Force Analysis Convention

1 - frame of the machine

2 - input gear

3, 4, ... successive gears.

a, b, .. for shafts.

F_{23} - force exerted by gear 2 on gear 3

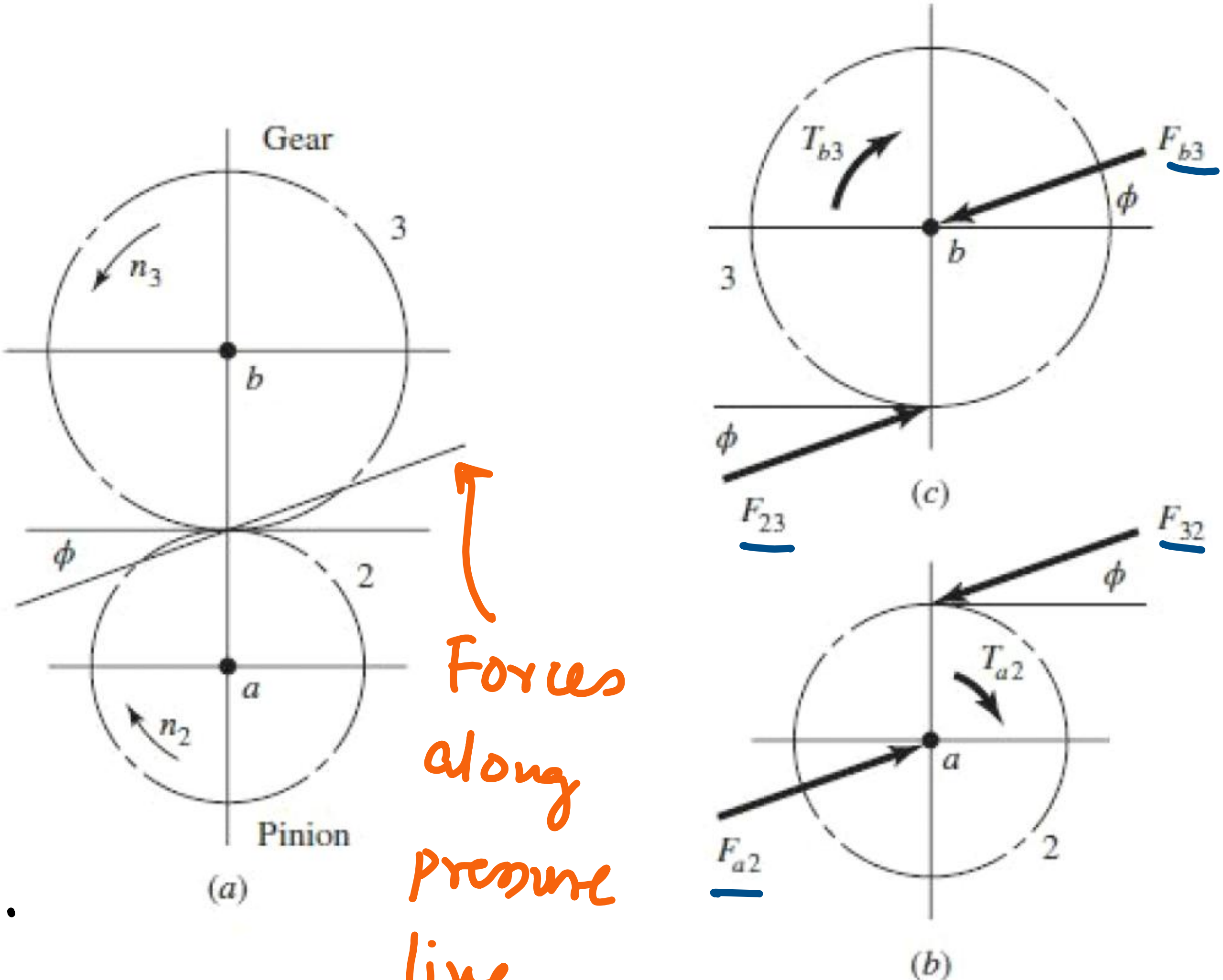
F_{2a} - force exerted by gear 2 on shaft a

F^t - tangential force

F^r - radial force

Force Analysis of Spur Gears (1/2)

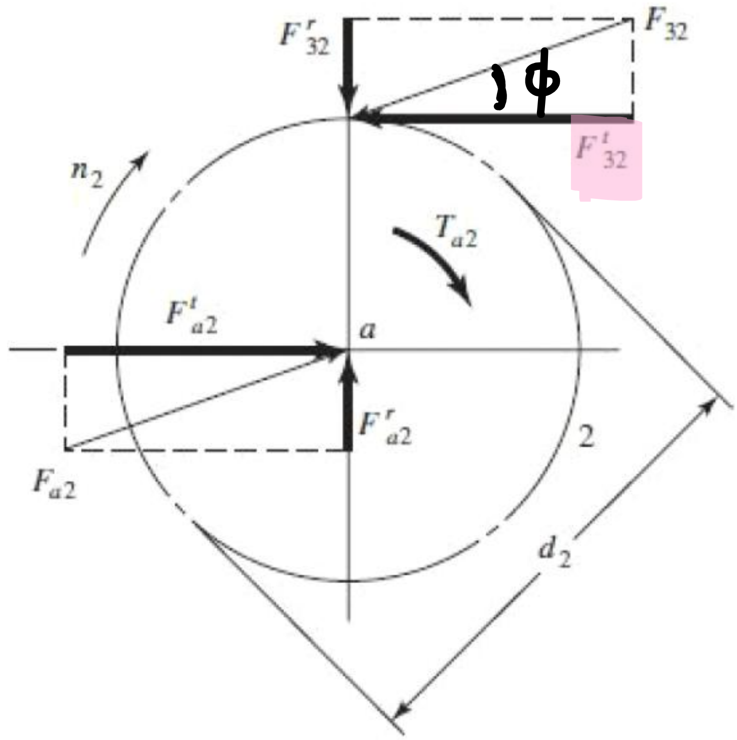
FBD



Forces along pressure line

Force Analysis of Spur Gear (2/2)

