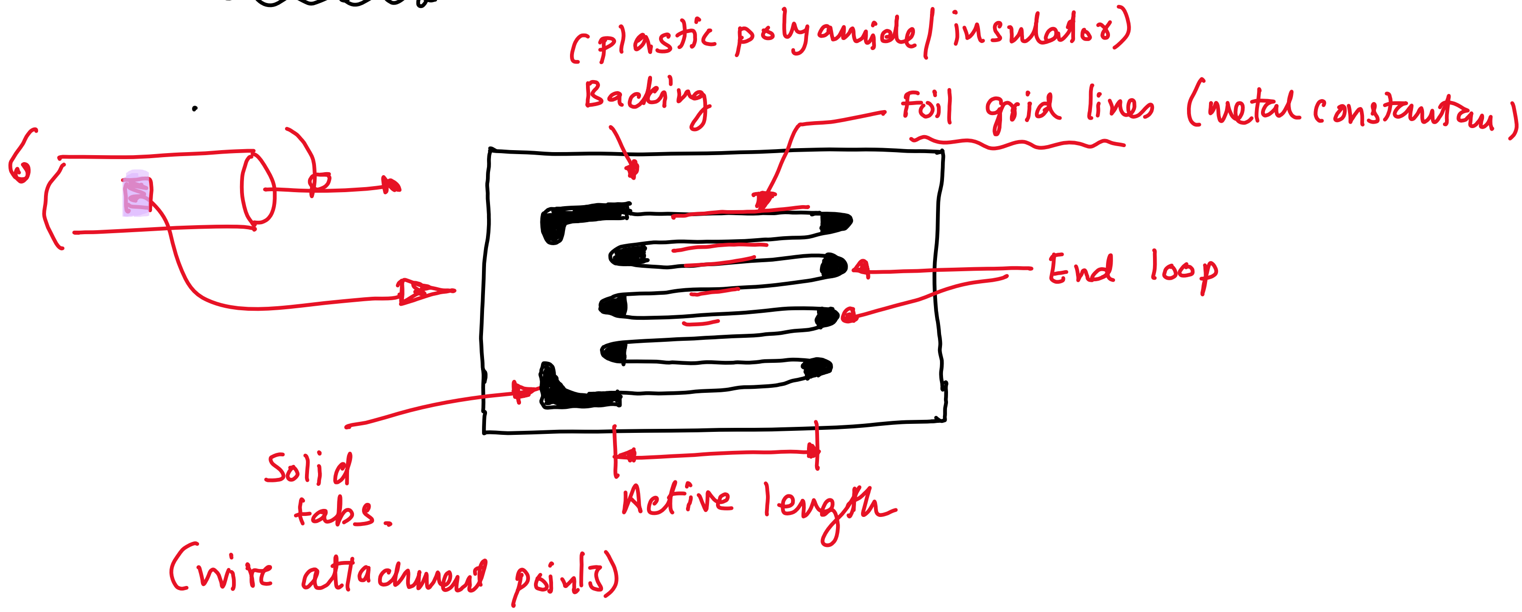
Stress and Strain measurement

- Strain gauge measure strain
- Stress is estimated using strain measurement and using the principles of solid mechanics

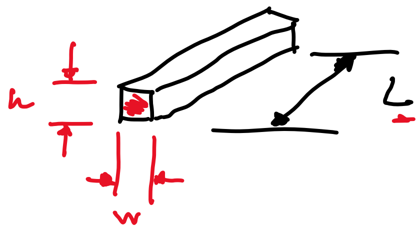
Strain gauge (5-15mm in size)



- strain gauge is attached using epoxy / cyanoacrylate
Backing provides surface for bonding and insulation from the surface the strain gauge is attached.
- lead wires are soldered to the solder tabs.
- when the object on which the strain gauge is attached is loaded, it bends. This deforms the strain gauge. The deformation causes the resistance of the strain gauge to change. By measuring the change in resistance, it is possible to measure the strain.
- NOTE: strain gauge measures average strain over the area it is attached.

- Note: strain gauge measures average strain over the area it is attached.

