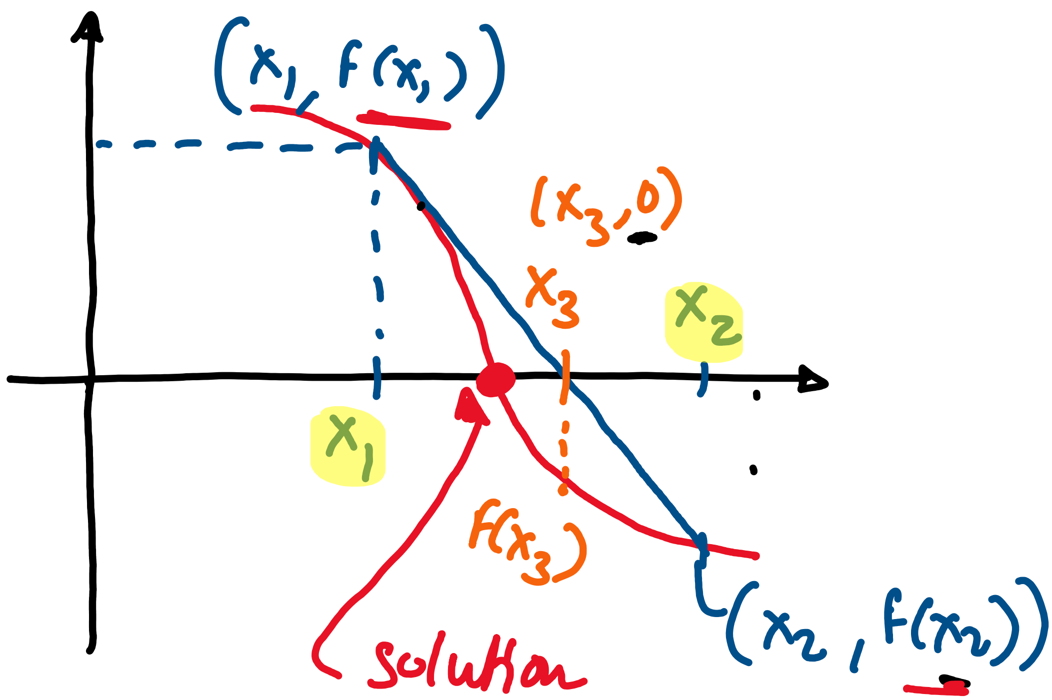
Regular - Falsi

① Initialization:

Needs 2 inital guesses that bracket the root (same bisection)

② Search Direction



$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(x_2)-0}{x_2-x_3}$$

$$\boxed{x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}}$$

③ Termination

$|f(x_3)| < \varepsilon$ stop $\{\varepsilon \approx 1e^{-3}\}$

or

iteration_count > max_iteration

max_iteration ≈ 100

EXAMPLE PROBLEM

Use Regular-Falsi to compute the roots of the equation $f(x) = x^2 - 3x$ upto
(a) 2 iterations
(b) compute code till convergence to 1e-3.

Use an initial guess [1, 4]

Iteration 1 $\quad x_1 = 1 \; ; \; x_2 = 4$

$$f(x_1 = 1) = 1^2 - 3(1) = -2$$

$$f(x_2 = 4) = 4^2 - 3(4) = 4$$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{(1)(4) - 4(-2)}{4 - (-2)}$$

$$= 12/6 = 2$$

$$f(x_3) = 2^2 - 3(2) = -2$$

$$f(x_2)\,f(x_3) = (4)(-2) < 0 \qquad x_1 = x_3$$

Iteration 2 $\quad x_1 = 2 \qquad x_2 = 4$

$$f(x_1 = 2) = -2$$

$$f(x_2 = 4) = 4$$

$$x_3 = \frac{(2)(4) - (4)(-2)}{4 - (-2)} = \frac{16}{6} = 2.667$$

$$f(x_3) = (2.667)^2 - 3(2.667) = -0.8881$$

$$f(x_2)\,f(x_3) = (4)(-0.8881) < 0 \qquad x_1 = x_3 = 2.667$$

$f(x_2) f(x_3) = (4)(-0.607) < 0 \qquad x_1 = x_3 = 2.687$

$x_1 = 2.687 \qquad x_2 = 4$