# Taylor series

Consider the function $f(x)$. The Taylor series gives a polynomial expansion of $f(x)$ at $x = x_i$

$x$ should be close to $x_i$

$$\boxed{f(x) = f(x_i) + (x-x_i) f'(x_i) + \frac{(x-x_i)^2}{2!} f^2(x_i) + \frac{(x-x_i)^3}{3!} f^3(x_i) + \dots \frac{(x-x)^n}{n!} f^n(x_i) + \dots}$$

$n! = (n)(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$

e.g. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$f^n(x)$ is the $n^{th}$ derivative of $f$ with respect to $x$.

e.g. $f^3(x) = \dfrac{d^3 f}{dx^3}$

**Example:** Do a Taylor series expansion of $\sin(x)$ at $x=0$ upto 8 terms

**Solution**

$$f(x) = f(x_i) + \frac{(x-x_i)}{2!} f'(x_i) + \dots + \frac{(x-x_i)^7}{7!} f^7(x_i)$$

$x_i = 0$

$f(x_i) = \sin x_i = \sin(0) = 0$

$f^1(x_i) = \cos(x_i) = \cos(0) = 1$

$f^2(x_i) = -\sin(x_i) = -\sin(0) = 0$

$f^3(x_i) = -\cos(x_i) = -\cos(0) = -1$

$f^4(x_i) = \quad \dots \quad = 0$

$f^5(x_i) = \quad \dots \quad = 1$

$f^6(x_i) = \quad \dots \quad = 0$

$f^7(x_i) = \quad \dots \quad = -1$

Complete this

$$f(x) = \cancel{0} + (x-0)(1) + \cancel{\frac{(x-0)^2}{2!}(0)} + \frac{x^3}{3!}(-1) + \dots$$

sin x

$$\cancel{\frac{x^4}{4!}(0)} + \frac{x^5}{5!}(1) + \cancel{\frac{x^6}{6!}(0)} + \frac{x^7}{7!}(-1)$$

$$\boxed{\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}}$$

EXAMPLE:

Compute the value of $\sin(x)$ at $x = 0.1$ using $2^{nd}$, $4^{th}$, $6^{th}$, $8^{th}$ order terms. Compare against the actual value

Solution

## Solution

Exact : $\sin(0.1) = 0.0998334166468$

$n=2$ : $\sin(0.1) = x = 0.1$

$n=4$ : $\sin(0.1) = x - \dfrac{x^3}{6} = 0.1 - \dfrac{0.1^3}{6} = 0.099833\bar{3}$

$n=6$ : $\sin(0.1) = 0.1 - \dfrac{0.1^3}{6} + \dfrac{0.1^5}{120} = 0.09983341\overline{66}$

$n=8$ : $\sin(0.1) = 0.1 - \dfrac{0.1^3}{6} + \dfrac{0.1^5}{120} - \dfrac{0.1^7}{5040}$

$= 0.0998334166468$