FBD of Pinion



$$\tan\phi = \frac{F^r_{32}}{F^t_{32}}$$

$$T_{a2} = F^t_{32}\frac{d_2}{2} \quad (1)$$

$$H = T_{a_2}\,\omega_2 \quad (2)$$

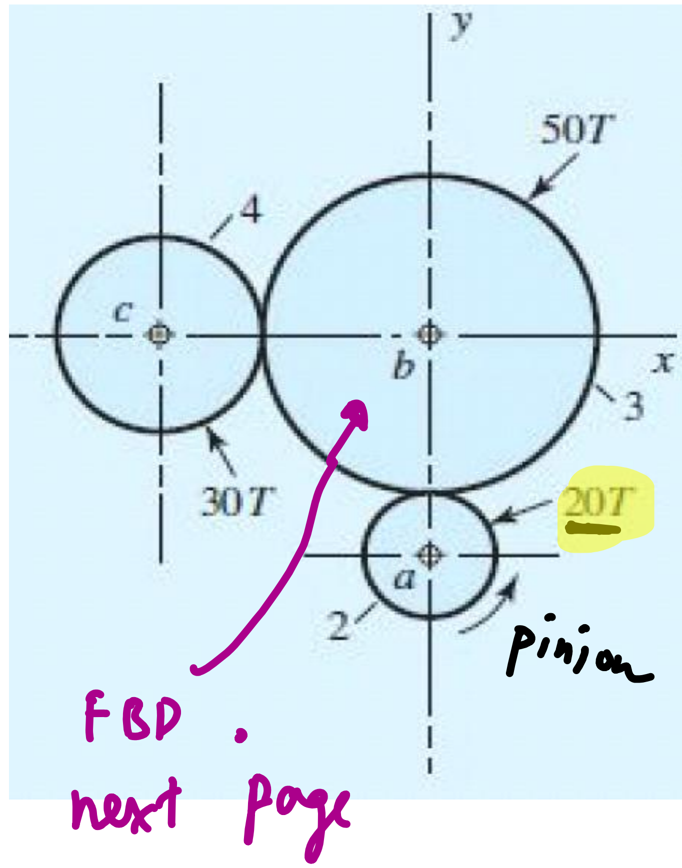
$$H = F^t_{32}\frac{d_2\,\omega_2}{2}$$

H - power

Subs (1) in (2)

Q1

Pinion 2 runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20 degrees full-depth system and have a module m = 2.5 mm. Draw a free body diagram and show all forces acting on ~~it~~ idler



$\omega_2 = 1750$ rev/min

$H = 2.5$ kW

$m = 2.5$ mm

$H = T_{a_2}\,\omega_2$

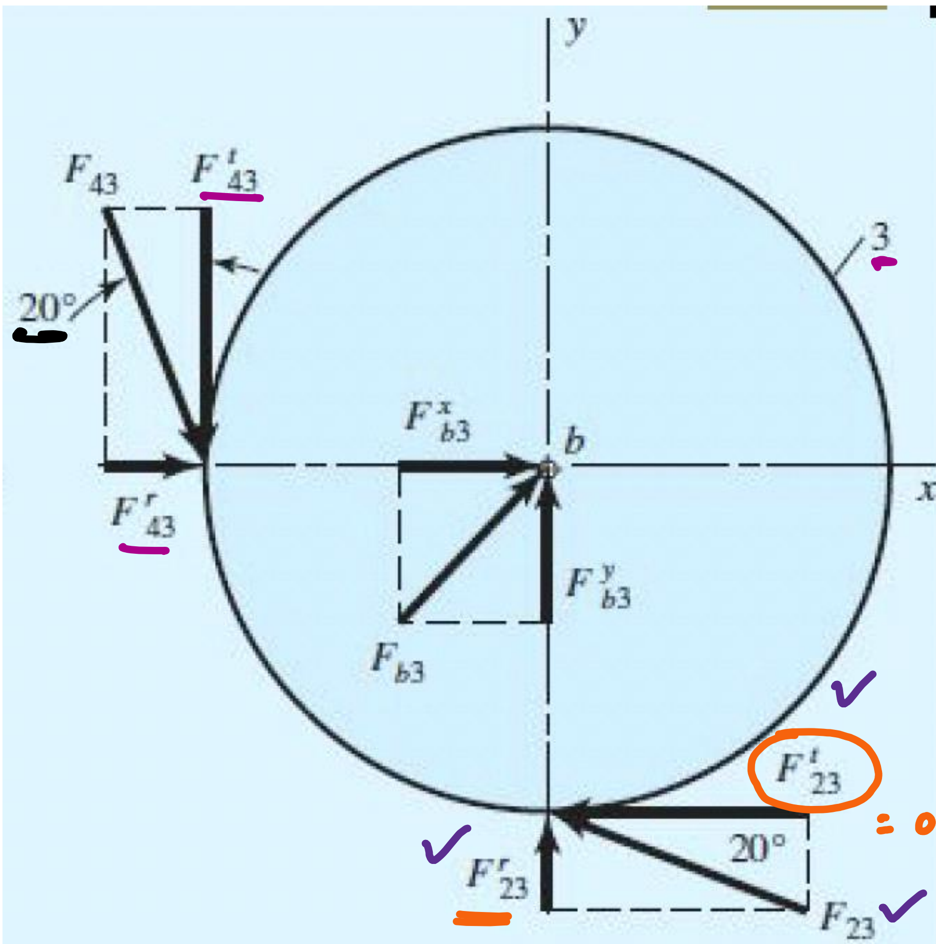
$H = \left(F_{23}^{t}\,\dfrac{d_2}{2}\right)\omega_2$

$d_2 = m N_2 = 2.5(20)$

$= 5$ mm

$2.5(10^3) = F_{23}^{t}\left(\dfrac{5(10^{-3})}{2}\right)\dfrac{1750}{}\dfrac{(2\pi)}{60}\dfrac{\text{rad}}{\text{s}}$

