~~Modified Euler method~~, Heun's method

Predictor – Corrector method

Predict:

$$y^*_{i+1} = y_i + \Delta x \, F(x_i, y_i)$$ – same as Euler

Correction

$$y_{i+1} = y_i + \frac{\Delta x}{2}\left[f(x_i, y_i) + f(x_i, y^*_{i+1})\right]$$

Truncation error $\propto O(\Delta x^3)$

Total error $\propto O(\Delta x^2)$

EXAMPLE: $\frac{dy}{dx} = f(x,y) = -3y$

IC $y(x=0) = 1$

Using a step size $= 0.1$ compute the solution for

① By hand $0 \le x \le 0.3$

② Using code $0 \le x \le 1$

Use Heun's method

$\frac{dy}{dx} = -3y$ (Analytical solution)

$$\int \frac{dy}{y} = -3\int dx$$

$$\ln y = -3x + C$$

$x=0 \quad y=1 \quad \ln(1) = -3(0) + C$

$\Rightarrow C = 0$

$\ln y = -3x \Rightarrow \boxed{y = e^{-3x}}$

Heun's method

$$y^*_{i+1} = y_i + \underbrace{\Delta x}_{0.1}\, f(x_i, y_i) \qquad dy^* = -3y_i$$

$$y_{i+1} = y_i + \frac{\Delta x}{2}\left[f(x_i, y_i) + f(x_i, y^*_{i+1}) \right] \qquad dy$$

$f(x_i, y_i) = -3y_i$ $\qquad$ $f(x_i, y^*_{i+1}) = -3y^*_{i+1}$

x_i	y_i	dy^*	y^*_{i+1}	dy	y_{i+1}	y_{exact}
0	1	-0.3	0.7	-0.255	0.745	0.7408
0.1	0.745	-0.2235	0.5215	-0.19	0.555	0.5488
0.2						