

Numerical solution of Partial Differential Equations

$$A \frac{\partial^2 \phi}{\partial x^2} + B \frac{\partial^2 \phi}{\partial x \partial y} + C \frac{\partial^2 \phi}{\partial y^2} + D \left(x, y, \phi, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right) = 0$$

Classification

$$B^2 - 4AC < 0 \quad \text{Elliptic PDE}$$

$$B^2 - 4AC = 0 \quad \text{Parabolic PDE}$$

$$B^2 - 4AC > 0 \quad \text{Hyperbolic PDE}$$

Heat conduction

$$\propto \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad \text{Parabolic}$$

$$B=0 ; A=1 ; C=0$$

$$B^2 - 4AC = 0$$

Poisson's Equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \beta(x, y)$$

$$B^2 - 4AC = 0 - 4(1)(1) < 0 \quad \text{Elliptic}$$

Wave equation

$$(1) \frac{\partial^2 \phi}{\partial t^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0$$

$$B^2 - 4AC = 0 - 4(1)(-c^2) > 0 \quad \text{Hyperbolic}$$