$\boxed{F_{23}^{t} = 0.546 \text{ kN}}$



$$F^r_{23} = F^t_{23} \tan 20^\circ$$

$$F^r_{23} = 0.546 \tan 20^\circ$$

$$\boxed{F^r_{23} = 0.199 \text{ kN}}$$

$$F_{23} = \sqrt{(F^r_{23})^2 + (F^t_{23})^2}$$

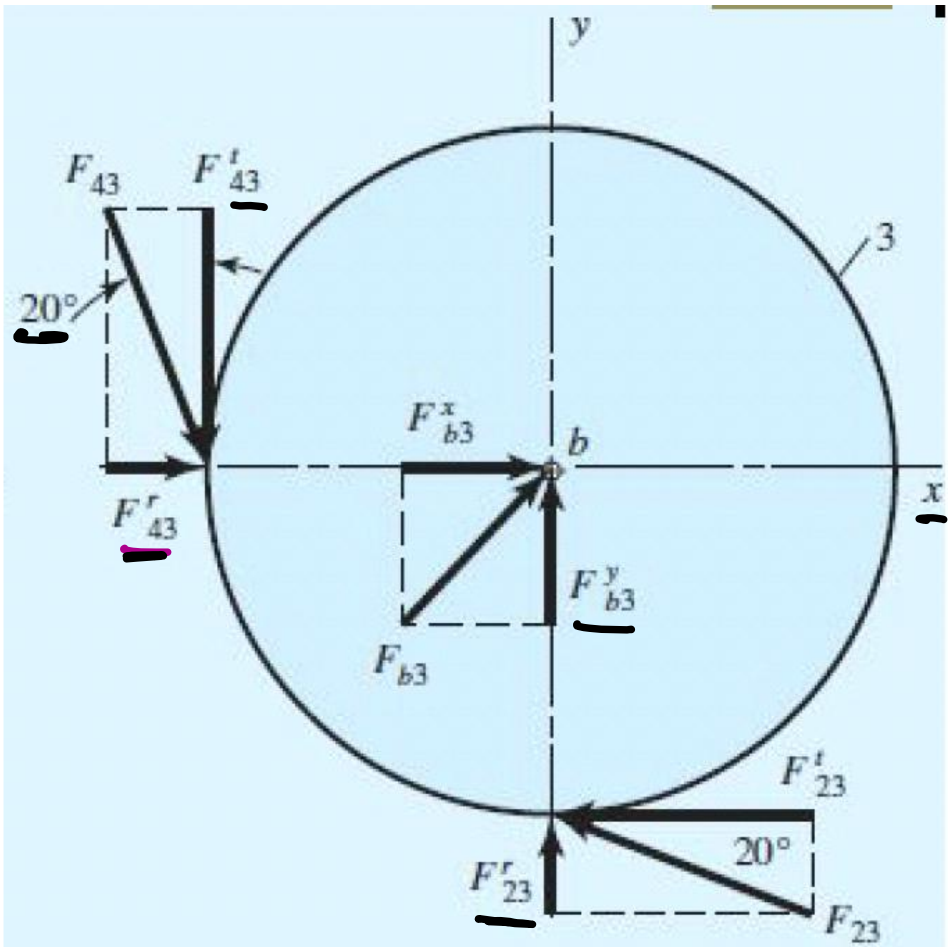
$$\boxed{F_{23} = 0.581 \text{ kN}}$$

$$\Sigma M_b = -F^t_{23} \frac{d_3}{2} + F^t_{43} \frac{d_2}{2} = 0$$

$$\boxed{F^t_{43} = F^t_{23} = 0.546 \text{ kN}}$$

$$\boxed{F^r_{43} = F^t_{43} \tan 20^\circ = 0.199 \text{ kN}}$$

$$\boxed{F_{43} = 0.581 \text{ kN}}$$



$$\Sigma F_x = F^x_{b3} + F^r_{43} - F^t_{23} = 0$$

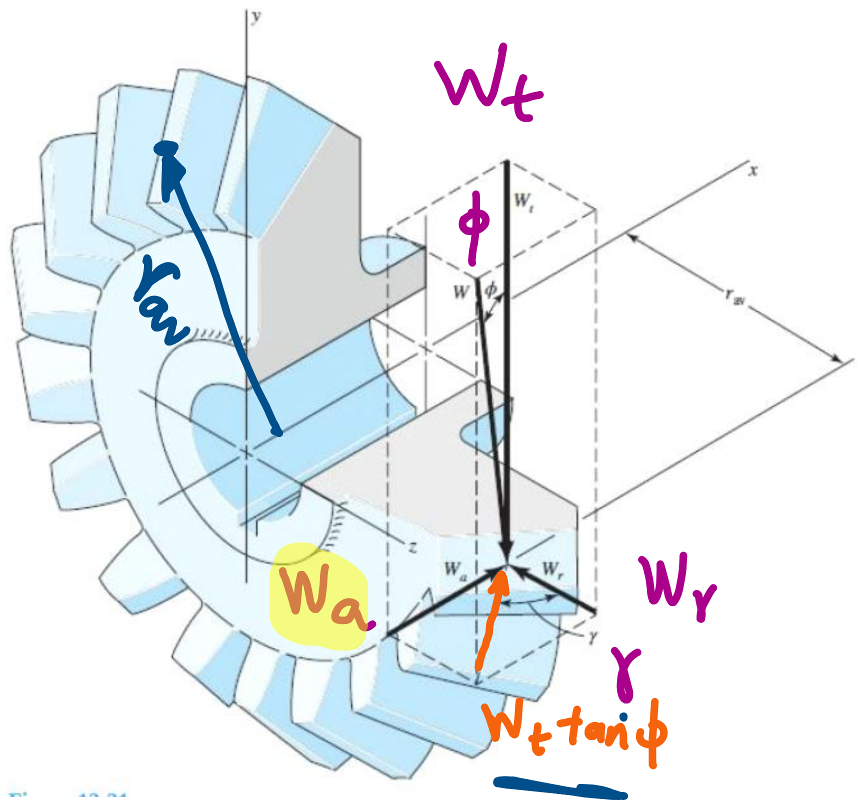
$$F^x_{b3} + 0.199 - 0.546 = 0 \quad \boxed{F^x_{b3} = 0.347\ \text{KN}}$$

$$\Sigma F_y : F^r_{23} + F^y_{b3} - F^t_{43} = 0$$

$$0.199 + F^y_{b3} - 0.546 = 0 \quad \boxed{F^y_{b3} = 0.347\ \text{KN}}$$

$$\boxed{F_{b3} = \sqrt{(F^x_{b3})^2 + (F^y_{b3})^2} = 0.491}$$

Force Analysis of Bevel Gears



W_t, W_a, W_r

tangential, axial, radial forces

A bevel gear does not have constant radius, so we use the radius r_{av} at the mid-point for Power/Torque calculation

