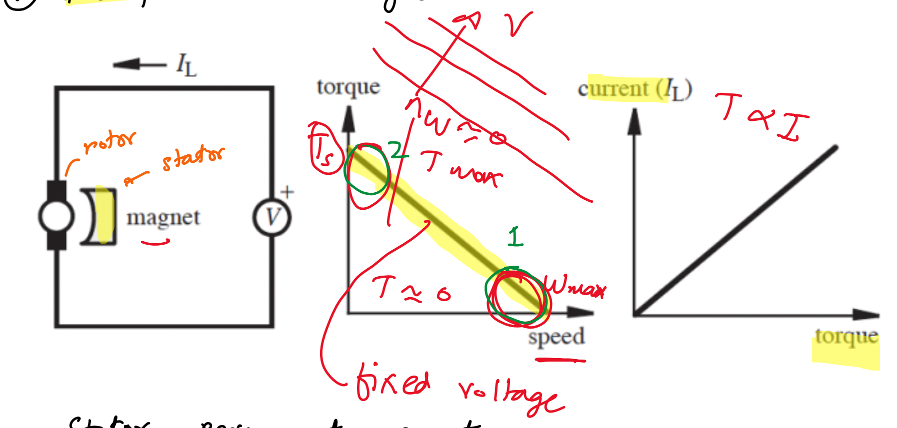
Torque is the average of n = 6 coils.

The 2 important specs for a DC motor are

① Stall Torque (T_s) : Torque corresponding to a speed of zero.

② No load speed (ω_{max}): The speed corresponding to no load or no torque

① DC permanent magnet



Stator – permanent magnet

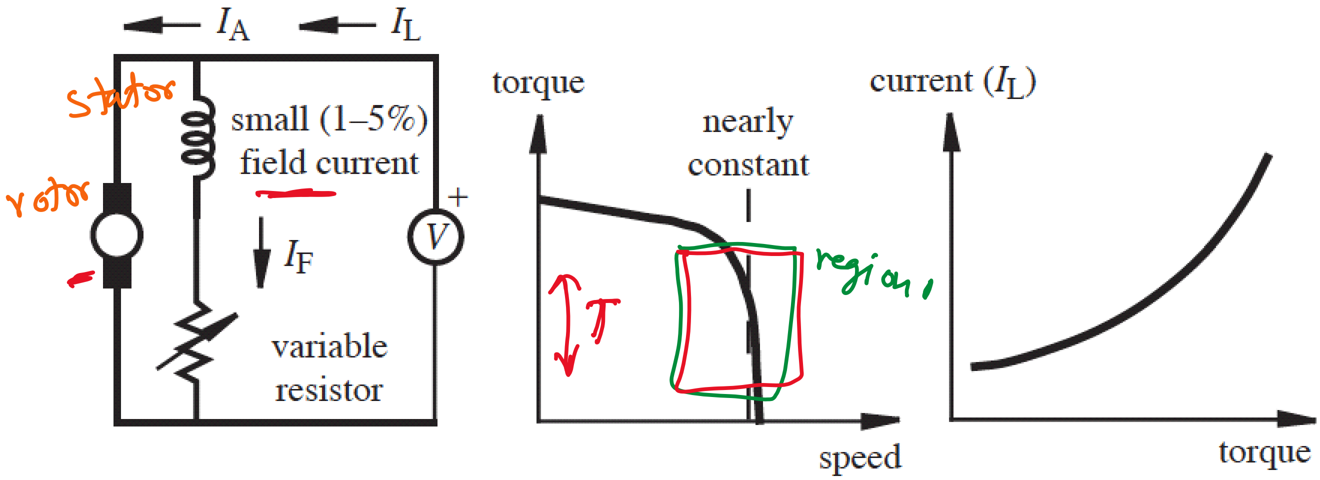
Rotor – coils

I-T curve – Torque ∝ current

T-ω curve – High speed / Low torque (region 1)
High Torque / Low speed (region 2)