Theory



$$\boxed{R = \frac{\rho l}{A}} \quad \text{— (1)}$$

Area = wh

✓ ρ = resistivity

✓ R = resistance

✓ L = length

✓ A = cross-sectional area

Taking the differential of (1)

$$\Rightarrow \quad \delta R = \frac{l}{A}\delta\rho + \frac{\rho}{A}\delta l - \frac{\rho l}{A^2}\delta A$$

Dividing by $R = \rho l / A$

$$\frac{\delta R}{R} = \frac{1}{\left(\frac{\rho l}{A}\right)}\left[\frac{l}{A}\delta\rho + \frac{\rho}{A}\delta l - \frac{\rho l}{A^2}\delta A\right]$$

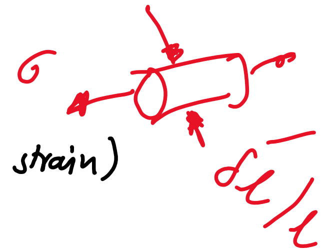
$$\rightarrow \quad \frac{\delta R}{R} = \frac{\delta\rho}{\rho} + \frac{\delta l}{l} - \frac{\delta A}{A} \quad \text{— (2)}$$

We know that $A = wh$

$$\delta A = h\,\delta w + w\,\delta h$$

$$\Rightarrow \quad \frac{\delta A}{A} = \frac{\delta w}{w} + \frac{\delta h}{h}$$

$$\frac{\delta l}{l} = \varepsilon_l \quad \text{(longitudinal strain)}$$



$$\frac{\delta w}{w} = \varepsilon_{t_1} \quad \& \quad \frac{\delta h}{h} = \varepsilon_{t_2} \quad \text{(transverse strain)}$$

But $\boxed{\nu = -\frac{\varepsilon_{t_1}}{\varepsilon_l} = -\frac{\varepsilon_{t_2}}{\varepsilon_l}}$ (ν = Poisson's ratio)

Thus, $\boxed{\frac{\delta w}{w} = -\nu \frac{\delta l}{l}}$ and $\boxed{\frac{\delta h}{h} = -\nu \frac{\delta l}{l}}$ — (3)

Substitute (2) in $\boxed{\delta A/A = \delta w/w + \delta h/h}$

$$\frac{\delta A}{A} = -2\nu \frac{\delta l}{l} \qquad \text{— (4)}$$

Substitute (4) in $\delta R/R = \delta\rho/\rho + \delta l/l - \delta A/A$

$$\frac{\delta R}{R} = \frac{\delta \rho}{\rho} + (1+2\nu)\frac{\delta l}{l}$$

Dividing by $\delta l/l = \varepsilon_l$

$$\frac{\delta R/R}{\varepsilon_l} = \frac{\delta\rho/\rho}{\varepsilon_l} + (1+2\nu) = F$$

(gage factor) $\simeq$ constant for a strain gage

Thus

$$\boxed{\delta R = F \varepsilon_l R}$$

Change in resistance $\propto$ Strain

or

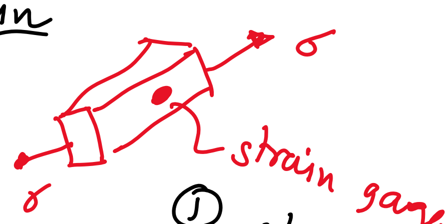
$$\boxed{\varepsilon_l = \frac{\delta R/R}{F}}$$

? measured

Typically, $R = 120\,\Omega$, $F = 2$ for metal foil gauge. δR is measure (next section) and ε_l is estimated from above equation