$$T = W_t \, r_{av} \Rightarrow H = T\omega$$

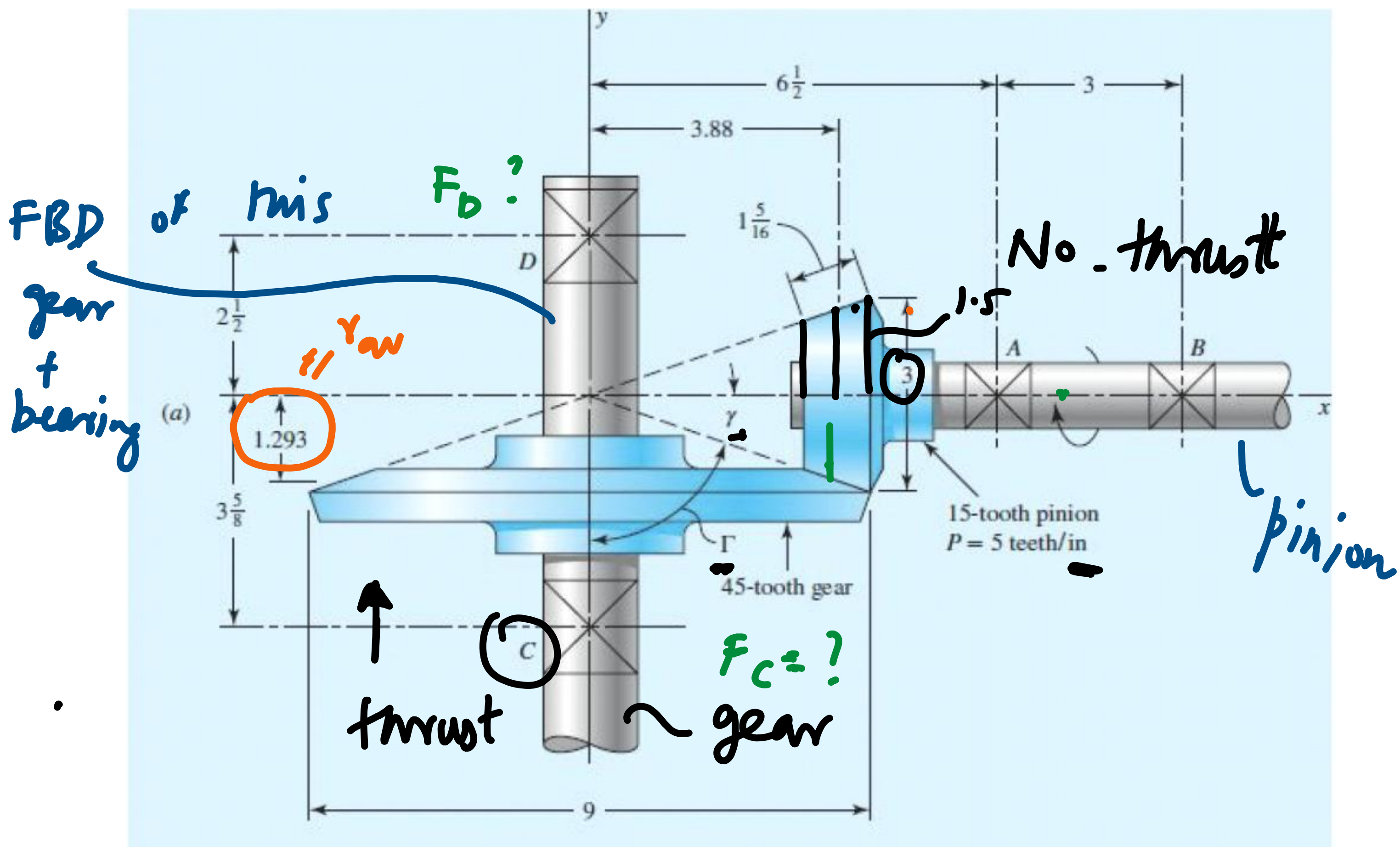
$$H = F \cdot V$$

$$W_r = (W_t \tan\phi)\cos\gamma$$

$$W_a = (W_t \tan\phi)\sin\gamma$$

Q2

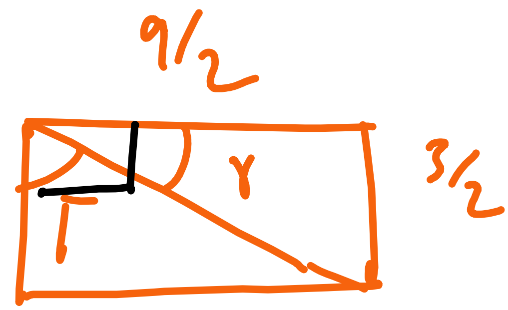
The bevel pinion rotates at 600 rev/min in the direction shown and transmits 5 hp to the gear. For simplicity, the teeth have been replaced with pitch cones. Assume that only bearing C can take thrust loads. Compute the bearing forces on the gear shaft.



$\omega_3 = 600$ rev/min

$H = 5$ hp

Compute W_t, W_r, W_a



$$\tan\gamma = \frac{3/2}{9/2} \Rightarrow \gamma = 18.4^\circ$$

$$\Gamma = 90^\circ - 18.4^\circ = 71.6^\circ$$

$$P = 5\ hp = 5(550)\ ft\cdot lb/s = 2750\ ft\cdot lb/s$$

$$\omega_3 = 600\ \frac{rev}{min} = 600\ \frac{2\pi}{60} = 62.8\ rad/s$$

$$P = T_3\omega_3 \Rightarrow 2750 = T_3(62.8) \Rightarrow T_3 = 43.789$$

$$T_3 = W_t r_{av} \Rightarrow 43.789 = W_t\left(\frac{1.293}{12}\right) \Rightarrow \boxed{W_t = 406\ lbf}$$

convert in to ft

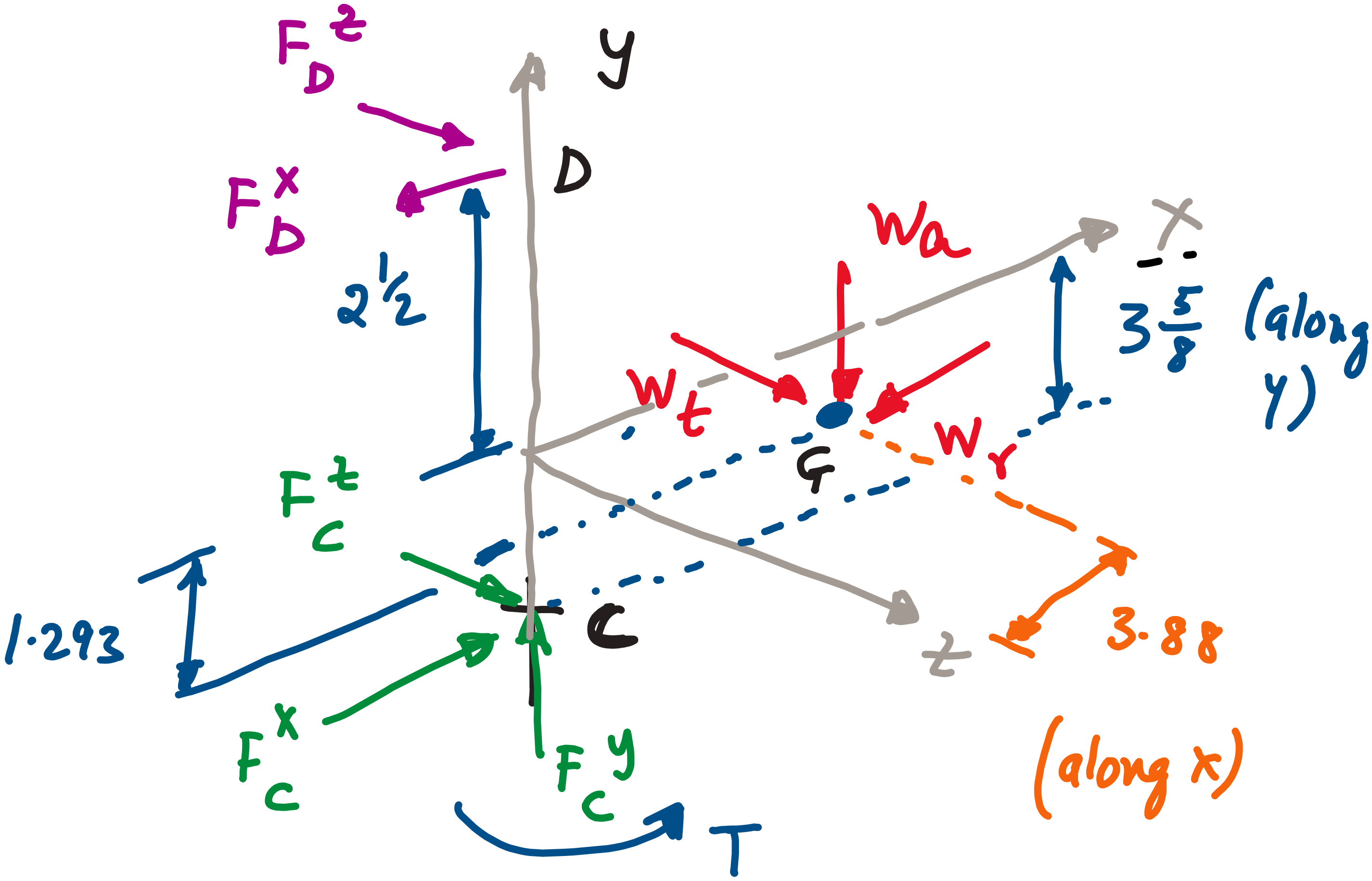
$$W_r = W_t \tan\phi \cos\Gamma = 406 \tan 20^\circ \cos 71.6^\circ$$

