# Relaxation method

This method can help improve the rate of convergence of the Gauss-Seidel method

Update rule

$$x^{i+1} = (1-w)\, x^{i} + w\, \bar{x}^{i+1}$$

previous iteration (under $x^{i}$)

Gauss-Seidel update (under $\bar{x}^{i+1}$)

$w$ - user chosen constant

$0 < w < 1$ Successive under relaxation (SUR)

$1 < w < 2$ Successive over relaxation (SOR)

EXAMPLE:

Solve for $x_1, x_2, x_3$ using Relaxation method

$$2x_1 + x_2 + x_3 = 7 \quad -(1)$$

$$x_1 - 3x_2 + x_3 = -2 \quad -(2)$$

$$2x_1 + 2x_2 - x_3 = 3 \quad -(3)$$

Use an initial guess $x_1^0 = x_2^0 = x_3^0 = 0$, $W = 0.8$

Solve up to 3 iterations.

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$$x_1 = (7 - x_2 - x_3)/2$$

$$x_2 = (-2 - x_1 - x_3)/(-3)$$

$$x_3 = (3 - 2x_1 - 2x_2)/(-1)$$

$$x_1^0 = x_2^0 = x_3^0 = 0$$

Iteration 1

Gauss-Seidel

$$x_1 = (7 - x_2 - x_3)/2$$

$$x_2 = (-2 - \underline{x_1} - \underline{x_3})/(-3)$$

$$x_3 = (3 - 2\underline{x_1} - 2\underline{x_2})/(-1)$$

$$\bar{x}_1^1 = (7-0-0)/2$$
$$= 3.5$$

$$x_1^1 = (1-w)\underline{x_1^0} + w\,\bar{x}_1^1$$
$$= (1-0.8)0 + 0.8(3.5) = \underline{2.8}$$

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$$\bar{x}_2^1 = (-2 - 2.8 - 0)/(-3) = \underline{1.6}$$

$$x_2^1 = (1-w)x_2^0 + w\,\bar{x}_2^1$$
$$= (1-0.8)0 + 0.8(1.6) = \underline{1.28}$$

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$$\bar{x}_3^1 = (3 - 2(2.8) - 2(1.28))/(-1) = \underline{5.16}$$

$$x_3^1 = (1-w)x_3^0 + w\,\bar{x}_3^1$$
$$= (1-0.8)(0) + 0.8(5.16) = 4.128$$

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$$x_1^1 = 2.8\ ;\ x_2^1 = 1.28\ ;\ x_3^1 = 4.128$$

compute these for iterations 2 and 3.

Check your calculations with the table given below

| iteration | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| 1 | 2.8 | 1.28 | 4.128 |
| 2 | 1.1968 | 2.2093 | 3.8753 |
| 3 | 0.6055 | 2.1701 | 2.8160 |