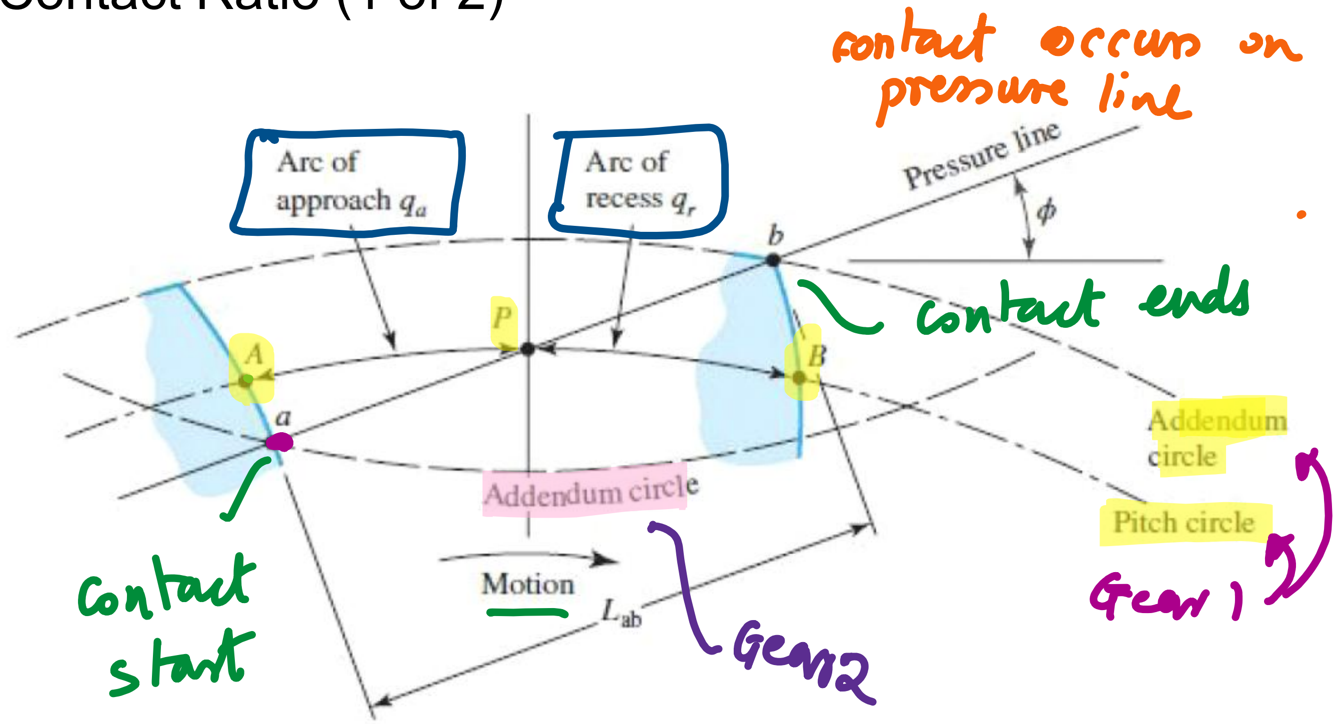
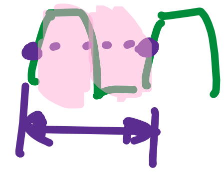
Contact Ratio (1 of 2)



- contact starts at 'a' and ends at 'b'
- 'a' & 'b' are where the pressure line intersects the addendum circle
- Teeth drawn at 'a' and 'b' intersect the pitch circle at A and B.
- AP is the arc of approach (q_a)
  PB is the arc of recess. (q_r)

  AP+PB is the arc of action $\boxed{(q_t)}$

(p) pitch



Contact Ratio (2 of 2)