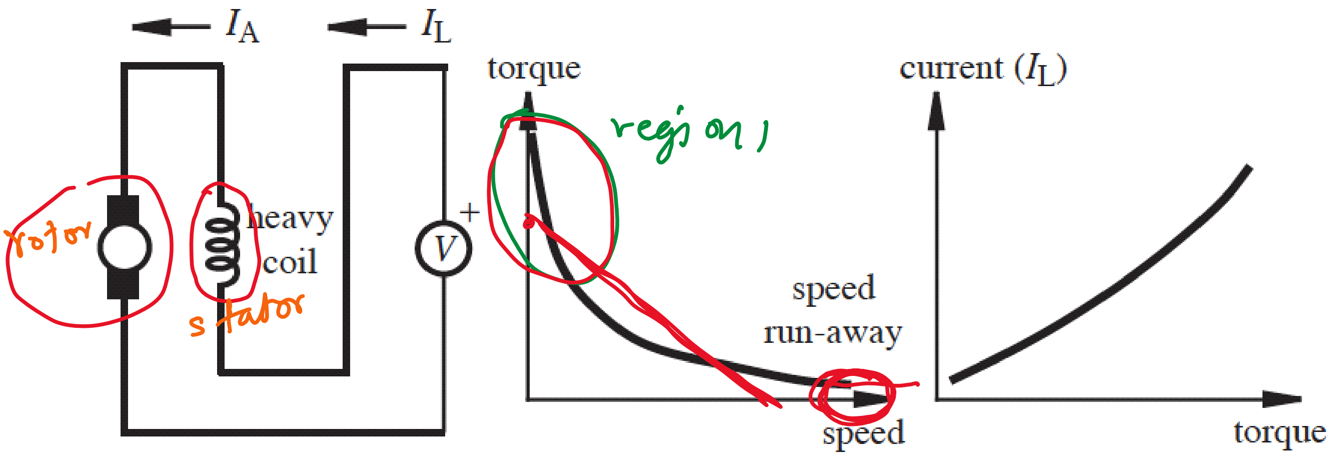
- These have limited power due to limits on magnetic strength
- Used in low power application (< 5hp)
- Can be brushed, brushless, stepper motor
- When coupled with encoder in position control mode they are called servos.

② DC shunt motor



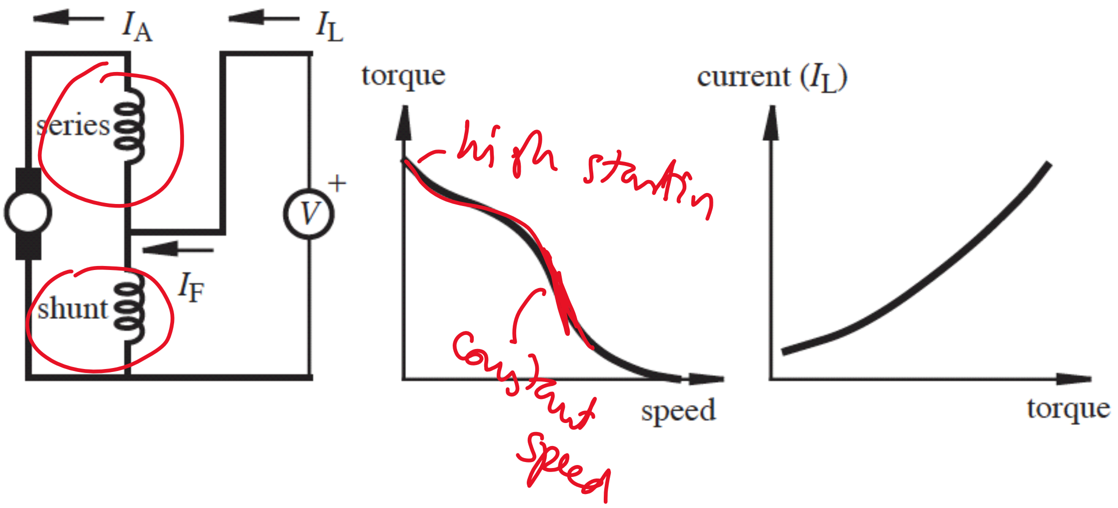
- No permanent magnet (instead uses field windings/stator)
- Armature (rotor) and field windings in parallel
- Constant speed at different loads (region 1)
- Applications that need constant speed, fans & blowers.

③ DC series motor.



