② Gauss - Jordan Elimination

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Use a_{11} as a pivot to eliminate a_{21}
(same as gauss elimination)

$$1\,x_1 + \frac{a_{12}}{a_{11}}x_2 = \frac{b_1}{a_{11}}$$
$$+ a'_{22}x_2 = b'_2$$

① pivot = 1
② elimination

Use a'_{22} to eliminate (a_{12}/a_{11})

$$x_1 + 0 = b''_1$$
$$\left(\frac{a'_{22}}{a'_{22}}\right)x_2 = \frac{b'_2}{a'_{22}}$$

$\frac{a'_{22}}{a'_{22}} = 1$

$$\begin{cases} x_1 + 0 = b''_1 \\ x_2 = \dfrac{b'_2}{a'_{22}} \end{cases}$$

EXAMPLE:

Solve the following set of equations

$$2x_1 + x_2 + x_3 = 7$$

$$x_1 - 3x_2 + x_3 = -2$$

$$2x_1 + 2x_2 - x_3 = 3$$

Use Gauss-Jordan elimination

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 7 \\ 1 & -3 & 1 & -2 \\ 2 & 2 & -1 & 3 \end{array}\right]$$

A | b

goal

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & c_1 \\ 0 & 1 & 0 & c_2 \\ 0 & 0 & 1 & c_3 \end{array}\right]$$

$R_1 = R_1/2$

$$\begin{bmatrix} 1 & 0.5 & 0.5 & 3.5 \\ 1 & -3 & 1 & -2 \\ 2 & 2 & -1 & 3 \end{bmatrix}$$

$R_2 = R_2 - \frac{1}{1} R_1$; $R_3 = R_3 - \frac{2}{1} R_1$

$$\begin{bmatrix} 1 & 0.5 & 0.5 & 3.5 \\ 0 & -3.5 & 0.5 & -5.5 \\ 0 & 1 & -2 & -4 \end{bmatrix}$$

$R_2 = R_2/-3.5$

$$\begin{bmatrix} 1 & 0.5 & 0.5 & 3.5 \\ 0 & 1 & -0.1428 & 1.5714 \\ 0 & 1 & -2 & -4 \end{bmatrix}$$

$R_1 = R_1 - \frac{0.5}{1} R_2$; $R_3 = R_3 - \frac{1}{1} R_2$

$$\begin{bmatrix} 1 & 0 & 0.5714 & 2.7143 \\ 0 & 1 & -0.1428 & 1.5714 \\ 0 & 0 & -1.8572 & -5.5714 \end{bmatrix}$$

$$R_3 = R_3 / -1.8572$$

$$\begin{bmatrix} 1 & 0 & 0.5714 & 2.7143 \\ 0 & 1 & -0.1428 & 1.5714 \\ 0 & 0 & 1 & 3.0001 \end{bmatrix}$$

$$R_1 = R_1 - \frac{0.5714}{1} R_3 \; ; \quad R_2 = R_2 - \frac{(-0.1428)}{1} R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1.0001 \\ 0 & 1 & 0 & 1.9998 \\ 0 & 0 & 1 & 3.0001 \end{array}\right]$$

I $\quad \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

$x_1 = 1.0001$
$x_2 = 1.9998$
$x_3 = 3.0001$

Actual
$x_1 = 1$
$x_2 = 2$
$x_3 = 3$