Experimental determination of strain

Wheatstone Bridge

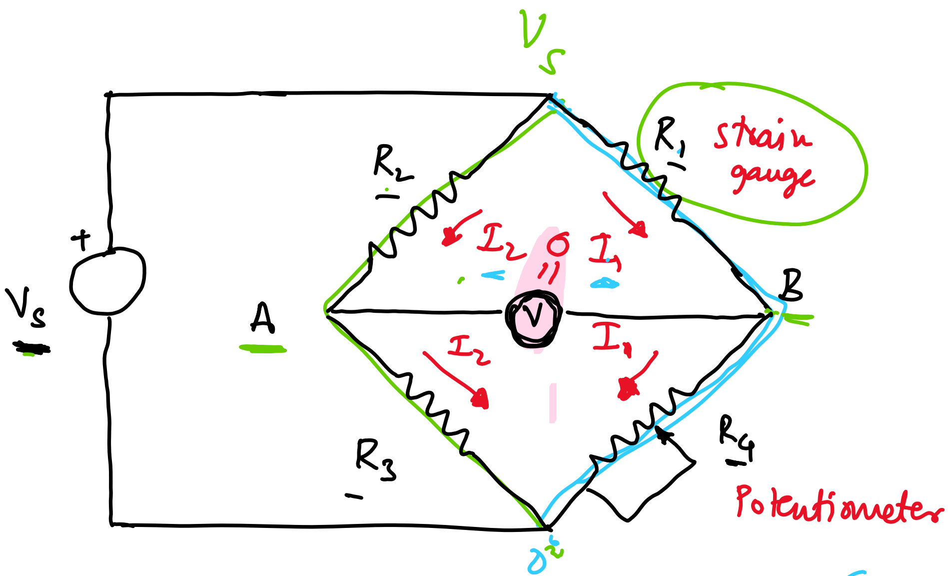


(1) stretching
(2) ε_l
(3) δR

$$\varepsilon_e = \frac{\delta R / R}{F}$$

measure

$F \approx 2$



Procedure

(1) Static Balance mode

$$V_A = V_s - V_A = I_1 R_1$$

$$V_s - V_B = I_2 R_2$$

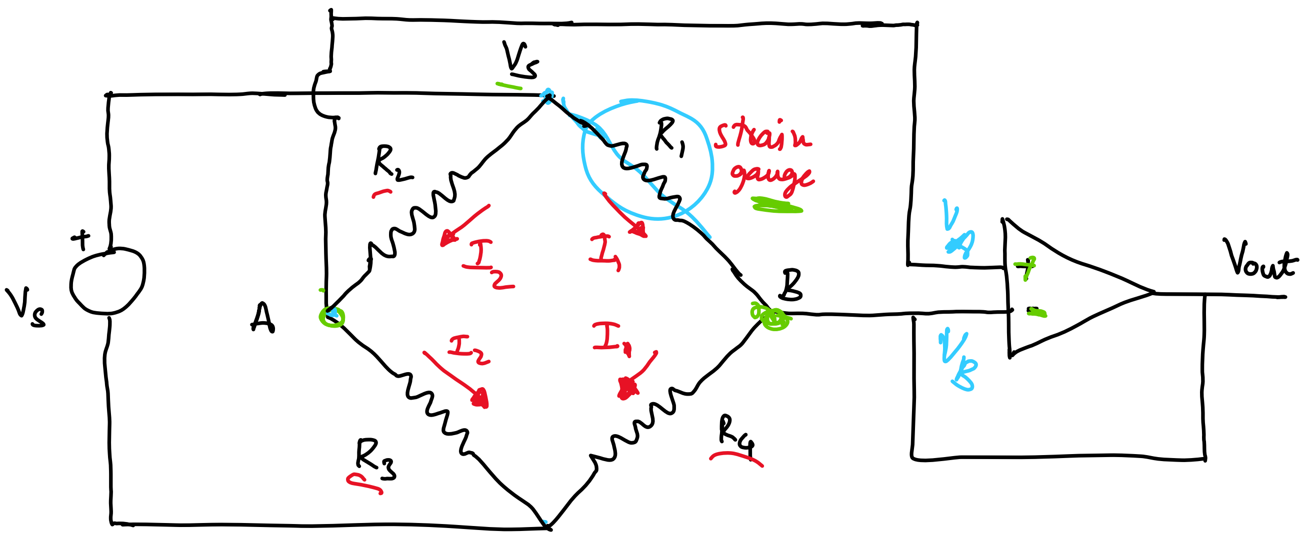
pot R_4 is adjusted till the voltmeter (V) reads 0. This implies that $V_A = V_B$

$$\Rightarrow I_1 R_1 = I_2 R_2 \quad - (1)$$

Also, $I_1 = \dfrac{V_s}{R_1 + R_4}$ $\quad I_2 = \dfrac{V_s}{R_2 + R_3}$ $\quad - (2)$