- armature (rotor) and field windings (stator) in ~~parallel~~ series
- no load speed is infinite
- provide high starting torque (region 1)

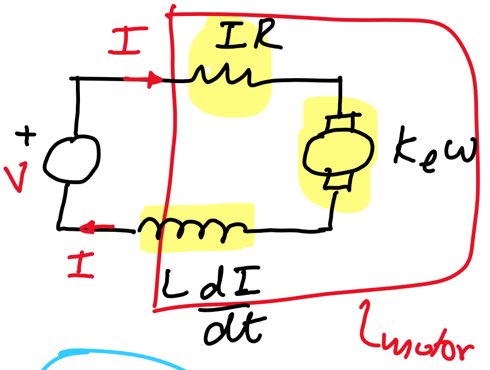
④ DC compound motor



- Series and parallel field windings
- Used when there is a need for high torque as well as constant speed such as punch presses

- 2, ③, ④ : These do not reverse their motion when the polarity is flipped,

Permanent Magnet DC motor



I^2R

$T \propto I$

R - resistance of windings & brushes

L - inductance of windings

ω - speed of the motor

I - current in the motor

✓ k_e - electrical constant of the motor

✓ k_t - torque constant of the motor

Power balance

$$\Rightarrow \underbrace{\cancel{I}V}_{\text{Power in}} = \cancel{I}\underbrace{(IR)}_{V_R} + \cancel{I}\underbrace{\left(\overset{0}{\cancel{L\frac{dI}{dt}}}\right)}_{V_L} + \cancel{I}\underbrace{(k_e\omega)}_{V_\omega} \quad - ①$$

Torque balance

$$\Rightarrow T = \underbrace{J}_{\text{inertia}}\underbrace{\frac{d\omega}{dt}}_{\text{acceleration}} + \underbrace{T_f}_{\text{friction torque}} \quad —— ②$$

Motor constitutive law

$$T = k_t I \quad —— ③$$

Put ③ in ① & assume steady state $\left(dI/dt = 0\right)$

$$\Rightarrow V = \left(\frac{T}{k_t}\right)R + k_e\omega$$

Re-arranging

$$\Rightarrow \boxed{T = -\left(\frac{k_e k_t}{R}\right)\omega + \left(\frac{k_t}{R}\right)V} \quad \text{— (IV)}$$

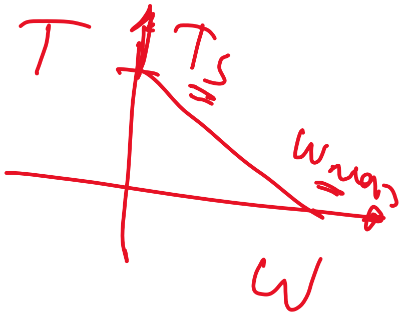
To compute stall torque, $T = T_s$ and $\omega = 0$

$$(*) \quad \boxed{T_s = \left(\frac{k_t}{R}\right)V} \quad \text{— (V)}$$

To compute no load speed, $\omega = \omega_{max}$ and $T = 0$

$$(*) \quad \boxed{\omega_{max} = \frac{V}{k_e}} \quad \text{— (VI)}$$

Put (V) and (VI) in (IV)



$$\boxed{T = T_s\left(1 - \frac{\omega}{\omega_{max}}\right)} \quad \text{— (VII)}$$

— simplify (IV)

The power, P, is given by $P = T\omega$

$$\boxed{P = T_s\left(\omega - \frac{\omega^2}{\omega_{max}}\right)}$$

Let us compute the extremum value of Power

$$\frac{dP}{d\omega} = T_s\left(1 - \frac{2\omega}{\omega_{max}}\right) = 0$$

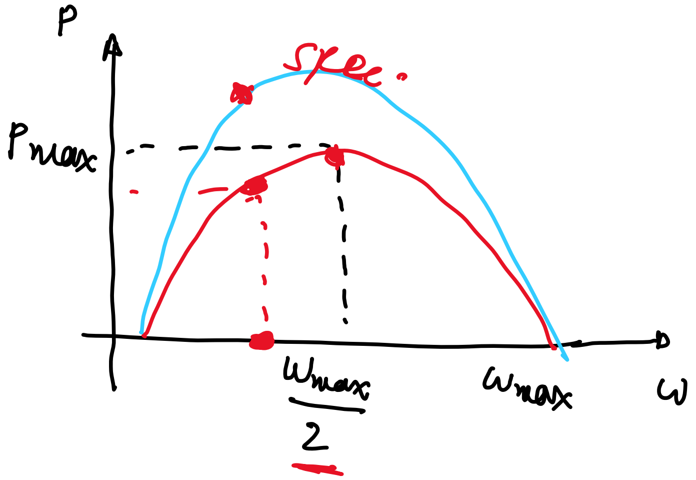
$$\Rightarrow \boxed{\omega = \frac{\omega_{max}}{2}}$$

maximize

$$\checkmark \quad \frac{d^2P}{d\omega^2} = T_s\left(0 - \frac{2}{\omega_{max}}\right) < 0$$

