Truncation errors

let $y(x) = \int_a^x f(x)\,dx \longrightarrow$

$y' = f(x)$ — ①

$y'' = f'(x)$ — ②

$y''' = f''(x)$ — ③

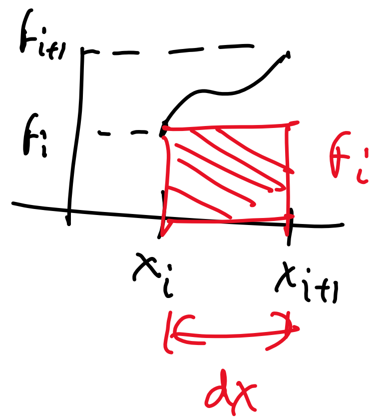
Taylor series about $y_i = y(x_i)$

$$y_{i+1} = y_i + \Delta x \, \underline{y_i'} + \frac{\Delta x^2}{2!} y_i'' + \frac{\Delta x^3}{3!} y_i''' + O[(\Delta x^4)]$$

① ↓ ② ↓ ③ ↓

$$④ \rightarrow \underline{y_{i+1}} = \underline{y_i} + \Delta x \, f_i + \frac{\Delta x^2}{2!} f_i' + \frac{\Delta x^3}{3!} f_i'' + O[\Delta x^4]$$

$$\overbrace{y_{i+1}}^{\text{exact}} - y_i = \int_{x_i}^{x_{i+1}} f(x)$$



Rectangular rule = $\underline{\underline{\Delta x \, f_i}}$

$$\text{Truncation error} = \underbrace{(y_{i+1} - y_i)}_{\text{exact}} - \underbrace{\Delta x f_i}_{\text{Rectangular rule}}$$

$$= \frac{\Delta x^2}{2!} f_i' + \frac{\Delta x^3}{3!} f_i'' + O[\Delta x^4]$$

Truncation error $\propto \Delta x^2$ **Rectangular**

proportional

Rule	Truncation Error (TE)
Rectangular	$\underline{\Delta x^2}$
$\underline{\text{Trapezoidal}}$	$\underline{\underline{\Delta x^3}}$
Simpson's 1/3	Δx^5
Simpson's 3/8	$\underline{\Delta x^5}$

Implications

Tolerance : $\underline{\underline{1e^{-5}}}$ (This is how much you want your calculation to be)

Simpsons	$\underline{\Delta x} = 10^{-1} = 0.1$	$TE = \Delta x^5 = 10^{-5}$
Trapezoidal	$\Delta x = 10^{-5/3} = 0.02$	$TE = \Delta x^3 = 10^{-5}$
Rectangular	$\Delta x = 10^{-5/2} = 0.003$	$TE = \Delta x^2 = 10^{-5}$

same accuracy