

Gauss-seidel method

Intuition

$$\rightarrow a_{11} x_1 + a_{12} x_2 = b_1 \quad \text{--- (1)}$$

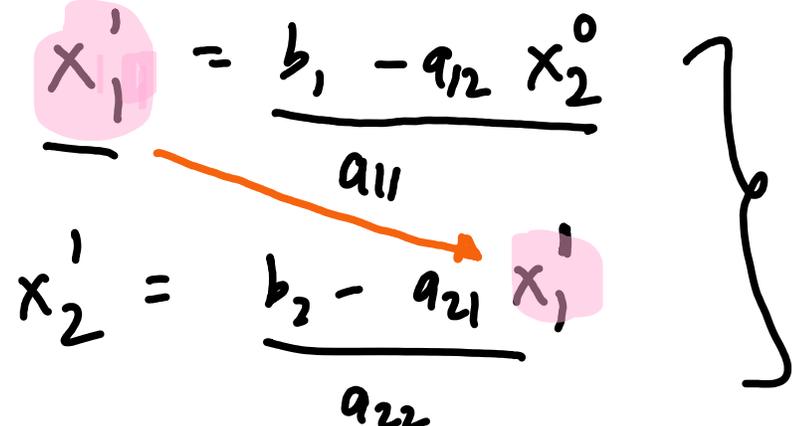
$$\rightarrow a_{21} x_1 + a_{22} x_2 = b_2 \quad \text{--- (2)}$$

$$\text{From (1)} \rightarrow x_1 = \frac{b_1 - a_{12} x_2}{a_{11}}$$

$$\text{From (2)} \rightarrow x_2 = \frac{b_2 - a_{21} x_1}{a_{22}}$$

Start with an initial guess $\underline{x_1^0, x_2^0}$

Iteration 1:

$$\underline{x_1^1} = \frac{b_1 - a_{12} x_2^0}{a_{11}}$$
$$x_2^1 = \frac{b_2 - a_{21} x_1^1}{a_{22}}$$


EXAMPLE:

Solve for x_1, x_2, x_3 using Gauss-Seidel

$$2x_1 + x_2 + x_3 = 7 \quad - (1)$$

$$x_1 - 3x_2 + x_3 = -2 \quad - (2)$$

$$2x_1 + 2x_2 - x_3 = 3 \quad - (3)$$

Use an initial guess $x_1^0 = x_2^0 = x_3^0 = 0$

Solve up to 3 iterations

Solution: $x_1 = \frac{7 - x_2 - x_3}{2}$ From (1)

$$x_2 = \frac{-2 - \overset{\downarrow}{x_1} - x_3}{-3}$$
 From (2)

$$x_3 = \frac{3 - 2x_1 - 2x_2}{-1}$$
 From (3)

Iteration 1 $x_1 = (7 - 0 - 0)/2 = 3.5$

$$x_2 = (-2 - 3.5 - 0)/-3 = 1.8333$$

$$x_3 = [3 - 2(3.5) - 2(1.8333)]/-1 = 7.6667$$

Iteration 2

$$x_1 = 3.5; x_2^1 = 1.8333; x_3^1 = 7.6667$$

$$x_1 = \frac{7 - (1.8333) - 7.6667}{2} = -1.25$$

$$x_2 = \frac{-2 - (-1.25) - 7.6667}{-3} = 2.8056$$

$$x_3 = \frac{3 - 2(-1.25) - 2(2.8056)}{-1} = 0.1111$$

$$x_1 = \frac{7 - x_2 - x_3}{2}$$

$$x_2 = \frac{-2 - x_1 - x_3}{-3}$$

$$x_3 = \frac{3 - 2x_1 - 2x_2}{-1}$$

Iteration 3

$$x_1 = 2.0416$$

$$x_2 = 1.3842$$

$$x_3 = 3.8516$$

Iter	x_1	x_2	x_3
1	3.5	1.8333	7.6667
2	-1.25	2.8056	0.1111
3	2.0416	1.3842	3.8516