Boolean Algebra

Fundamental laws

① OR

$A + 0 = A$ (A = input)

$A + 1 = 1$

$A + A = A$

$A + \bar{A} = 1$

② AND

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot A = A$

$A \cdot \bar{A} = 0$

③ NOT

$\bar{\bar{A}} = A$

② Commutative laws

$$A + B = B + A$$

→ $A \cdot B = B \cdot A$

③ Associative laws

$$(A+B) + C = A + (B+C)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C) = (B \cdot C) \cdot A$$

④ Distributive laws

→ $\underline{A} \cdot (\underline{B+C}) = A \cdot B + A \cdot C$

$\underline{\underline{A + B \cdot C}} = (A+B) \cdot (A+C)$ – Prove this.

① Simplify

② Truth table

To prove: $A + (B \cdot C) \stackrel{?}{=} (A + B) \cdot (A + C)$

A	B	C	$B \cdot C$	$A + B \cdot C$	$A + B$	$A + C$	$(A+B)\cdot(A+C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

5. Other Useful Identities

$$A + (A \cdot B) = A$$

$$A \cdot (A + B) = A$$

$$A + \bar{A} \cdot B = A + B$$

$$(A + B) \cdot (A + \bar{B}) = A$$

$$A \cdot B + B \cdot C + \bar{B} \cdot C = A \cdot B + C$$

$$A \cdot B + A \cdot C + \bar{B} \cdot C = A \cdot B + \bar{B} \cdot C$$

Prove these
1. Simplify
2. Truth table

EXAMPLE 1:

Prove $A + A \cdot B = A$

Left side: $A + A \cdot B$

$$= A \cdot 1 + A \cdot B$$

$$= A \cdot (1 + B)$$

$$= A \cdot 1$$

$$= A \quad = \text{Right side}$$

EXAMPLE 2

Prove this: $(A+B)\cdot(A+\overline{B}) = \underline{A}$

Left side: $\underbrace{(A+B)}_{C}\cdot(A+\overline{B})$

$= C\cdot A + C\cdot\overline{B}$ $\quad \{(A+B)\cdot C = A\cdot C + B\cdot C\}$

$= (A+B)\cdot A + (A+B)\cdot\overline{B}$

$= A + B\cdot A + A\cdot\overline{B} + B\cdot\overline{B}$

$A + A\cdot B + A\cdot\overline{B} + 0$

$= A + A\cdot(\underbrace{B+\overline{B}}_{1})$

$= \underline{A}$ (Right side)

⑥ De Morgans law

$\overline{A+B+C+\dots} = \overline{A}\cdot\overline{B}\cdot\overline{C}\dots$

$\overline{A\cdot B\cdot C\cdot} = \overline{A}+\overline{B}+\overline{C}+\dots$

Convert OR/NOR to AND/NAND

EXAMPLE 3:

Simplify the boolean expression to minimize the number of logical expressions

$$Y = C + A \cdot B + A \cdot C + B \cdot \bar{A}$$

Solution:

3 '+' : 3 OR Gates

3 '·' : 3 AND Gates

1 '‾' : 1 NOT Gate

7432 (1)
7408 (1)
704 (1)

3 chips.

$$Y = C + A \cdot B + A \cdot C + B \cdot \bar{A}$$

$$= C + A \cdot C + B \cdot (A + \bar{A})$$

$$= C + A \cdot C + B \cdot 1$$

$$= 1 \cdot C + A \cdot C + B$$

$$= (1 + A) \cdot C + B$$

$$= B + C$$ — 1 OR Gate

Use 1 7432