From (1) & (2) $\quad \dfrac{R_1}{R_1 + R_4} = \dfrac{R_2}{R_2 + R_3} \Rightarrow \dfrac{R_1}{R_4} = \dfrac{R_2}{R_3}$

$$\boxed{R_1 = \frac{R_2 R_4}{R_3}} \quad - (3)$$



② Dynamic Balance mode

Next, the component is loaded. This causes the strain gauge to bend. The resistance of the gauge changes to $R_1 + \delta R_1$. The output voltage, $V_{out} \neq 0$

$$\left.\begin{aligned} V_s - V_A &= I_2 R_2 \\ V_s - V_B &= I_1 (R_1 + \delta R_1) \end{aligned}\right\} \longrightarrow V_A - V_B = I_1(R_1 + \delta R_1) - I_2 R_2$$

But $$V_{out} = V_A - V_B = I_1 (R_1 + \delta R_1) - I_2 R_2 \quad -④$$

But $$I_1 = V_s / (R_1 + \delta R_1 + R_4) \;;\; I_2 = V_s / (R_2 + R_3)$$

$$\Rightarrow \boxed{V_{out} = V_s \left[\frac{R_1 + \delta R_1}{R_1 + \delta R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right]}$$

Re-arranging

$$\boxed{\frac{\delta R_1}{R_1} = \frac{\frac{R_4}{R_1}\left(\frac{V_{out}}{V_s} + \frac{R_2}{R_2+R_3}\right)}{\left[1 - \frac{V_{out}}{V_s} - \frac{R_2}{R_2+R_3}\right]} - 1}$$

R_2, R_3, R_4, R_1, V_s are known

V_{out} is measured

δR_1 can then be estimated from the equation

Finally, using $\varepsilon_l = \frac{\delta R / R}{F}$ one can estimate strain ε_l

Force and Torque sensor

- Force / Torque sensor have a strain gauge
- The bending of the strain gauge causes its resistance to change. The resistance change can be measured to compute the strain

σ $\quad \sigma = Y\,\varepsilon$

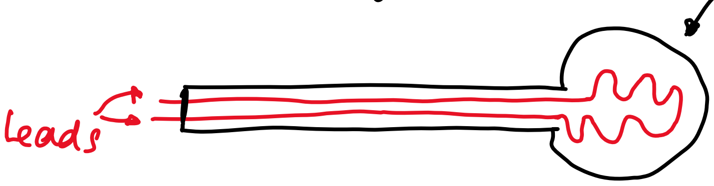
ε_l - The strain can be converted to stress using Young's modulus
- Stress is converted to Force / Torque using cross sectional area and moment arm. $\tau = (\sigma A) r$ ← moment arm

$F = \sigma A$

- Manufacturers list the output as mV/V. For example 2mV/V means for a supply voltage of 1V (denominator) will produce an output of 2mV (numerator)
- Force / Torque sensor needs to be calibrated before they can be used.

Force sensing resistor

Force application



- No force, infinite resistance / Force applied, resistance decreases to several hundred ohms.
- Use voltage divider to measure resistance $\propto$ force
- cheap but low accuracy.

- cheap but low accuracy,

Temperature measurement

① Electrical Resistance Thermometer

Principle: Resistance changes when subject to a temperature

ⓐ Resistance temperature device

- Uses a metal
  - ~ Platinium, large temperature range (−220°C to 750°C), expensive
  - ~ Nickel or Copper, cheap but limited range

$$R = R_0 \{1 + \alpha (T - T_0)\}$$

T_0, T

T, T_0 — temperatures

R, R_0 — resistances at temperature T, T_0 respectively

α — material constant

ⓑ Thermistor

- Uses ceramic or polymer

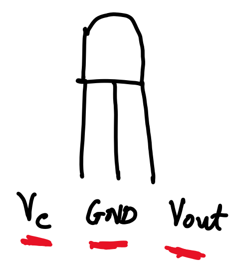
$$R = R_0 e^{\{\beta (\frac{1}{T} - \frac{1}{T_0})\}}$$