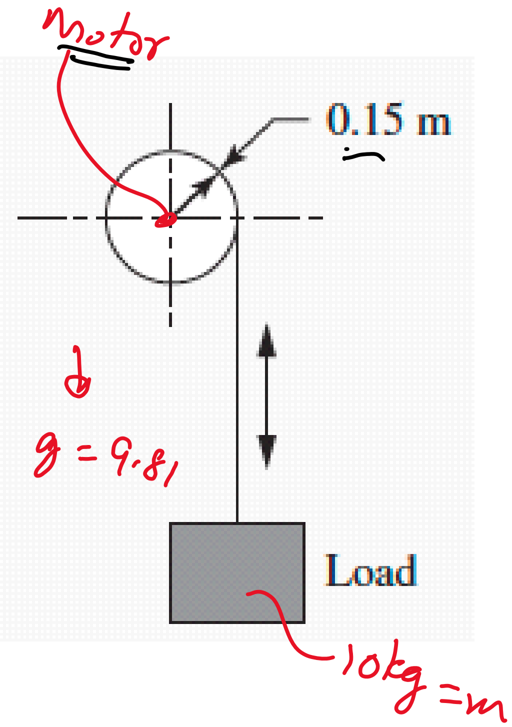
P achives maximum value at $\omega = \omega_{max}/2$

$$\boxed{P_{max} = \frac{T_s\,\omega_{max}}{4}}$$



A 0.25 hp (1 hp = 746 W) DC-geared motor is used to lift a mechanical load of 10 kg using a pulley arrangement as shown below. The no-load motor speed is 300 rpm and the starting torque is 23.8 Nm. The frictional resistance in the pulley drive is 2 Nm. Neglecting inertia of the rotor, the pulley, and the cable, determine

a) The initial acceleration of this load
b) The steady-state lifting speed of the load
c) The output horsepower of the motor at steady state



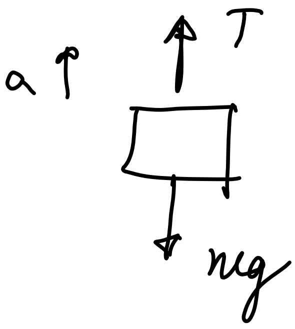
$\omega_{max} = 300$ rpm

✓ $T_s = 23.8$

✓ $T_f = 2$

FBD

(a)



$$\tau_{motor} = \overset{\checkmark}{T_s} - \overset{\checkmark\ M}{T_f}$$

$$M_o = \overset{?}{T}(0.15) = \overset{?}{\tau_{motor}}$$

$$T - mg = m\overset{?}{a}$$

3 equation
3 unknowns

$$\boxed{a = 4.73\ m/s^2}$$

ⓑ When $a=0$, mass is moving at steady speed

$$T - mg = m\overset{0}{\cancel{a}}$$

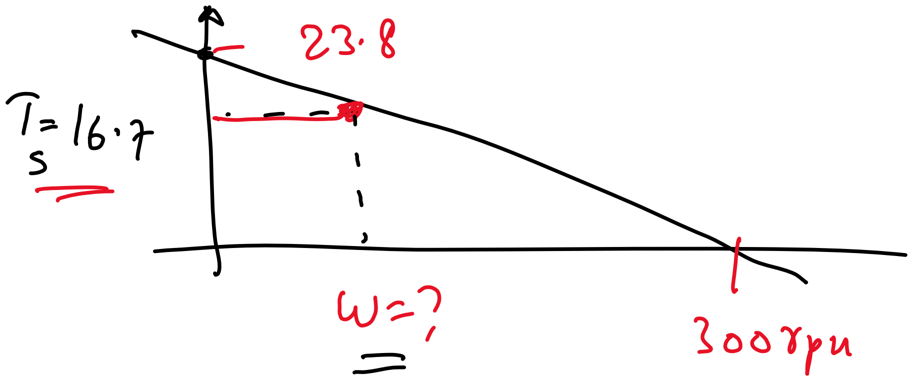
$$T = mg \checkmark$$

$$\tau_{motor} = T(0.15)$$

$$\tau_{motor} = T_s - T_f$$

$$\begin{aligned} T_s &= \tau_{motor} + T_f \\ &= T(0.15) + T_f \\ &= mg(0.15) + T_f \end{aligned}$$

$$T_s = 16.7 \text{ Nm}$$



$$\frac{T}{\underline{\underline{23.8}}} + \frac{\omega}{\underline{\underline{300}}} = 1$$

