(ii) Laasoren Method / Implicit method

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

$$\alpha \left[\frac{T_{i+1,j+1} - 2T_{i+1,j} + T_{i+1,j-1}}{\Delta x^2} \right] = \frac{T_{i+1,j} - T_{i,j}}{\Delta t}$$

Simplify this equation

$$-F\,T_{i+1,j+1} + (1+2F)\,T_{i+1,j} - F\,T_{i+1,j-1} = T_{i,j}$$

$$F = \frac{\alpha \Delta t}{\Delta x^2}$$

Implicit because we cannot solve for $T_{i+1,j+1}$, $T_{i+1,j}$, $T_{i+1,j-1}$ explicitly

Unconditionally stable

EXAMPLE: $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ $(\alpha = 0.2)$

IC: $T(X,0) = 100X(1-X)$

BC: $T(0,t) = T(1,t) = 0$

Assuming $\Delta X = 0.25$; $\Delta t = 0.1$, compute the temperature profile for

$0 \leq t \leq 0.5$ and $0 \leq X \leq 1$

use Laasonen method

$$F = \frac{\alpha \Delta t}{\Delta x^2} = \frac{(0.2)(0.1)}{(0.25)^2} = 0.32$$

$T_{i,j}$ i = time index

j = position index

$t_i = i\Delta t$ $i = 0, 1, 2, 3, 4, 5$

$t_i = 0, 0.1, 0.2, 0.3, 0.4, 0.5$

$x_j = j\Delta x$ $\quad j = 0, 1, 2, 3, 4$

$x_j = 0, 0.25, 0.5, 0.75, 1.0$

Temperature profile

i \ j	t	0	1	2	3	4
		0	0.25	0.5	0.75	1
0	0	0	18.75	25	18.75	0
1	0.1	0	15.595	21.3297	15.595	0
2	0.2	0	13.0397	18.0946	13.0397	0
3	0.3	0	10.9366	15.3012	10.9366	0
4	0.4	0	9.1889	12.9159	9.1889	0
5	0.5	0	7.7281	10.8914	7.7281	0

IC: $T(0,x) = 100x(1-x) = T_{0,j}$

$T_{0,0} = 100(0)(1-0) = 0$

$T_{0,1} = 100(0.25)(1-0.25) = 18.75$

$T_{0,2} = 100(0.5)(1-0.5) = 25$

$T_{0,3} = 100(0.75)(1-0.75) = 18.75$

$T_{0,4} = 100(1)(1-1) = 0$

From (I)

$$-0.32\,T_{1,2} + 1.64\,T_{1,1} = 18.75$$

$$-0.32\,T_{1,3} + 1.64\,T_{1,2} - 0.32\,T_{1,1} = 25$$

$$1.64\,T_{1,3} - 0.32\,T_{1,2} = 18.75$$

$$\begin{bmatrix} 1.64 & -0.32 & 0 \\ -0.32 & 1.64 & -0.32 \\ 0 & -0.32 & 1.64 \end{bmatrix} \begin{bmatrix} T_{1,1} \\ T_{1,2} \\ T_{1,3} \end{bmatrix} = \begin{bmatrix} 18.75 \\ 25 \\ 18.75 \end{bmatrix}$$

$$A \quad X = \quad b$$

$$X = A \backslash b$$

$$T_{1,1} = T_{1,3} = 15.595$$

$$T_{1,2} = 21.33$$

Put $i = 1$; $j = 1, 2, 3$

$$\begin{bmatrix} 1.64 & -0.32 & 0 \\ -0.32 & 1.64 & -0.32 \\ 0 & -0.32 & 1.64 \end{bmatrix} \begin{bmatrix} T_{2,1} \\ T_{2,2} \\ T_{2,3} \end{bmatrix} = \begin{bmatrix} 15.595 \\ 21.33 \\ 15.595 \end{bmatrix}$$

$T_{2,1} = T_{2,3} = 13.0397$

$T_{2,2} = 18.0946$

Repeat for $i = 2, 3, 4$