① Euler's method:

consider $\frac{dy}{dx} = f(x, y)$

initial condition $y(x = x_0) = y_0$

e.g. $\frac{dv}{dt} = a$ 　　 v = velocity
a = acceleration

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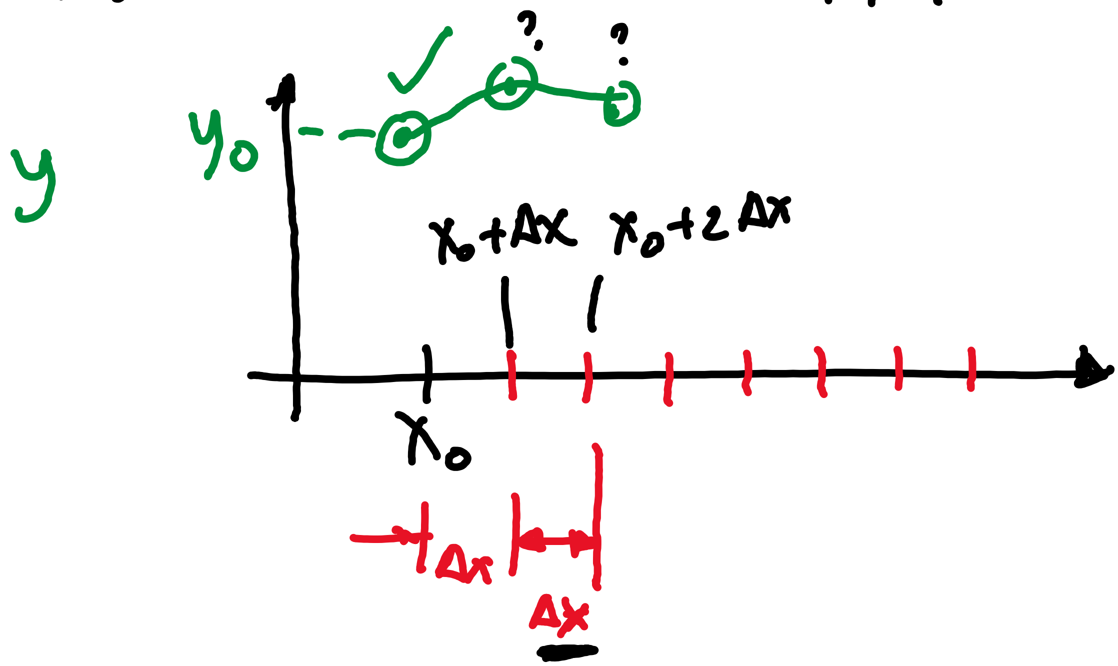
choose a step size Δx

$x_i = x_0 + i\,\Delta x$ 　　 $i = 0, 1, 2, \ldots n$



$$\frac{dy}{dx} = f(x,y) \Rightarrow \frac{y_{i+1}-y_i}{\Delta x} = f(x_i,y_i)$$

Forward difference formula

Solve for y_{i+1}

$$\boxed{y_{i+1} = y_i + \Delta x\, f(x_i,y_i)}$$ Euler's method

Truncation error

Taylor series

ignored

$$y_{i+1} = y_i + \Delta x \frac{dy}{dx} + \frac{\Delta x^2}{2!}\frac{d^2y}{dx^2} + \cdots$$

$$= y_i + \Delta x\, f(x,y) + O(\Delta x^2)$$

Trunction error $= y_{i+1} - y_i - \Delta x\, f(x_i,y_i)$

$$\boxed{TE = O(\Delta x^2)}$$

Total error $= \int TE\, \underline{dx} = O(\Delta x^2)\left(\frac{1}{\Delta x}\right) = O(\Delta x)$

EXAMPLE: $\frac{dy}{dx} = f(x,y) = -3y$

IC $y(x=0) = 1$

Using a step size $= 0.1$ compute the solution for

① By hand $0 \le x \le 0.3$

② Using code $0 \le x \le 1$

Euler's method: $y_{i+1} = y_i + \Delta x \, f(x_i, y_i)$

$\Delta x = 0.1$ $x_0 = 0$; $x_i = x_0 + i\Delta x$

$i = 0, 1, 2, 3 \Rightarrow$

x_0,	$x_0 + \Delta x$,	$x_0 + 2\Delta x$,	$x_0 + 3\Delta x$
0,	0.1 ,	0.2 ,	0.3
x_0,	x_1	x_2 ,	x_3

i	x_i	y_i	$\Delta x\, f(x_i, y_i)$ $-3\Delta x\, y_i$	$y_{i+1} = y_i + \Delta x\, f(x_i, y_i)$ y_{i+1}
0	0	1	$-3(0.1)(1) = -0.3$	$1 - 0.3 = 0.7$ ✓
1	0.1	0.7	$-3(0.1)(0.7) = -0.21$	$0.7 - 0.21 = 0.49$ ✓
2	0.2	0.49	$-3(0.1)(0.49) = -0.147$	$0.49 - 0.147 = 0.343$ ✓