② Adam-Moulton Method (Implicit)

$$y_{i+1} = y_i + \Delta x\, f_{i+1}$$

$i+1$ not i ⟵ Euler

Adam-Moulton order 1
Adam's 1st close formula

Example: $f_i = y_i^2$

$$y_{i+1} = y_i + \Delta x\, y_{i+1}^2$$

$$y_{i+1} - \Delta x\, y_{i+1}^2 = y_i$$

$$-\Delta x\, y_{i+1}^2 + y_{i+1} - y_i = 0$$

Quadratic equation in y_{i+1}

Example: $f_i = e^{y_i}$

$$y_{i+1} = y_i + \Delta x\, e^{y_{i+1}}$$

? ? ✓

$$y_{i+1} - \Delta x\, e^{y_{i+1}} - y_i = 0$$

$$F(y_{i+1}) = 0$$

Root finding e.g.

$F(y_{i+1}) = 0$ Root finding e.g. Newton Raphson

$$y_{i+1} = y_i + \Delta x \left[\frac{f_i}{2} + \frac{f_{i+1}}{2} \right] + O(\Delta x^3)$$

Adam's 2nd closed formula
Adam–Moulton 2nd order formula

Generalized Adam-Moulton formula

$$y_{i+1} = y_i + \Delta x \sum_{m=0}^{n-1} \beta^*_{nm} f_{i+1-m} + O\left[(\Delta x)^{n+1}\right]$$

TABLE 9.2

The Values of the Coefficient b^*_{nm} for the Adams–Moulton Formulas, for n up to 6

n	β^*	$m=0$	1	2	3	4	5
1	β^*_{1m}	1					
2	$2\beta^*_{2m}$	1	1				
3	$12\beta^*_{3m}$	5	8	−1			
4	$24\beta^*_{4m}$	9	19	−5	1		
5	$720\beta^*_{5m}$	251	646	−264	106	−19	
6	$1440\beta^*_{6m}$	475	1427	−798	482	−173	27

EXAMPLE:

ODE: $\frac{dy}{dx} = f(x,y) = x - y$; IC: $y(x=0) = 1$

Use Adam-Moulton method of order 2 with a step size of $\Delta x = 0.1$ to compute the solution for

$$0 \leq x \leq 0.5$$

$$y_{i+1} = y_i + \Delta x \left[\frac{f_i}{2} + \frac{f_{(i+1)}}{2} \right]$$

$$y_{i+1} = y_i + \frac{\Delta x}{2} \left[x_i - y_i + x_{i+1} - y_{i+1} \right]$$

$$y_{i+1} \left[1 + \frac{\Delta x}{2} \right] = y_i \left[1 - \frac{\Delta x}{2} \right] + \Delta x \left[\frac{x_i + x_{i+1}}{2} \right]$$

$$y_{i+1} = \frac{y_i \left[1 - \frac{\Delta x}{2} \right] + \Delta x \left[\frac{x_i + x_{i+1}}{2} \right]}{1 + \frac{\Delta x}{2}}$$

$$y_{i+1} = \frac{y_i\left[1-\frac{\Delta x}{2}\right] + \Delta x\left[\frac{x_i + x_{i+1}}{2}\right]}{1+\frac{\Delta x}{2}}$$

Put $i=0$

$$y_1 = \frac{y_0\left(1-\Delta x/2\right) + \Delta x\left[x_0 + x_1\right]/2}{1+\Delta x/2}$$

$$y_1 = \frac{(1)\left[1-0.1/2\right) + 0.1\left[0+0.1\right]/2}{1+0.1/2}$$

$$y_1 = 0.9095$$

$$i=1 \;;\; y_2 = \frac{y_1\,(1-\Delta x/2) + \Delta x\,(x_1+x_2)/2}{1+\Delta x/2}$$

$$y_2 = \frac{0.9095\,(1-0.5/2) + 0.5\,(0.1+0.2)/2}{1+0.1/2}$$

$$y_2 = 0.8372$$

x_i	y_i	y_{i+1}	Method
0	1	0.9095	Adam-Moulton
0.1	0.9095	0.8372	Adam- Moulton
0.2	0.8372	0.7813	Adam – Moulton
0.3	0.7813	0.7402	Adam - Moulton
0.4	0.7402	0.7126	Adam- Moulton

↑ Solution