Gaussian elimination with partial pivoting

① Partial pivoting

Use the largest value in a column as a pivot

② Elimination

③ Back-substitution

EXAMPLE:

Solve using Gauss elimination

$$\begin{aligned} x_2 + x_3 &= 3 \\ 2x_1 + x_2 &= 8 \\ x_1 + 2x_2 + x_3 &= 8 \end{aligned}$$

$$\begin{bmatrix} 0 & 1 & 1 & 3 \\ 2 & 1 & 0 & 8 \\ 1 & 2 & 1 & 8 \end{bmatrix}$$

— Augmented matrix.

R1 C2 has the largest value (2). Use this as the pivot

$R_1 \leftrightarrow R_2$ (Partial pivot)

$$\begin{bmatrix} 2 & 1 & 0 & 8 \\ 0 & 1 & 1 & 3 \\ 1 & 2 & 1 & 8 \end{bmatrix}$$

Elimination $R_3 = R_3 - \frac{1}{2} R_1$

$$\begin{bmatrix} 2 & 1 & 0 & 8 \\ 0 & 1 & 1 & 3 \\ 0 & 1.5 & 1 & 4 \end{bmatrix}$$

Since $R_3 C_2 = 1.5$ is the largest value

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & 1 & 0 & 8 \\ 0 & 1.5 & 1 & 4 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$$R_3 = R_3 - \frac{1}{1.5} R_2$$

$$\begin{bmatrix} 2 & 1 & 0 & 8 \\ 0 & 1.5 & 1 & 4 \\ 0 & 0 & 0.3333 & 0.3333 \end{bmatrix}$$

✓

$$2x_1 + x_2 = 8 \quad \text{—(i)}$$

$$1.5x_2 + x_3 = 4 \quad \text{—(ii)}$$

$$+0.3333 x_3 = 0.3333 \quad \text{—(iii)}$$

(iii) $x_3 = 1$

(ii) $1.5x_2 + 1 = 4 \Rightarrow x_2 = 2$

(i) $2x_1 + 2 = 8 \Rightarrow x_1 = 3$

$$x_1 = 3 \; ; \; x_2 = 2 \; ; \; x_1 = 1$$