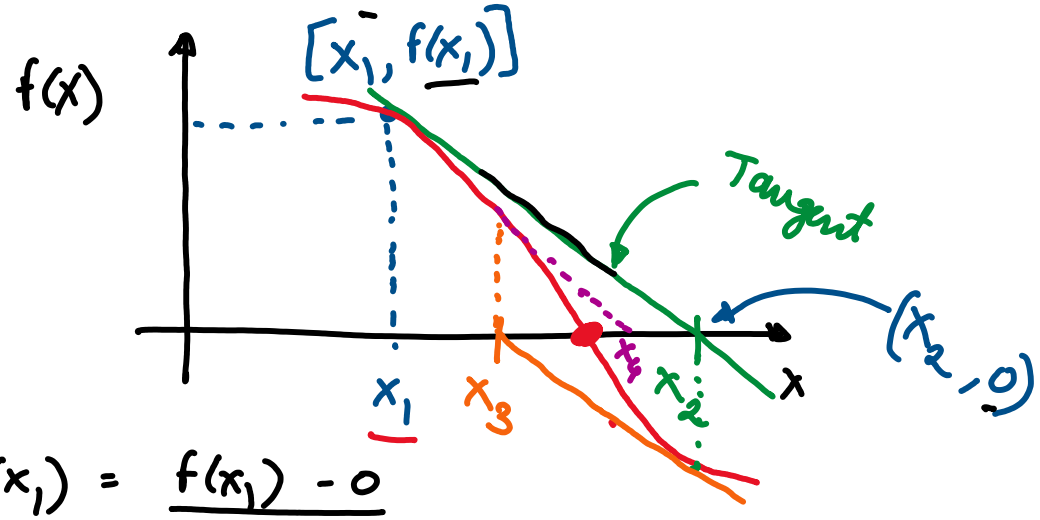
③ Newton Raphson's method

(i) Initialization

Needs one initial guess.

(ii) Search Direction



$$f'(x_1) = \frac{f(x_1) - 0}{x_1 - x_2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

(iii) Termination: Same as Bisection / Regular Falsi

EXAMPLE: Compute the root of the function $f(x) = x^2 - 3x$ using Newton Raphson method. Use an initial guess of 4.

(a) Compute 3 iterations by hand

(b) Write a computer code that will terminate when $|f(x)| < 1e-3$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad ; \quad f(x) = x^2 - 3x, \quad f'(x) = 2x - 3$$

Iteration 1: $x_1 = 4$

$$f(x_1) = 4^2 - 3(4) = 4$$

$$f'(x_1) = 2(4) - 3 = 5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4 - \frac{4}{5} = 3.2$$

$$f(x_2) = 3.2^2 - 3(3.2) = 0.64 > 1e-3$$

$$x_1 = x_2 = 3.2$$

Iteration 2 $x_1 = 3.2$

$$f(x_1) = 3.2^2 - 3(3.2) = 0.64$$

$$f'(x_1) = 2(3.2) - 3 = 3.4$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.2 - \frac{0.64}{3.4} = 3.011$$

$$f(x_2) = (3.011)^2 - 3(3.011) = 0.0033 > 1e\text{-}3 \quad (0.001)$$

$$x_1 = x_2 = 3.011$$

Iteration 3

$$x_1 = 3.011$$

$$f(x_1) = 0.033$$

$$f'(x_1) = 2(3.011) - 3 = 3.022$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.011 - \frac{0.033}{3.022}$$

$$x_2 = 3.00008$$

$$f(x_2) = (3.00008)^2 - 3(3.00008)$$

$$= 0.0002 = 2 \times 1e\text{-}4 < 1e\text{-}3$$

Converged.

Root is 3.00008

HW 4 due Sep 19 Thu

Mock Exam 1 Sep 26 Thu +3 pts.

Solution posted 24 hrs before Sep 26 Thu

Exam 1 : HW 3 & HW 4 Wed Sep 25

Tested on