$$\boxed{W_r = 46.6\ lbf}$$

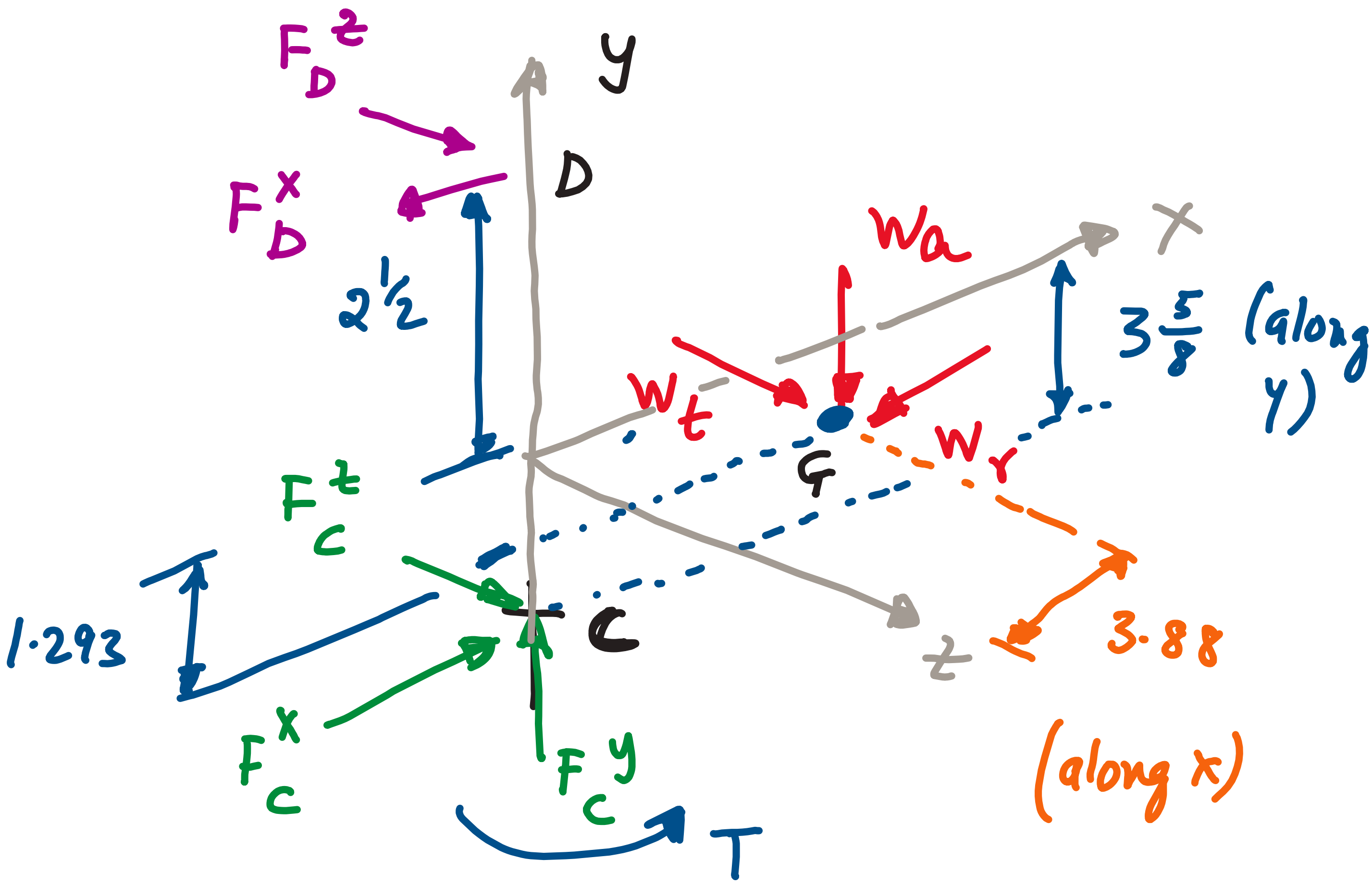
$$W_a = W_t \tan\phi \sin\Gamma = 406 \tan 20^\circ \sin 71.6^\circ$$

$$\boxed{W_a = 140\ lbf}$$

FBD Q2



Unknowns: F_D^z, F_D^x, F_C^z, F_C^x, F_C^y, T (6)



$$\Sigma M_D^x = -\left(2\tfrac{1}{2} + 3\tfrac{5}{8}\right)F_C^z - \left(2\tfrac{1}{2} + 1.293\right)W_t = 0$$