- β = constant

accuracy within 0.01°C

- range −90°C to 130°C

② Temperature Sensor integrated circuit

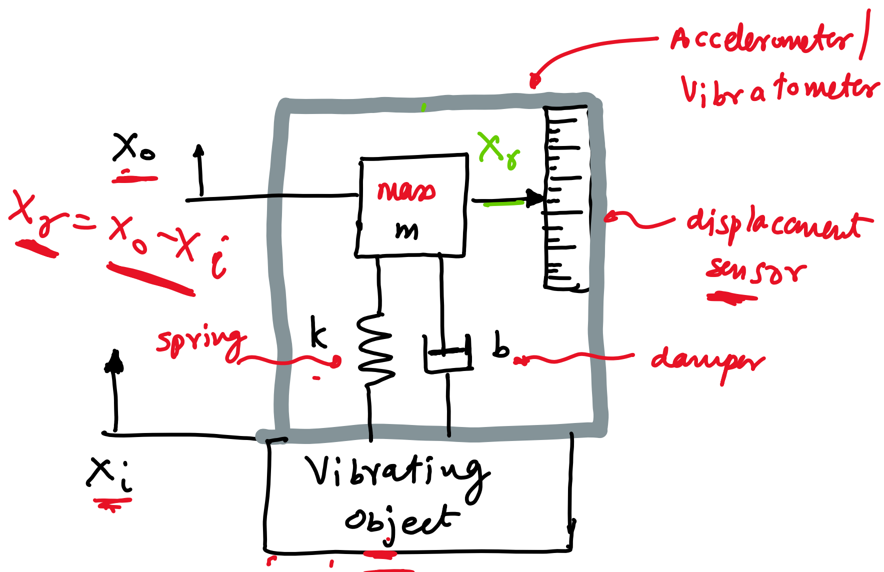


- LM 35C; 3 leads; V_c, GND, V_{out}
- Needs to be calibrated before use
- Uses a transistor for temperature measurement

Forward voltage of PN junction depends on the temperature

Forward voltage of PN junction depends on the temperature

Vibration and Acceleration Measurement

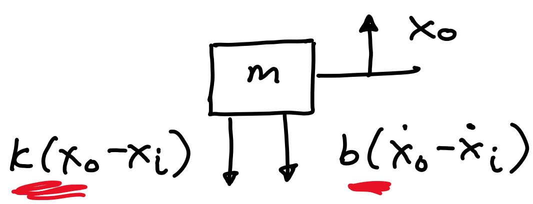


By appropriate choices of mass (m), spring constant (k), damping constant (b) we can use this device as

(a) measure acceleration of the object $\ddot{x}_i$ (Accelerometer)

(b) measure displacement of the object x_i (Vibrometer)

FREE BODY DIAGRAM



$\uparrow kx_{eq.}$

$\downarrow mg$

Using Newtons law

mg $\otimes$

$$\Sigma f_{ext} = m\ddot{x}_o$$

✓ $$-b(\dot{x}_o - \dot{x}_i) - k(x_o - x_i) = m\ddot{x}_o$$

The displacement sensor measures the relative displacement between the object and the mass (m)

$$x_r = x_o - x_i$$

mass

Thus, putting $x_r = x_o - x_i \Rightarrow x_o = x_r + x_i$ in

$$\Rightarrow -b(\dot{x}_o - \dot{x}_i) - k(x_o - x_i) = m\ddot{x}_o$$

$$\Rightarrow -b\dot{x}_r - kx_r = m\ddot{x}_r + m\ddot{x}_i$$

$$\Rightarrow \boxed{m\ddot{x}_r + b\dot{x}_r + kx_r = -m\ddot{x}_i}$$

x_r — measured

$X_i = ?$ & constants, estimate object

Let $\omega_n = \sqrt{k/m}$ $\quad \xi = b/2\sqrt{km}$

ω_n^2

$$\ddot{x}_r + 2\xi\omega_n\dot{x}_r + \cancel{k}x_r = -\ddot{x}_i$$

If the input is $x_i(t) = X_i \sin(\omega t)$ then the output will be $x_r(t) = X_r \sin(\omega t + \phi)$ $\{X_r, \phi$ are unknown$\}$

