① Exact fit with n-degree polynomial

Given $(n+1)$ data points : $(x_0, y_0), (x_1, y_1), \ldots$
$\ldots (x_n, y_n)$

n^{th} order polynomial

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

$a_0, a_1, \ldots a_n$ are unknown constants

$$y_0 = a_0 + a_1 x_0 + a_2 x_0^2 + \ldots \quad a_n x_0^n \quad - ①$$

$$y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + \ldots \quad a_n x_1^n \quad - ②$$

$$\vdots \qquad \vdots$$

$$y_n = a_0 + a_1 x_n + a_2 x_n^2 + \ldots \quad a_n x_n^n \quad - (n+1)$$

$$\underbrace{\begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^n \\ \vdots & & & & & \\ 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^n \end{bmatrix}}_{(n+1)(n+1)} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}}_{(n+1)\times 1} = \underbrace{\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}}_{(n+1)\times 1}$$

$$\underbrace{A}_{(n+1)(n+1)} \; \underbrace{X}_{(n+1)\times 1} = \underbrace{b}_{(n+1)\times 1}$$

Solve for X

Gauss-Jordan
Gauss-Seidel
$\vdots$

EXAMPLE

Given the following data

x	0	1	2
y	2	0	4

Generate a polynomial fit to the data

Solution

3 data points

2 degree polynomial

$$\boxed{y = a_0 + a_1 x + a_2 x^2}$$

Substitute

$$2 = a_0 + a_1(0) + a_2(0)^2$$
$$0 = a_0 + a_1(1) + a_2(1)^2$$
$$4 = a_0 + a_1(2) + a_2(2)^2$$

$$\begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0^2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$a_0 = 2$; $a_1 = -5$; $a_2 = 3$ $\Rightarrow$ $\boxed{y = 2 - 5x + 3x^2}$