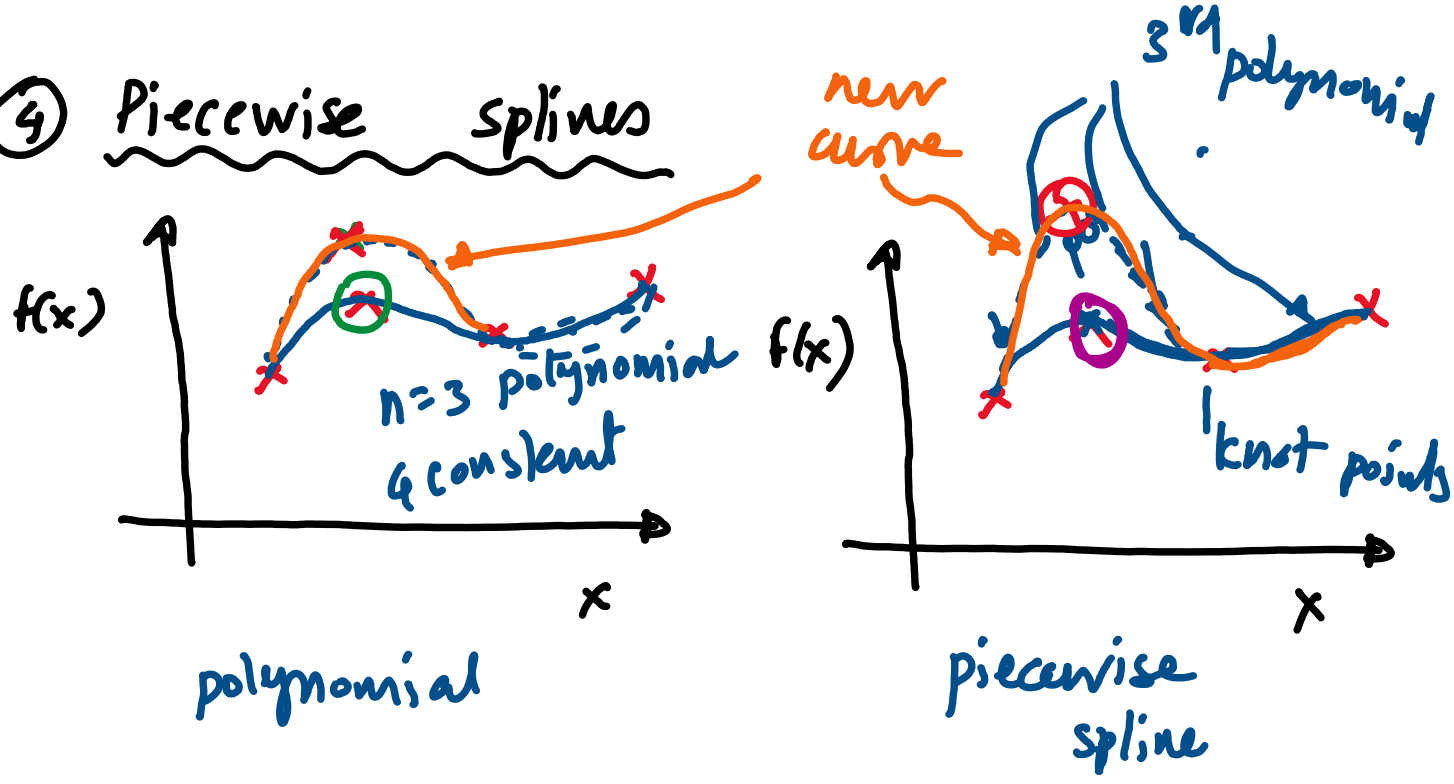
④ Piecewise splines



Given data points

$[x_0, f(x_0)], [x_1, f(x_1)], \ldots [x_n, f(x_n)]$

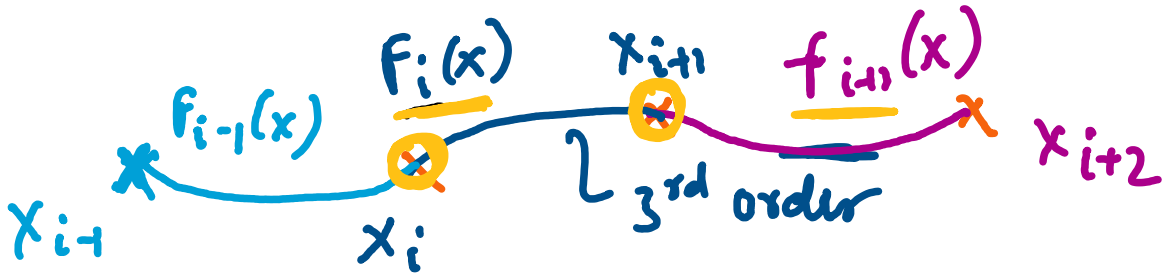
$(n+1)$ data points

Assume a 3rd order polynomial passes through points x_i and x_{i+1}

$$f_i(x) = a_{i0} + a_{i1}x + a_{i2}x^2 + a_{i2}x^3 \checkmark$$

function **spline** Use this ✓

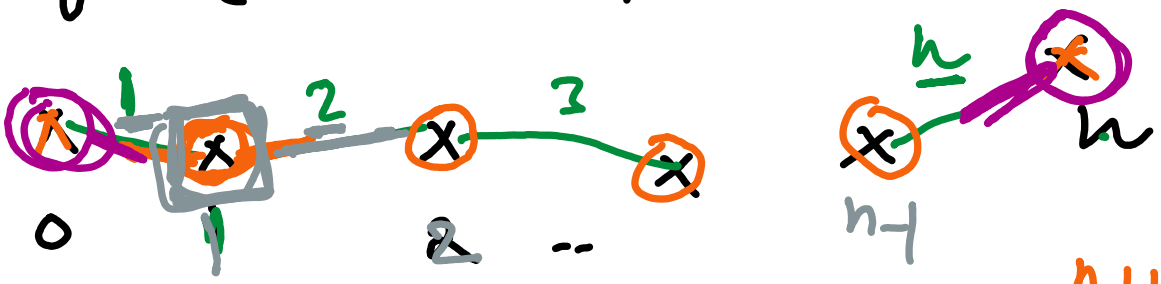
$$f_i(x) = a_{i0} + a_{i1}(x - x_i) + a_{i2}(x - x_i)^2 + a_{i3}(x - x_i)^3$$



$$f_i(x) = a_{i0} + a_{i1}(x - x_i) + a_{i2}(x - x_i)^2 + a_{i3}(x - x_i)^3$$

- $f_i(x_i) = a_{i0}$
- $f_i(x_{i+1}) = a_{i0} + a_{i1}(x_{i+1} - x_i) + a_{i2}(x_{i+1} - x_i)^2 + \cdots + a_{i3}(x_{i+1} - x_i)^3$
- $f_i'(x_{i+1}) = f_{i+1}'(x_{i+1})$
- $f_i''(x_{i+1}) = f_{i+1}''(x_{i+1})$
- $f_i'(x_i) = f_{i-1}'(x_i)$
- $f_i''(x_i) = f_{i-1}''(x_i)$

Compute constants & equations for (n+1) knot points



$n+1-2 = (n-1)$

n - polynomial

4 - constants / polynomial

$4n$ - constants

$2(n-1)$ condition on intermediate points

$n-1$ f' condition

$n-1$ f'' condition

2 conditions at end-points

$2n-2 + n-1 + n-1 = 4n-2$ equations

We have $4n$ constants, but only $4n-2$ equations.

We need to specify 2 more conditions in order to fit the spline(s)

Here are different ways to specify 2 conditions

① Natural spline:

$$f''(x_0) = 0 \quad \text{and} \quad f''(x_n) = 0$$

② Clamped condition: spline ✓

$$f'(x_0) = 0 \quad \text{and} \quad f'(x_n) = 0$$

③ Not-a-knot default spline

$$f_1'''(x_1) = f_2'''(x_1) \; ; \; f_{n-1}'''(x_{n-1}) = f_n'''(x_{n-1})$$