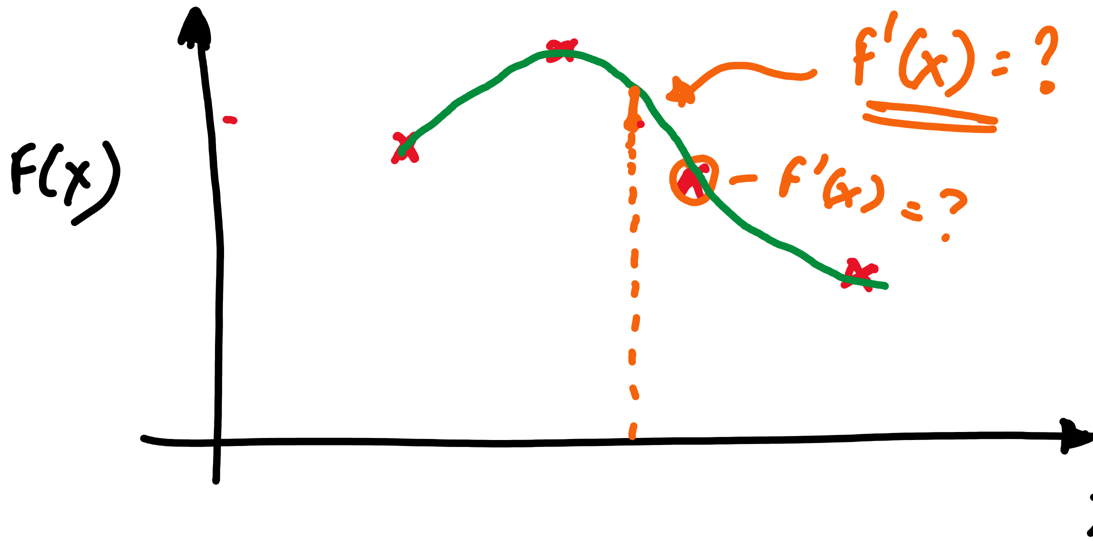
Polynomial Representation



① There 4 points given

$[x_1, f(x_1)]$, $[x_2, f(x_2)]$

$[x_3, f(x_3)]$, $[x_4, f(x_4)]$ } Ⓘ

② 3^{rd} order polynomial will pass through 4 points

$$F(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

③ Solve for a_0, a_1, a_2, a_3 using Ⓘ

④ $f'(x) = a_1 + 2a_2 x + 3a_3 x^2$

EXAMPLE

The value of $f(x)$ at $x = 0, 1, 2$ are $2, -1, -2$. Compute $f'(x=0.5)$

Solution

① The curve passes through

$$[x, f(x)] = (0, 2) \quad (1, -1) \quad (2, -2)$$

② Choose a 2^{nd} order polynomial

$$f(x) = a_0 + a_1 x + a_2 x^2$$

③

$$2 = a_0 + a_1(0) + a_2(0)$$

$$\Rightarrow a_0 = 2$$

$$-1 = a_0 + a_1(1) + a_2(1)$$

$$-1 = 2 + a_1 + a_2$$

$$\Rightarrow a_1 + a_2 = -3 \qquad \text{— (II)}$$

$$-2 = a_0 + a_1(2) + a_2(2)^2$$

$$-2 = 2 + 2a_1 + 4a_2$$

$$\Rightarrow 2a_1 + 4a_2 = -4$$

$$\Rightarrow a_1 + 2a_2 = -2 \quad \text{— III}$$

From II and III

$$\boxed{\begin{array}{l} a_0 = 2 \\ a_1 = -4 \\ a_2 = 1 \end{array}}$$

$$a_1 + a_2 = -3$$

$$a_1 + 2a_2 = -2$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

$$f(x) = a_0 + a_1 x + a_2 x^2$$

$$= 2 - 4x + x^2$$

$$f'(x) = 0 - 4 + 2x$$

$$f'(0.5) = -4 + 2(0.5)$$

$$= -4 + 1$$

$$= -3$$

$$\boxed{f'(0.5) = -3}$$

Partial Derivatives

Function the depend on more than 1 variable : e.g. $f(x,y)$

We want to compute $\frac{\partial f}{\partial x}$ you hold $y = \text{constant}$.

e.g. $f(x,y) = \underline{x}^2 y$

$$\frac{\partial F}{\partial x} = 2xy$$

$$\frac{\partial F}{\partial y} = x^2$$

$$F(x_i, y_j) \longrightarrow F_{i,j}$$

Forward difference

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\boxed{\begin{aligned} & f_{i+1,j} \\ & = f(x_i + \Delta x, y_j) \end{aligned}}$$

$$\left.\frac{\partial f}{\partial y}\right|_{i,j} = \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

Backward Difference

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i,j} - f_{i-1,j}}{\Delta x}$$

$$\left.\frac{\partial f}{\partial y}\right|_{i,j} = \frac{f_{i,j} - f_{i,j-1}}{\Delta y}$$

Central difference

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x}$$

$$\left.\frac{\partial f}{\partial y}\right|_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y}$$

Second Central Difference

$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{i,j} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{\Delta x^2}$$

$$\left.\frac{\partial^2 f}{\partial y^2}\right|_{i,j} = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\Delta y^2}$$