Multi-step methods

- They need information at multiple time steps.

① Adam-Bashforth method: (Explicit method)

$$\boxed{y_{i+1} = y_i + \Delta x \left[\frac{3}{2} f(x_i, y_i) - \frac{1}{2} f(x_{i-1}, y_{i-1}) \right] + O(\Delta x^3)}$$

Adam-Bashforth 2nd order
Adam's 2nd open formula

$$\boxed{y_{i+1} = y_i + \Delta x \left[\frac{23}{12} f(x_i, y_i) - \frac{16}{12} f(x_{i-1}, y_{i-1}) + \frac{5}{12} f(x_{i-2}, y_{i-2}) \right] + O(\Delta x^4)}$$

Adam-Bashforth 3rd order
Adam's 3rd open formula

Generalized Adam-Bashforth method

$$\boxed{y_{i+1} = y_i + \Delta x \sum_{m=1}^{n} \beta_{nm} f_{i+1-m} + \left[O(\Delta x)^{n+1}\right]}$$

where $f_{i+1-m} = f(x_{i+1-m}, y_{i+1-m})$

TABLE 9.1
The Values of the Coefficient β_{nm} for the Adams–Bashforth Method, for n up to 6

n	⊠	$m = 1$	2	3	4	5	6
1	β_{1m}	1					
2	$2\ \beta_{2m}$	3	–1				
3	$12\ \beta_{3m}$	23	–16	5			
4	$24\ \beta_{4m}$	55	–59	37	–9		
5	$720\ \beta_{5m}$	1901	–2774	2616	–1274	251	
6	$1440\ \beta_{6m}$	4277	–7923	9982	–7298	2877	–475

EXAMPLE:

ODE : $\frac{dy}{dx} = f(x,y) = x - y$; IC : $y(x=0) = 1$

Use Adam-Bashforth method of order 2 with a step size of $\Delta x = 0.1$ to compute the solution for

$$0 \le x \le 0.5$$

Adam-Bashforth 2nd order

$$y_{i+1} = y_i + \Delta x \left[\frac{3}{2} \underset{f(x_i, y_i)}{f_i} - \frac{1}{2} \underset{f(x_{i-1}, y_{i-1})}{f_{i-1}} \right]$$

$x_i = -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5$

$$\cancel{y_1 = y_0 + \Delta x \left[\frac{3}{2} f_0 - \frac{1}{2} f_{-1} \right]}$$

Issue: data not available

$$y_2 = y_1 + \Delta x \left[\frac{3}{2} f_1 - \frac{1}{2} f_0 \right]$$

Use single-step method e.g. Euler to compute this.

Use Euler's method to compute y_1

$$y_1 = y_0 + \Delta x \, f_0 = (x-y)$$

$$= 1 + 0.1\,(0-1)$$

$$y_1 = 0.9$$

Use Adam - Bashforth for $i \geq 1$

$$y_2 = y_1 + \Delta x \left[\frac{3}{2} f_1 - \frac{1}{2} f_0\right]$$

$$= y_1 + \Delta x \left[\frac{3}{2}(x_1 - y_1) - \frac{1}{2}(x_0 - y_0)\right]$$

$$y_2 = 0.9 + 0.1\left[\frac{3}{2}(0.1 - 0.9) - \frac{1}{2}(0-1)\right]$$

$$y_2 = 0.83$$

x_i	y_i	y_{i+1}	Method
0	1	0.9	Euler
0.1	0.9	0.83	Adam-Bashforth
0.2	0.83	0.7755	Adam-Bashforth
0.3	0.7755	0.7357	Adam-Bashforth
0.4	0.7357	0.7091	Adam-Bashforth

solution