

## Lagrange interpolation

This method like the polynomial fit uses  $n^m$  order polynomial but uses a smarter way of writing the polynomial that avoids the computation of the inverse.

### EXAMPLE

Given the following data

|   |   |   |   |
|---|---|---|---|
| x | 0 | 0 | 2 |
| y | 2 | 0 | 4 |

Generate a polynomial fit to the data using Lagrange interpolation.

Solution :

$$y = b_0 \frac{(x-1)(x-2)}{(0-1)(0-2)} + b_1 \frac{(x-0)(x-2)}{(1-0)(1-2)} + b_2 \frac{(x-0)(x-1)}{(2-0)(2-1)}$$

|   |   |   |   |
|---|---|---|---|
| x | 0 | ? | 2 |
| y | 2 | 0 | 4 |

$$y = b_0 \frac{(x-1)(x-2)}{1} + b_1 (x-0)(x-2) \\ + b_2 (x-0)(x-1)$$


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Substituting

$$x=0 ; y=2$$

$$2 = b_0 (0-1)(0-2) + 0 + 0 \Rightarrow b_0 = 1$$

$$x=1 ; y=0$$

$$0 = b_1 (1-0)(1-2) \Rightarrow b_1 = 0$$

$$x=2 ; y=4$$

$$4 = b_2 (2-0)(2-1) \Rightarrow b_2 = 2$$


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$$y = 1(x-1)(x-2) + 0 + 2(x-0)(x-1)$$

$$= x^2 - 3x + 2 + 2x^2 - 2x$$

$$\boxed{y = 2x^2 - 5x + 2}$$