$$\frac{16.7}{23.8} + \frac{\omega}{300} = 1$$

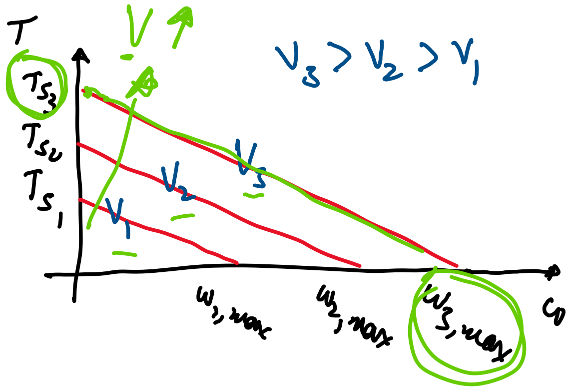
$$\omega = \underline{\underline{89.5}} \text{ rpm}$$

$$V = \omega r = 1.41 \text{ m/s}$$

ⓒ $P = T\omega = 16.7(89.5) = 0.21\ \underline{\underline{hp}}$

Controlling a DC motor

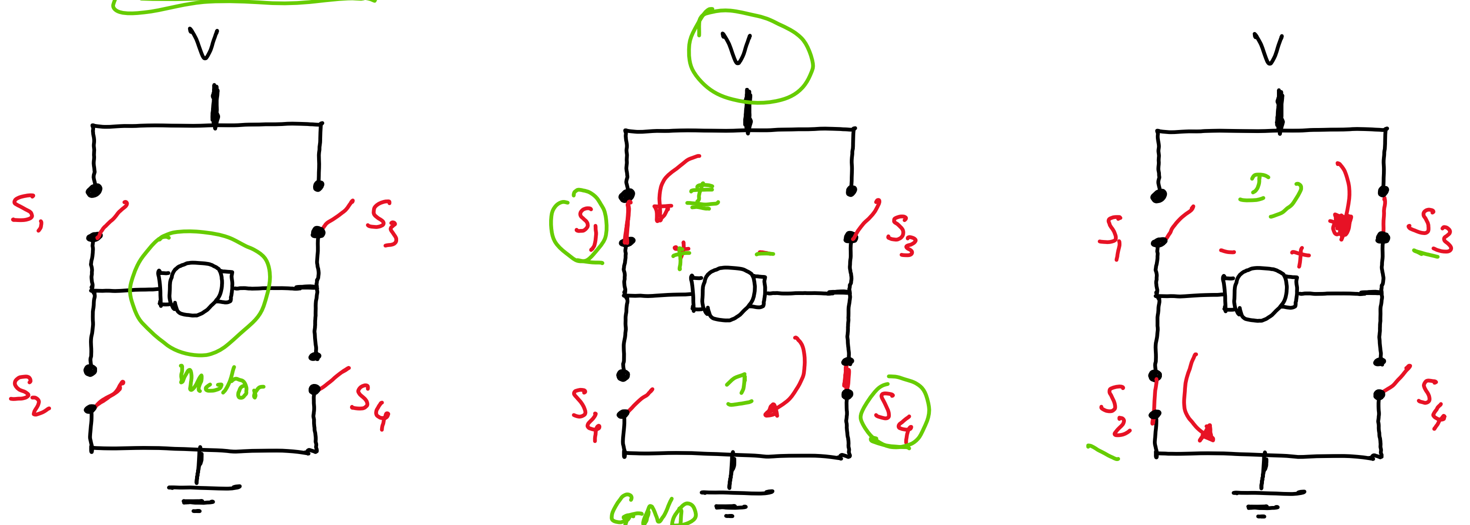
The T-ω curve of the motor at different voltages is shown to the right



Thus, one can modulate the voltage to achive higher speeds at the same torque and vice versa.

We have seen the use of a transistor to modulate the voltage. But this does not let us change the direction of the voltage. A better way of modulating the magnitude as well as direction of voltage is through an H-bridge.

H-bridge



① Direction control

(i) S_1, S_4 closed to make the motor spin one way

(ii) S_2, S_3 closed to make the motor spin other way

② Speed control   pulse width modulation

The speed control is achieved by controlling the frequency of closing/opening the switch. Let T be a time constant associated with switching. The H-bridge can switch on-off much faster than T. To achive an average voltage of 0.2 V (V= supply voltage), the H-bridge closes the switch for 0.2T and keeps the switch open for 0.8T. This results in an average voltage of 0.2V.