① Linear Regression

$$F(x) = ax + b \qquad a, b \text{ constants}$$

$$\min_{a,b} E = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

From calculus:

$$\frac{dE}{da} = \sum_{i=1}^{n} 2(\underline{y_i - ax_i - b})(-x_i) = 0$$

$$\Rightarrow -2 \sum_{i=1}^{n} (x_i y_i - a x_i^2 - b x_i) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - a \sum_{i=1}^{n} x_i^2 - b \sum_{i=1}^{n} x_i = 0$$

① — $$\boxed{a \sum_{i=1}^{n} x_i^2 + b \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i}$$

$$\frac{dE}{db} = \sum_{i=1}^{n} 2(\underline{\underline{y_i - ax_i - b}})(-1) = 0$$

$$\frac{df}{db} = \sum_{i=1}^{n} 2(y_i - ax_i - b)(-1) = 0$$

$$= -2\left[\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i - bn\right] = 0$$

$$② \quad \boxed{a\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i}$$

Solve for a, b from ①, ②

$$\boxed{\begin{aligned} b &= \frac{(\sum x_i y_i)(\sum x_i) - (\sum x_i^2)(\sum y_i)}{(\sum x_i)^2 - n(\sum x_i^2)} \\ \\ a &= \frac{\sum x_i \sum y_i - n\sum x_i y_i}{(\sum x_i)^2 - n(\sum x_i^2)} \end{aligned}}$$

Fit a straight line to the data shown on the right

x	y
-2.0000	-2.7664
-1.6000	-1.8747
-1.2000	-2.7127
-0.8000	-1.7671
-0.4000	0.1816
0.0000	1.0231
0.4000	2.0852
0.8000	1.3998
1.2000	3.6906
1.6000	3.0207
2.0000	4.9748

Solution

$$y = f(x) = ax + b$$

$$b = \frac{\left(\sum x_i y_i\right)\left(\sum x_i\right) - \left(\sum x_i^2\right)\left(\sum y_i\right)}{\left(\sum x_i\right)^2 - n\left(\sum x_i^2\right)}$$

$$a = \frac{\sum x_i \sum y_i - n \sum x_i y_i}{\left(\sum x_i\right)^2 - n\left(\sum x_i^2\right)}$$

III

$n = 11$ (# data points)

x_i	y_i	x_i^2	$x_i y_i$
-2	-2.7664	4	5.5328
-1.6	-1.8747	2.56	2.9995
-1.2	-2.7127	1.44	3.2552
-0.8	-1.7671	0.64	1.4137
-0.4	0.1816	0.16	-0.0726
0	1.0231	0	0
0.4	2.0852	0.16	0.8341
0.8	1.3998	0.64	1.1199
1.2	3.6906	1.44	4.4287
1.6	3.0207	2.56	4.8331
2.0	4.9748	4.0	9.9496
0	7.255	17.6	34.2939

$\Sigma x_i = 0$

$\Sigma y_i = 7.255$

$\Sigma x_i y_i = 34.2939$

$\Sigma x_i^2 = 17.6$

$n = 11$

Substitute in III

$a = 1.9485$
$b = 0.6595$

$$\boxed{y = ax + b = 1.9485x + 0.6595}$$