

② Polynomial Fit

$$\rightarrow y = f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots + c_m x^m$$

$$\rightarrow E = \sum_{i=1}^n \left[y_i - \underbrace{[c_0 + c_1 x_i + c_2 x_i^2 + \dots + c_m x_i^m]}_{f(x_i)} \right]^2$$

$$\frac{dE}{dc_0} = 2 \sum_{i=1}^n [y_i - [c_0 + c_1 x_i + \dots + c_m x_i^m]] (-1) = 0$$

$$\frac{dE}{dc_1} = 2 \sum_{i=1}^n [y_i - [c_0 + c_1 x_i + \dots + c_m x_i^m]] (-x_i) = 0$$

$$\frac{dE}{dc_2} = 2 \sum_{i=1}^n [y_i - [c_0 + c_1 x_i + \dots + c_m x_i^m]] (-x_i^2) = 0$$

⋮

$$\frac{dE}{dc_m} = 2 \sum_{i=1}^n [y_i - [c_0 + c_1 x_i + \dots + c_m x_i^m]] (-x_i^m) = 0$$

(m+1) equations // (m+1) unknowns
 c_0, c_1, \dots, c_m

$$\begin{bmatrix}
 n & \sum x_i & \sum x_i^2 & & \sum x_i^m \\
 \sum x_i & \sum x_i^2 & \sum x_i^3 & & \sum x_i^{m+1} \\
 \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & & \sum x_i^{m+2} \\
 \vdots & \vdots & \vdots & & \vdots \\
 \sum x_i^m & \sum x_i^{m+1} & \sum x_i^{m+2} & & \sum x_i^{2m}
 \end{bmatrix}
 \begin{bmatrix}
 c_0 \\
 c_1 \\
 c_2 \\
 \vdots \\
 c_m
 \end{bmatrix}
 =
 \begin{bmatrix}
 \sum y_i \\
 \sum x_i y_i \\
 \sum x_i^2 y_i \\
 \vdots \\
 \sum x_i^m y_i
 \end{bmatrix}$$

$$\begin{matrix}
 (m+1) & (m+1) & (m+1) & (m+1)
 \end{matrix}$$

$$A C = b$$

$$C = A^{-1} b$$

$$C = (c_0, c_1, c_2, \dots, c_m)$$

Polynomial example

Obtain a 3rd order polynomial fit to the data shown on the right.

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

$$A c = b$$

	x	y
x_1	-2.0000	-2.0328
x_2	-1.6000	0.0946
x_3	-1.2000	-2.2934
\vdots	-0.8000	-1.7862
\vdots	-0.4000	0.4392
\vdots	0.0000	0.5463
\vdots	0.4000	1.5743
\vdots	0.8000	-0.0284
\vdots	1.2000	5.5692
\vdots	1.6000	6.8773
x_{11}	2.0000	15.4496

$$A = \begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \end{bmatrix} \quad b = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \sum x_i^3 y_i \end{bmatrix}$$

$$\sum x_i = x_1 + x_2 + \dots + x_{11} = 0 \quad \leftarrow$$

$$\sum x_i^2 = x_1^2 + x_2^2 + \dots + x_{11}^2 = 17.6 \quad \leftarrow$$

$$\sum y_i = y_1 + y_2 + \dots + y_{11} = 24.4093 \quad \leftarrow$$

$$\sum x_i y_i = x_1 y_1 + x_2 y_2 + \dots + x_{11} y_{11} = 57.1129 \quad \leftarrow$$

$$\begin{bmatrix} 11 & 0 & 17.6 & 0 \\ 0 & 17.6 & 0 & 50.1248 \\ 17.6 & 0 & 50.1248 & 0 \\ 0 & 50.1248 & 0 & 168.0589 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 24.4093 \\ 57.1129 \\ 75.3937 \\ 182.2007 \end{bmatrix}$$

A
 C
 b

$$C = A^{-1} b$$

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -0.4279 \\ 1.0453 \\ 1.6544 \\ 0.7724 \end{bmatrix}$$

$$y = -0.4279 + 1.0453x + 1.6544x^2 + 0.7724x^3$$