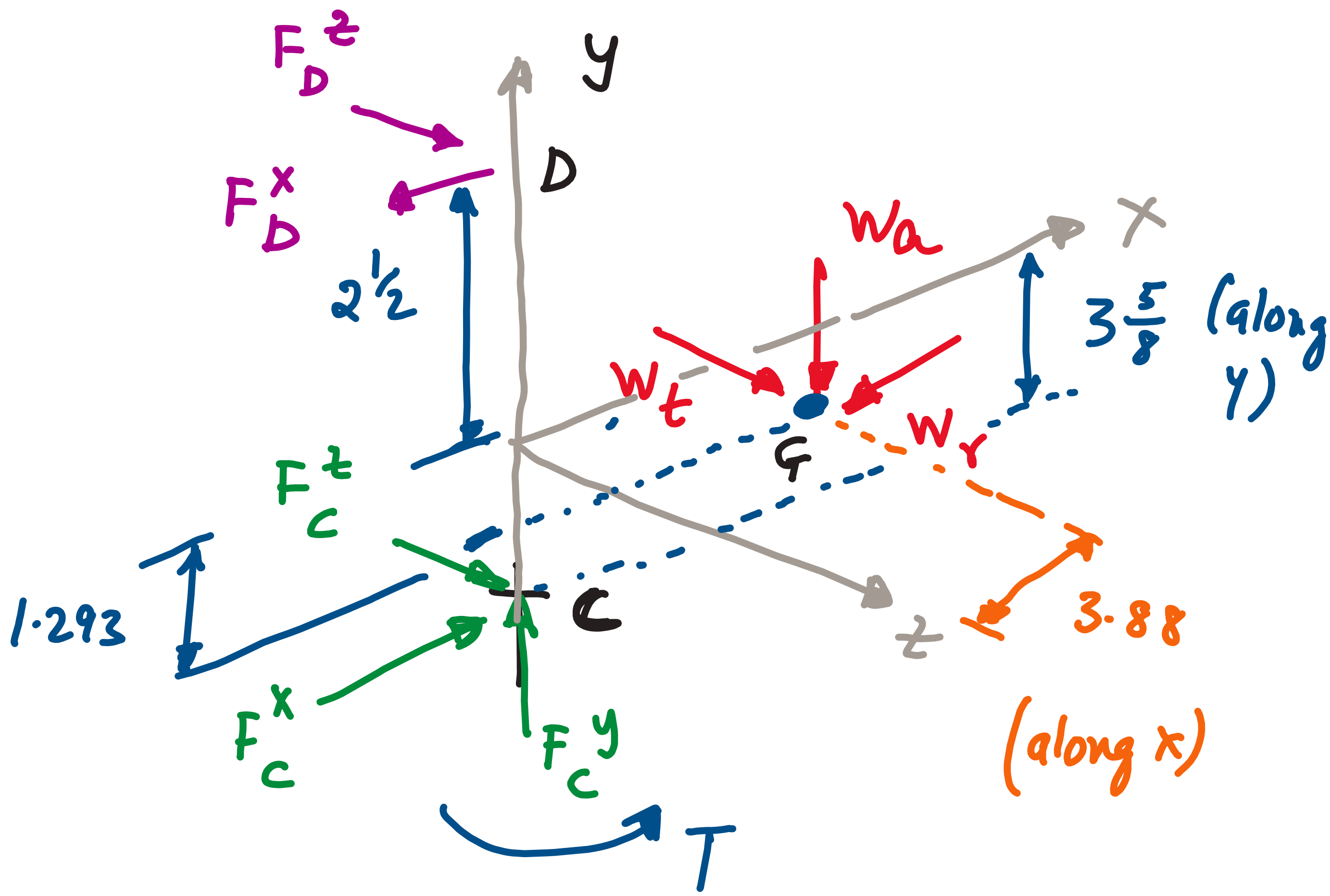
$$F_C^z = -251.42 \text{ lb.F}$$

$$\Sigma M_D^z = +F_C^x\left(2\tfrac{1}{2} + 3\tfrac{5}{8}\right) - W_r\left(2\tfrac{1}{2} + 1.293\right) - W_a(3.88) = 0$$

$$F_C^x = 117.5 \text{ lbF}$$

$$\Sigma M_D^y = T - W_t(3.88) = 0$$

$$T = 1575.28 \text{ lbf.in}$$



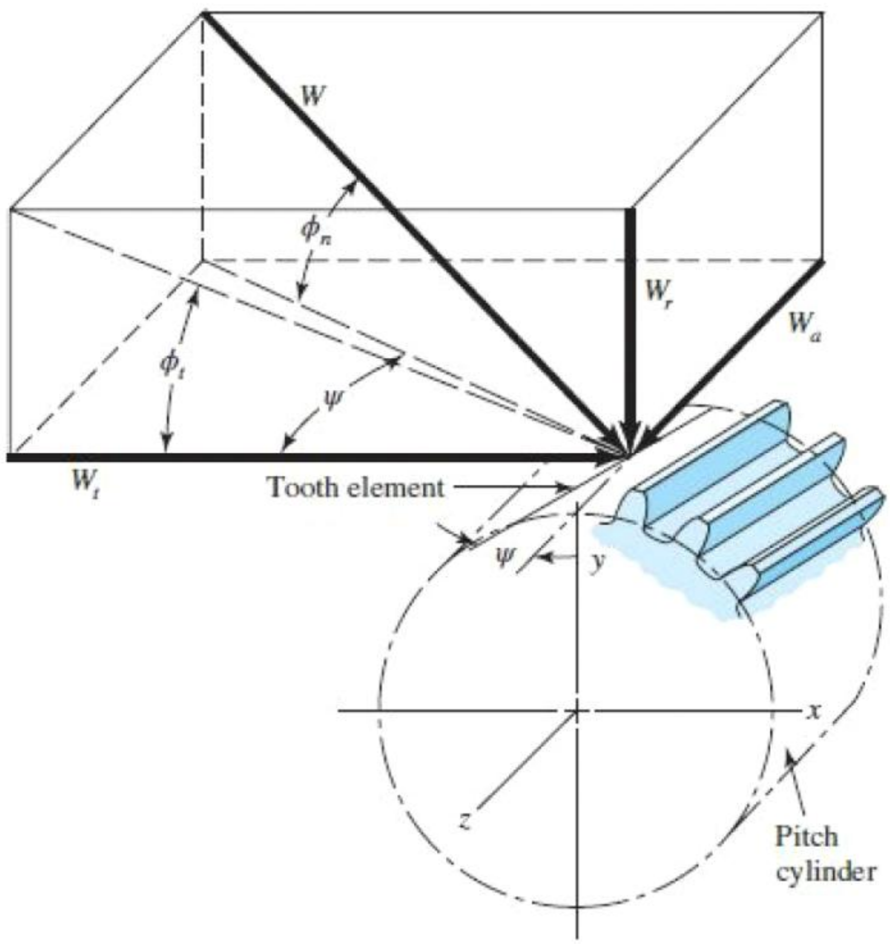
$$\Sigma F^x = F_C^x - W_r - F_D^x = 0 \Rightarrow F_D^x = 71.4 \text{ lbf}$$

$$\Sigma F^y = F_C^y - W_a = 0 \Rightarrow F_C^y = 140 \text{ lbf}$$

$$\Sigma F^z = F_C^z + W_t + F_D^z = 0 \Rightarrow F_D^z = -155 \text{ lbf}$$

$$\boxed{\begin{aligned} F_C &= 118\hat{i} + 140\hat{j} - 251\hat{k} \text{ lbf} \\ F_D &= -71.4\hat{i} - 155\hat{k} \text{ lbf} \\ T &= 1575 \text{ lbf.in} \end{aligned}}$$