We can solve for X_r & ϕ as follows

$$\frac{X_r}{X_i} = \frac{r^2}{[(1-r^2) + 4\xi^2 r^2]^{1/2}} \quad \text{— I}$$

$= \omega/\omega_n$

$$\phi = -\tan^{-1}\left(\frac{2\xi r}{1-r^2}\right) \quad \text{— II}$$

where $r = \dfrac{\omega}{\omega_n}$

① Use as accelerometer

$x_i = X_i \sin(\omega t)$

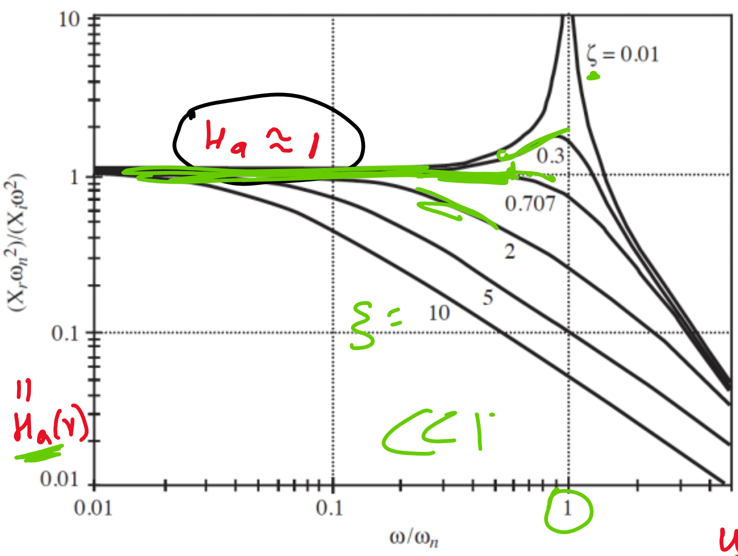
$$\ddot{x}_i = \underbrace{X_i \omega^2}_{\text{amplitude}} \sin(\omega t + \cancel{\phi})$$

$$X_r = \frac{X_i \omega^2/\omega_n^2}{\left[(1-r^2)^2 + 4\xi^2 r^2\right]^{1/2}} \qquad \text{From (I)}$$

$$\frac{X_i \omega^2}{X_r \omega_n^2} = \underbrace{\left[(1-r^2)^2 + 4\xi^2 r^2\right]^{1/2}}_{r,\ \xi} = \frac{1}{H_a(r)} \approx 1$$

$r, \xi \Rightarrow b, k$

We generate a plot of $H_a(r)$ as a function of r for various ξ



We choose ξ, r such that $H_a \approx 1$.

(i) $\frac{\omega}{\omega_n} << 1 \Rightarrow \omega_n$ large

(ii) $\xi = 0.707$ gives largest range where $H_a \approx 1$

$\frac{\omega}{\omega_n} << 1$ ω_n is large

Since $\omega_n = \sqrt{k/m}$ & we want large ω_n, choose large k, small m

Since m is small, the accelerometer can be kept small.

When $H_a \approx 1$ Acceleration of object $= X_i \omega^2 = X_r \omega_n^2$

measured by displacement sensor

accelerometer design.

