Ordinary Differential Equations (ODE)

ODE:

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots \frac{d^ny}{dx^n}\right) = 0 \quad \text{— I}$$

x – independent variable

y – dependent variable

To solve an ODE or order 'n', we need 'n' conditions.

e.g. $\underbrace{y(0), \frac{dy}{dx}(0), \frac{d^2y}{dx^2}(0) \ldots \frac{d^ny}{dx^n}(0)}$ are

Initial value problem; all conditions at $x=0$
(IVP)

$\underbrace{y(5), \frac{dy}{dx}(0), \frac{d^2y}{dx^2}(0), \ldots \frac{d^ny}{dx^n}(0)}$

Boundary value problem; atleast one not at $x=0$
(BVP)

IVP is easier to solve than a BVP

To solve ODE of order n, break it down to 'n' first order equations.

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots \frac{d^ny}{dx^n}\right) = 0$$

isolate

$$\frac{d^ny}{dx^n} = f\left(x, y, \frac{dy}{dx} \ldots\right)$$

$Y_0 = y$	$\frac{dY_0}{dx} = \cancel{\frac{dy}{dx}} = Y_1$
$Y_1 = \frac{dy}{dx}$	$\frac{dY_1}{dx} = \cancel{\frac{d^2y}{dx^2}} = Y_2$
$Y_2 = \frac{d^2y}{dx^2}$	$\vdots$
$\vdots$	$\frac{dY_{n-1}}{dx} = Y_n$
$Y_n = \frac{d^ny}{dx^n}$	$\frac{dY_n}{dx} = f(x, Y_0, Y_1, Y_2, \ldots Y_{n-1})$

EXAMPLE:
$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + y + x^2 = 0$$

Rewrite as 1st order equations

isolate

$$\frac{d^3y}{dx^3} = -2\frac{d^2y}{dx^2} - y - x^2 \quad \text{— (a)}$$

$Y_0 = y$; $Y_1 = \frac{dy}{dx}$; $Y_2 = \frac{d^2y}{dx^2}$; $\cancel{Y_3 = \frac{d^3y}{dx^3}}$

$$\frac{dY_0}{dx} = \frac{dy}{dx} = Y_1 \quad \text{— (1)}$$

$$\frac{dY_1}{dx} = \frac{d^2y}{dx^2} = Y_2 \quad \text{— (2)}$$

$$\frac{dY_2}{dx} = \frac{d^3y}{dx^3} = -2\frac{d^2y}{dx^2} - y - x^2 \quad \text{from (a)}$$

$$\frac{dY_2}{dx} = -2Y_2 - Y_0 - x^2 \quad \text{— (3)}$$

(1) – (2) – (3) are the necessary equations