Force Analysis Helical Gears



ϕ_n - pressure angle in normal direction

ϕ_t - pressure angle in tangential direction

ψ - helix angle

W_t is computed from power

$$\underline{W}_r = W_t \tan \phi_{\underline{t}}$$

$$W_a = W_t \tan \psi$$

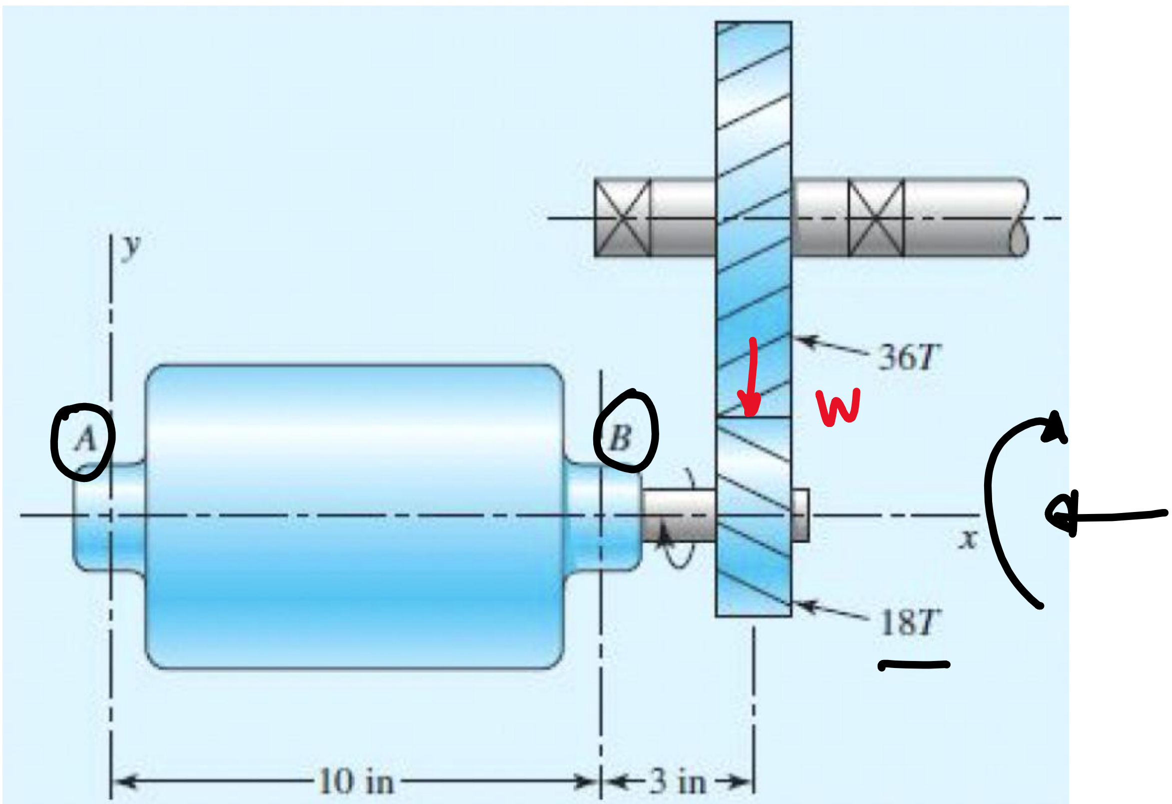
$$W = \frac{W_t}{\cos\phi_{\underline{n}} \cos\underline{\psi}}$$

Also note that

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

Q3

An electric motors transmits 1 hp at 1800 rev/min in the clockwise direction as viewed from the positive x-axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20 degrees, a helix angle of 30 degrees, and a normal diametral pitch of 12 teeth per in. Create a 3-dimensional sketch of the motor shaft and the pinion and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.



$P = 1\,hp$

$\omega = 1800\ rev/min$

$\phi_n = 20°$

$\psi = 30°$

$P_n = 12$

$\phi_t = ?$

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \Rightarrow \cos 30^\circ = \frac{\tan 20^\circ}{\tan\phi_t}$$

$$\phi_t = 22.8^\circ$$

$$p_t = p_n \cos\psi = 12 \cos 30^\circ = 10.39 \text{ teeth/in}$$

$$d = \frac{N}{p_t} = \frac{18}{10.39} = 1.723 \text{ in}$$

$$\omega = 1800 \frac{\text{rev}}{\text{min}} = \frac{1800\,(2\pi)}{(60)} = 163.2 \frac{\text{in}}{\text{s}}$$

$$P = 1\text{hp} = 550 \text{ lb.ft/s}$$

$$P = \left(W_t \frac{d}{2}\right)\omega \Rightarrow 550 = W_t \left(\frac{1.723}{12}\right) 163.2$$