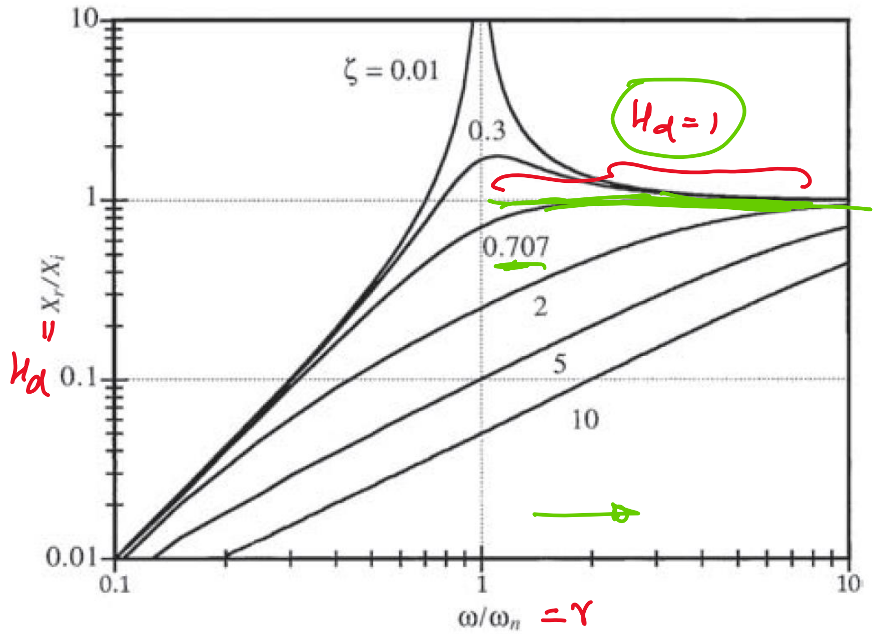
measured by displacement sensor

accelerometer design.

② Use as vibrometer

From (I)
$$\frac{X_r}{X_i} = \frac{r^2}{[(1-r^2)^2 + 4\xi^2 r^2]^{1/2}} = H_d(r)$$

Lets plot $H_d(r)$ as a function of r for various ξ



Choose r, ξ such that $H_d \approx 1$

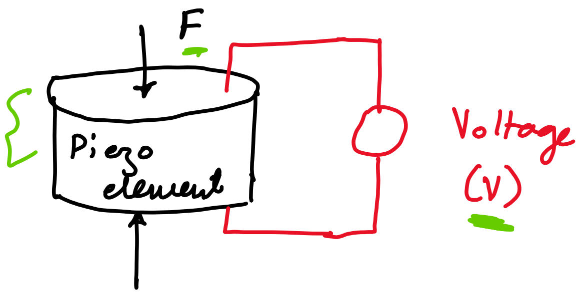
(i) $r >> 1$; $\omega/\omega_n >> 1$ ω_n should be small

(ii) $\xi = 0.707$ gives the largest range for $H_d \approx 1$

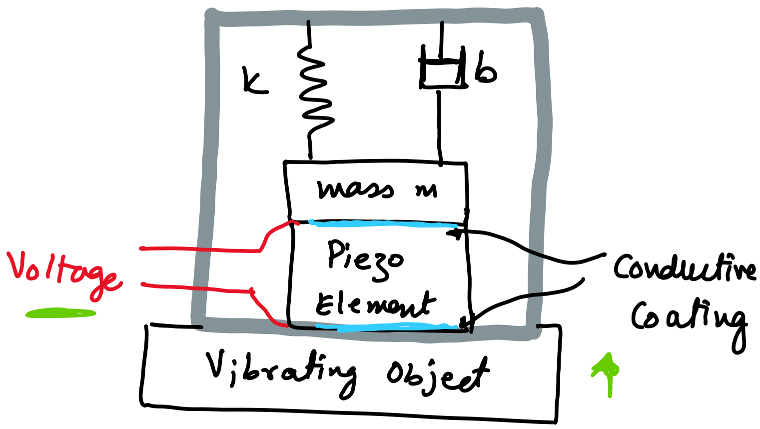
Since $\omega_n = \sqrt{k/m}$, choose small k, large m. As m is large, the vibrometer tends to be big. e.g. seismograph is a vibrometer that measure earthquake intensity

Displacement of object $= X_i = X_r$ displacement sensor

② Piezo-electric accelerometer



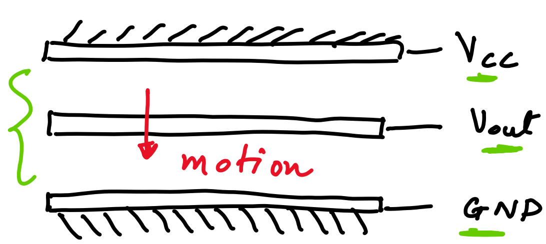
Principle : A piezoelement when subject to a force, it produces a voltage. Piezo-elements include quartz, tourmaline, rochelle salt, lead zirconate, barium titanate



Principle

- object vibrates
mass accelerates exerting force on the piezo element
- force on the piezo produces a voltage output

③ Integrated circuit accelerometer



- Based on capacitors
- top / bottom plates are fixed
- middle plate is attached to the moving object.
- This changes the capacitance & changes V_{out}

IMU
Inertial measurement unit.