Word problems

1. Define the problem
2. Write quasi-logic statements
3. Write Boolean expression
4. Simplify to use minimum number of gates.
5. Draw circuit diagram and/or truth table

EXAMPLE 1

A student in the class uses the following heuristic to decide if the student will attend class or not

The student will attend class (A) on Tuesdays and if the student got at least 6 hrs of sleep (C) or the student will attend class if there is an exam (B) and if the student got at least 6 hours of sleep. (C)

Develop a boolean expression for the students heuristic

① Y = output, attend class or not

A = input, A = 1 Tuesday; A = 0 Not Tuesday

B = input, B = 1 exam; B = 0 No exam

C = input; C = 1 ≥ 6 hr sleep; C = 0 ≤ 6 hrs sleep.

② Attend class (Y = 1) if it is Tuesday

[(A = 1) AND if ≥ 6 hrs sleep (C = 1)]

OR

[if there is an exam (B = 1) AND if ≥ 6 hrs sleep (C = 1)]

③ $Y = A \cdot C + B \cdot C$ ← 1 OR, 2 AND } 2 Chips

$Y = (A + B) \cdot C$ ← 1 OR, 2 AND } 2 chips

(5) Minimize Gates

$Y = (A + B) \cdot C$ ←

$= A \cdot C + B \cdot C$

$P \quad Q$

De Morgans

$\overline{P+Q} = \overline{P} \cdot \overline{Q}$

$\overline{\overline{P+Q}} = \overline{\overline{P} \cdot \overline{Q}}$

$P+Q = \overline{\overline{P} \cdot \overline{Q}}$

$Y = \overline{\overline{A \cdot C} \cdot \overline{B \cdot C}}$ (3, 2, 1)

$P \quad Q$

3 NAND Gates = 1 IC

Example 2

An investor in Mechatronics startups uses the following heuristics to invest in potential new companies

1) The startup is rated for its mechanical design content, either a bad or a good
2) The startup is rated for its business potential, either a bad or a good
3) The sales pitch by the founders of the company. This is rated on a scale of 0 to 3 with 0 being poor and 3 being excellent

The investor invests in the startup if the mechanical design content is good and the sales pitch is rated 3 or if the mechanical design content is good or the business potential is good and the sales pitch is rated either 2 or 3.

Develop a Boolean expression and then a Boolean circuit for the investors heuristics

① Define the problem in words.

Y = output. [=0 not investing ; =1 invest]

A = mechanical design [0 = bad ; 1 = good]

B = business content [0 = bad ; 1 = good]

C, D = rating

C = 0 D = 0 rating 0

C = 0 D = 1 rating 1

C = 1 , D = 0 rating 2

C = 1, D = 1 rating 3

② Write quasi-logical statements

Investor will invest if (Y=1)

(a) mech design is good (A=1) AND sales pitch is 3 (C=1, D=0)

OR

(b) mech. design is good (A=1) OR business potential is good (B=1) AND sales pitch is 2 or 3

(C=1, D=0) (C=1, D=1)

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③ Write a boolean expression

$$Y = A \cdot C \cdot D + (A + B) \cdot (C \cdot \bar{D} + C \cdot D)$$

(a) (b)

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④ Simplify the expression

$$Y = A \cdot C \cdot D + (A+B) \cdot (C \cdot \bar{D} + C \cdot D)$$

$$= A \cdot C \cdot D + (A+B) \cdot C(\bar{D} + D)$$

$$\bar{D} + D = 1$$

$$= A \cdot C \cdot D + (A+B) \cdot C$$

$$= A \cdot C \cdot D + A \cdot C \cdot 1 + B \cdot C$$

$$= A \cdot C \cdot (D+1) + B \cdot C$$

$$D + 1 = 1$$

$$= A \cdot C + B \cdot C \quad - 1 \text{ OR}, 2 \text{ AND} - 2 \text{ ICs}$$

$$= (A+B) \cdot C \quad - 1 \text{ OR}, 1 \text{ AND} - 2 \text{ ICs}$$

The expression is the same as previous example. Using De Morgan's law

$$Y = \overline{A \cdot C} \cdot \overline{B \cdot C} \quad 3 \text{ NAND's} - 1 \text{ IC}$$

Boolean expressions from truth table

Example

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

Write a Boolean expression for Y

There are 2 methods

① Sum-of-products method

e.g. $Y = (A \cdot B) + (\bar{A} \cdot B)$

i) Find rows where output is 1

(ii) Form products of inputs such that you get the required output (=1)

(iii) Sum all expression in (ii)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

(i) Row 2 → $Y=1$ $A=0$ $B=1$

(ii) $Y = \bar{A} \cdot B$

(iii) $Y = \bar{A} \cdot B$ (final)

(2) Product of sum method

e.g. $y = (A+B)\cdot(\bar{A}+B)$

(i) Find the rows with 0's

(ii) Form the sum of inputs such that you get the required output

(iii) Take the product of all boolean expressions in (ii)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

(i) row 1, row 3, row 4

→	Y = 0	Y = 0	Y = 0
→	A = 0	A = 1	A = 1
→	B = 0	B = 0	B = 1

(ii) $A+B$ $\quad \bar{A}+B$ $\quad \bar{A}+\bar{B}$

(iii) $Y = (A+B)\cdot(\bar{A}+B)\cdot(\bar{A}+\bar{B})$

(ANSWER)