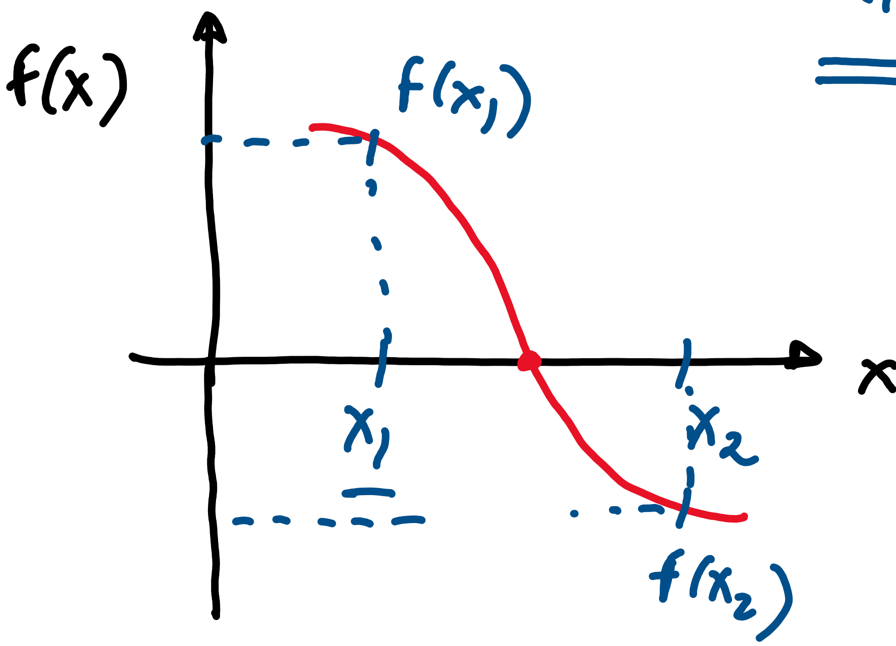
# I Bisection method

## 1 Initialization

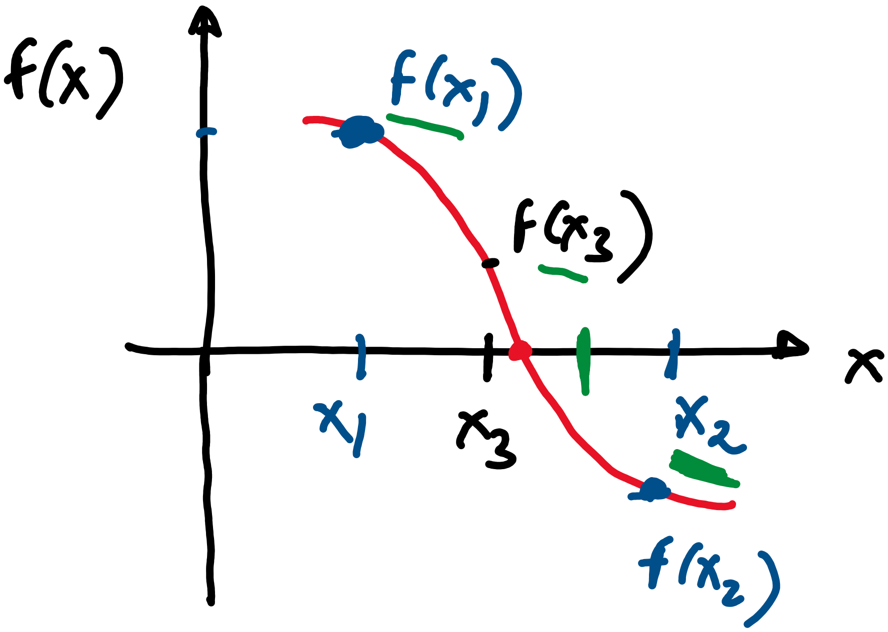
a) Need 2 guesses $(x_1, x_2)$

b) The 2 guesses need to bracket the root

$$f(x_1)\, f(x_2) < 0$$



## 2 Search Direction




$$x_3 = \frac{x_1 + x_2}{2}$$

Bisection

If $f(x_3)\,f(x_1) < 0$ $\quad x_2 = x_3 \quad (x_1, x_3)$

else $\quad x_1 = x_3 \quad (x_2, x_3)$

## 3 Termination

$\|f(x_3)\| < \varepsilon$ $\quad$ e.g. $\varepsilon = 10^{-3}$

stop

iter > max_iter $\quad$ e.g. max_iter=100

# Bisection Example

Using Bisection method, compute the root of the equation $f(x) = x^2 - 3x$. Use $[1, 4]$ as the initial guess. Do upto 3 iterations by hand and upto tolerance of 1e-3 using Octave

---

We know that the solution is $x = 0, 3$.

**Iteration 1** $\quad x_1 = 1 \qquad x_2 = 4$

$$f(x_1) = 1^2 - 3(1) = -2 \qquad f(x_2) = 4^2 - 3(4) = 4$$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1+4}{2} = 2.5$$

$$f(x_3) = 2.5^2 - 3(2.5) = -1.25$$

$$f(x_3) f(x_1) = (-1.25)(-2) > 0$$

$(2.5, 4)$

$$f(x_3) f(x_2) = (-1.25)(4) < 0 \qquad x_1 = x_3$$

$x_1$ is not needed

$x_1$ is not needed

## Iteration 2 $\quad (2.5, 4)$

$$f(x_1) = 2.5^2 - 3(2.5)$$
$$= -1.25$$

$$f(x_2) = 4^2 - 3(4)$$
$$= 4$$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{2.5 + 4}{2} = 3.25$$

$$f(x_3) = 3.25^2 - 3(3.25)$$
$$= 0.8125$$

$$f(x_1)\,f(x_3) = (-1.25)(0.8125) < 0 \qquad x_2 = x_3$$

$$(2.5, 3.25)$$

Iterations 3 : $(2.5, 3.25)$

$f(x_1) = f(2.5) = -1.25$

$f(x_2) = f(3.25) = 0.8125$

$x_3 = \frac{x_1 + x_2}{2} = \frac{2.5 + 3.25}{2} = 2.875$

$f(x_3) = 2.875^2 - 3(2.875)$
$= -0.359$

$f(x_2)\, f(x_3) < 0$ $\qquad x_1 = x_3$

$(2.875, 3.25)$