

# Iterative methods

## ① Jacobi Iterative method

### Intuition

$$a_{11} x_1 + a_{12} x_2 = b_1 \quad -\textcircled{1}$$

$$a_{21} x_1 + a_{22} x_2 = b_2 \quad -\textcircled{2}$$

$$\text{From } \textcircled{1} \quad x_1 = \frac{b_1 - a_{12} x_2}{a_{11}}$$

$$\text{From } \textcircled{2} \quad x_2 = \frac{b_2 - a_{21} x_1}{a_{22}}$$

Start with an initial guess  $x_1^0, x_2^0$

$$\text{Iteration 1: } x_1^1 = \frac{b_1 - a_{12} x_2^0}{a_{11}}$$

$$x_2^1 = \frac{b_2 - a_{21} x_1^1}{a_{22}}$$

Iteration 2:  $x_1^2 = \frac{b_1 - a_{12} x_2^1}{a_{11}}$

$$x_2^2 = \frac{b_2 - a_{21} x_1^1}{a_{22}}$$

keep doing this till convergence

Ⓐ  $|x_1^i - x_1^{i-1}| < \epsilon$

and  
 $|x_2^i - x_2^{i-1}| < \epsilon$

Ⓑ  $\sqrt{(x_1^i - x_1^{i-1})^2 + (x_2^i - x_2^{i-1})^2} < \epsilon$

$\epsilon$  = user defined tolerance

e.g.  $\epsilon \approx 10^{-6}$  or  $1e-6$

## EXAMPLE:

Solve for  $x_1, x_2, x_3$  using Jacobi method.

$$2x_1 + x_2 + x_3 = 7 \quad - \textcircled{1}$$

$$x_1 - 3x_2 + x_3 = -2 \quad - \textcircled{2}$$

$$2x_1 + 2x_2 - x_3 = 3 \quad - \textcircled{3}$$

Use an initial guess  $x_1^0 = x_2^0 = x_3^0 = 0$

Solve up to 3 iterations

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Solution:  $x_1 = \frac{7 - x_2 - x_3}{2}$  From  $\textcircled{1}$

$$x_2 = \frac{-2 - x_1 - x_3}{-3} \quad \text{From } \textcircled{2}$$

$$x_3 = \frac{3 - 2x_1 - 2x_2}{-1} \quad \text{From } \textcircled{3}$$

Iteration 1  $x_1 = (7 - 0 - 0)/2 = 3.5$

$$x_2 = (-2 - 0 - 0)/-3 = 0.6667$$

$$x_3 = \frac{(3 - 2(0) - 2(0))}{-1} = -3$$

$$x_1^1 = 3.5; x_2^1 = 0.6667; x_3^1 = -3$$

Iteration 2:

$$x_1 = \frac{7 - 0.6667 - (-3)}{2} = 4.6667$$

$$x_2 = \frac{-2 - 3.5 - (-3)}{-3} = 0.8333$$

$$x_3 = \frac{3 - 2(3.5) - 2(0.6667)}{-1} = 5.3334$$

$$x_1^2 = 4.6667; x_2^2 = 0.8333; x_3^2 = 5.3334$$

Iteration 3

$$x_1 = 0.4166$$

$$x_2 = 4.000$$

$$x_3 = 8.000$$

Iter	$x_1$	$x_2$	$x_3$
1	3.5	0.6667	-3
2	4.6667	0.8333	5.3333

$$x_1 = \frac{7 - x_2 - x_3}{2}$$

$$x_2 = \frac{-2 - x_1 - x_3}{-3}$$

$$x_3 = \frac{3 - 2x_1 - 2x_2}{-1}$$

2	4.6667	0.8333	5.3333
3	0.4167	4.0000	8.0000