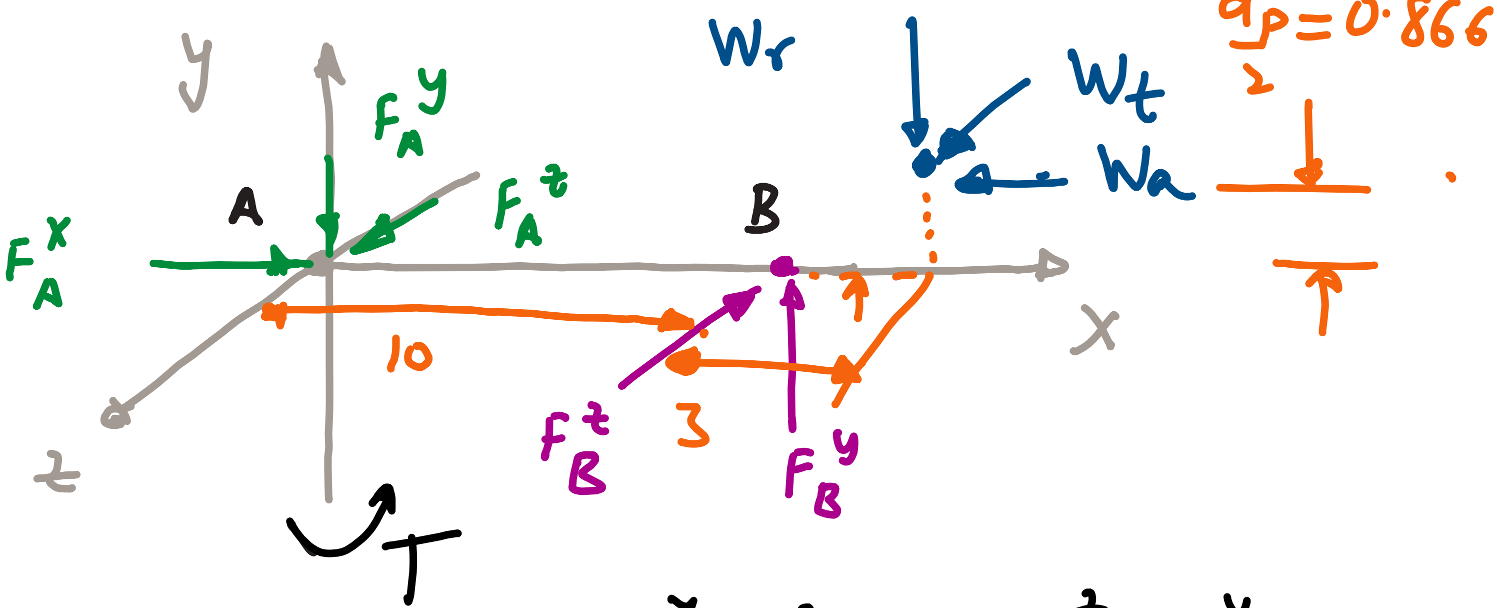
in to ft

$$\boxed{W_t = 40.4 \text{ lbf}}$$

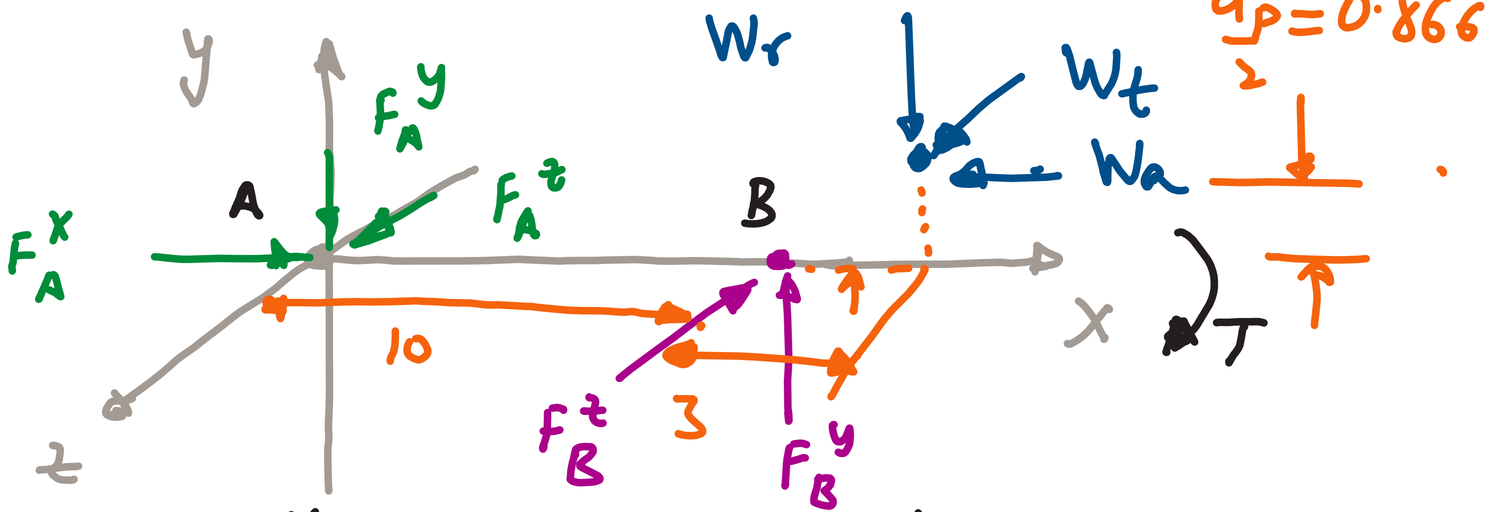
$$W_r = W_t \tan\phi_t = 40.4 \tan 22.8^\circ; \quad \boxed{W_r = 17 \text{ lbf}}$$

$$W_a = W_t \tan\psi = 40.4 \tan 30^\circ; \quad \boxed{W_a = 23.3 \text{ lbf}}$$

FBD Q3



Unknowns : $F_A^x, F_A^y, F_A^z, T, F_B^z, F_B^y$



$$\Sigma F_x = F_A^x - W_a = 0 \Rightarrow F_A^x = 23.3\ \text{lbf}$$

$$\Sigma M_A^x = -T + W_t\, 0.866 = 0 \Rightarrow T = 35\ \text{lbf.in}$$

$$\Sigma M_A^y = +F_B^z(10) - W_t(13) = 0 \Rightarrow F_B^z = 52.5\ \text{lbf}$$

$$\Sigma M_A^z = +F_B^y(10) - W_r(13) + \underbrace{W_a\, 0.866}_{=0} \Rightarrow F_B^y = 20.1\ \text{lbf}$$

$$\Sigma F_y = F_B^y - F_A^y - W_r = 0 \Rightarrow F_A^y = 3.1\ \text{lbf}$$

$$\Sigma F_z = F_A^z - F_B^z + W_t = 0 \Rightarrow F_A^z = -12.1\ \text{lbf}$$

$$\boxed{\begin{aligned} F_A &= (23.3\hat{i} - 3.1\hat{j} + 12.1\hat{k})\ \text{lbf} \\ F_B &= (20.1\hat{j} - 52.5\hat{k})\ \text{lbf} \\ T &= -35\hat{i}\ \text{lbf.in} \end{aligned}}$$

Final Exam

May 7. ERF 1003 (Lecture room)

8-10 AM || 4 letter size pages Front/Back

Comprehensive Exam : all HWs + all Lectures
all Exam so far

Finals:

50 points : 50% long (4 Questions)

50% MCQ/Fill in the blanks (12 Questions)

50% Based on HW 10-13 (new stuff)
+ relevant lectures

50% Based on HW 1-9
+ relevant lectures
+ Exam 1-3

long 1 Question on combined
looking