③ Tips for linearizing 'non-linear' functions

ⓐ $y = a e^{bx}$

Take $\log_e$ to the base e

$$\underbrace{\log_e y}_{z} = \underbrace{\log_e a}_{\bar{a}} + \underbrace{\log_e e^{bx}}_{bx}$$

$$z = \bar{a} + bx$$

x	y	$z = \log_e y$
x	⋮	⋮
⋮		

Use linear regression to solve for $\bar{a}$, b

$\bar{a} = \log_e a$

$a = e^{\bar{a}}$

ⓑ You could use the same method for

$$y = a x^b$$

$$\underbrace{\log y}_{z} = a + b \underbrace{\log x}_{\bar{x}}$$

$$z = a + b\bar{x}$$

x	y	$\bar{x}$	z

(c) $y = \dfrac{c_1}{c_2 + x}$

Multiply by $\dfrac{c_2 + x}{y}$

$$\left(\frac{c_2 + x}{\cancel{y}}\right) \cancel{y} = \left(\frac{\cancel{c_2 + x}}{y}\right) \frac{c_1}{\cancel{(c_2 + x)}}$$

$$c_2 + x = \frac{c_1}{y}$$

Divide by c_1 $\Rightarrow$ $\dfrac{c_2}{c_1} + \underbrace{\left(\dfrac{1}{c_1}\right)}x = \dfrac{1}{y}$

x	y	$z = 1/y$

$$b + a x = z$$

(d) Similar method can be used for

$$y = c_1 + \frac{c_2}{x}$$

Nonlinear fitting

x	y
0.0000	0.5790
0.2000	0.7203
0.4000	0.6340
0.6000	0.7973
0.8000	1.2253
1.0000	1.5090
1.2000	1.9423
1.4000	1.7626
1.6000	2.9008
1.8000	2.6405
2.0000	4.0626

Fit the equation

$$y = a e^{bx}$$

to the data shown on the right

$$\underbrace{\log y}_{z} = \underbrace{\log a}_{\bar{a}} + \underbrace{bx}_{bx}$$

$$z = \bar{a} + bx$$

$$\bar{a} = \frac{(\Sigma x_i z_i)(\Sigma x_i) - (\Sigma x_i^2)(\Sigma z_i)}{\left[(\Sigma x_i^2) - n\, \Sigma x_i^2\right]}$$

$$b = \frac{\Sigma x_i \Sigma z_i - n\, \Sigma x_i z_i}{\left[\Sigma x_i^2 - n\, \Sigma x_i^2\right]}$$

$n = 11$

x	y	$z = \log y$	x^2	xz
$\vdots$	$\vdots$			

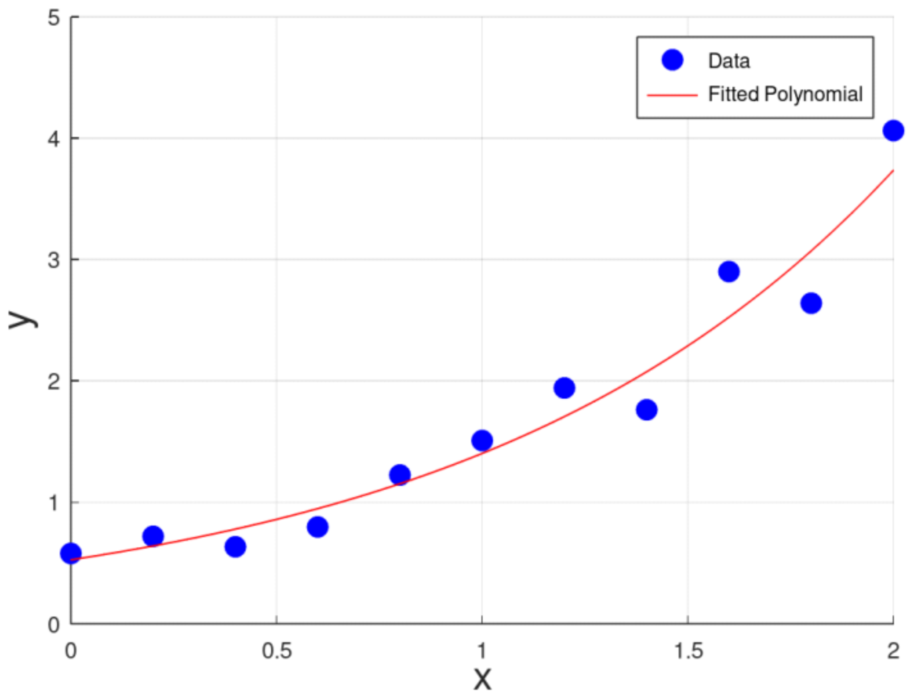
x	z	x^2	xz
0.0000	-0.5464	0.0000	-0.0000
0.2000	-0.3281	0.0400	-0.0656
0.4000	-0.4557	0.1600	-0.1823
0.6000	-0.2266	0.3600	-0.1359
0.8000	0.2032	0.6400	0.1625
1.0000	0.4115	1.0000	0.4115
1.2000	0.6639	1.4400	0.7967
1.4000	0.5668	1.9600	0.7935
1.6000	1.0650	2.5600	1.7040
1.8000	0.9710	3.2400	1.7478
2.0000	1.4018	4.0000	2.8036
Σx	Σz	Σx^2	Σxz

$$\bar{a} = -0.6407$$

$$b = 0.9794$$

$$\bar{a} = \log_e a \Rightarrow a = e^{\bar{a}} = e^{-0.6407} = 0.5270$$

$$\boxed{y = a e^{bx} = 0.5270\, e^{0.9794x}}$$


Data
Fitted Polynomial
y
x
0
1
2
3
4
5
0
0.5
1
1.5
2