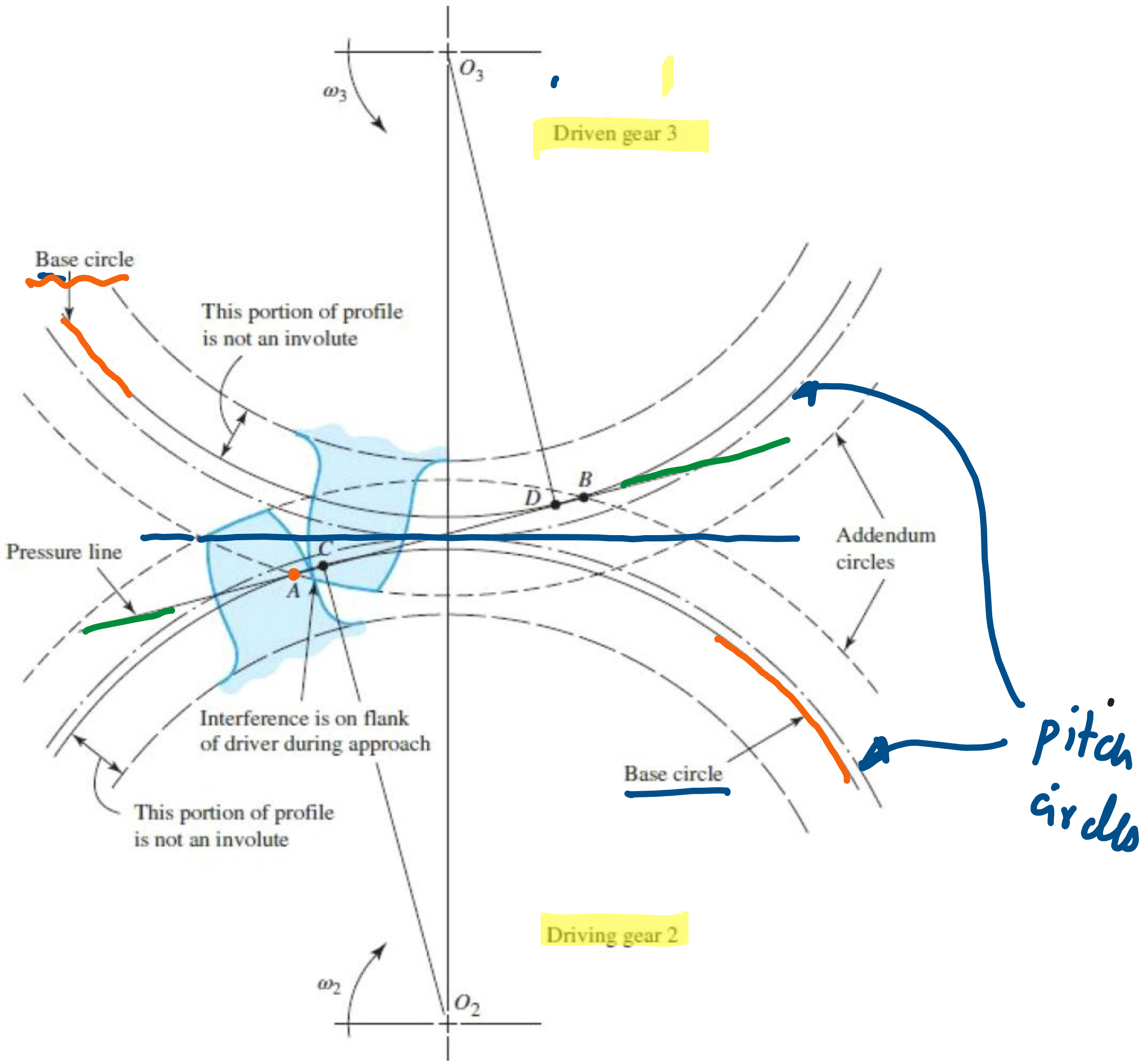
- If the arc of action is equal to the pitch. i.e., $q_t = p$. This means that exactly one tooth + space occupies the arc of action. In other words, when a tooth is beginning contact at 'a', the adjoining tooth is leaving contact at 'b'. Thus only one tooth will be in contact

- If $q_t = 1.2p$ implies that one tooth + 20% of the second tooth are in contact

- contact ratio (m_c) . $\boxed{m_c = \dfrac{q_t}{p}}$

- General recommendation is $1 \leq m_c \leq 1.2$ because inaccuracies in mounting might increase the m_c, increasing the possibility of impact between the teeth.

Interference (1 of 2)



- A and B on the pressure line are the initial & final contact points
- C and D are contacts on base circle
- C & D inside A & B leads to interference.

Interference (2 of 2)

- Interference is explained as follows.
- CD is the involute portion of the gear
- But AC and DB are outside the involute portion
- Hence there interference at AC & DB
- Since this is a geometric phenomenon, we can compute the minimum number of teeth to avoid interference

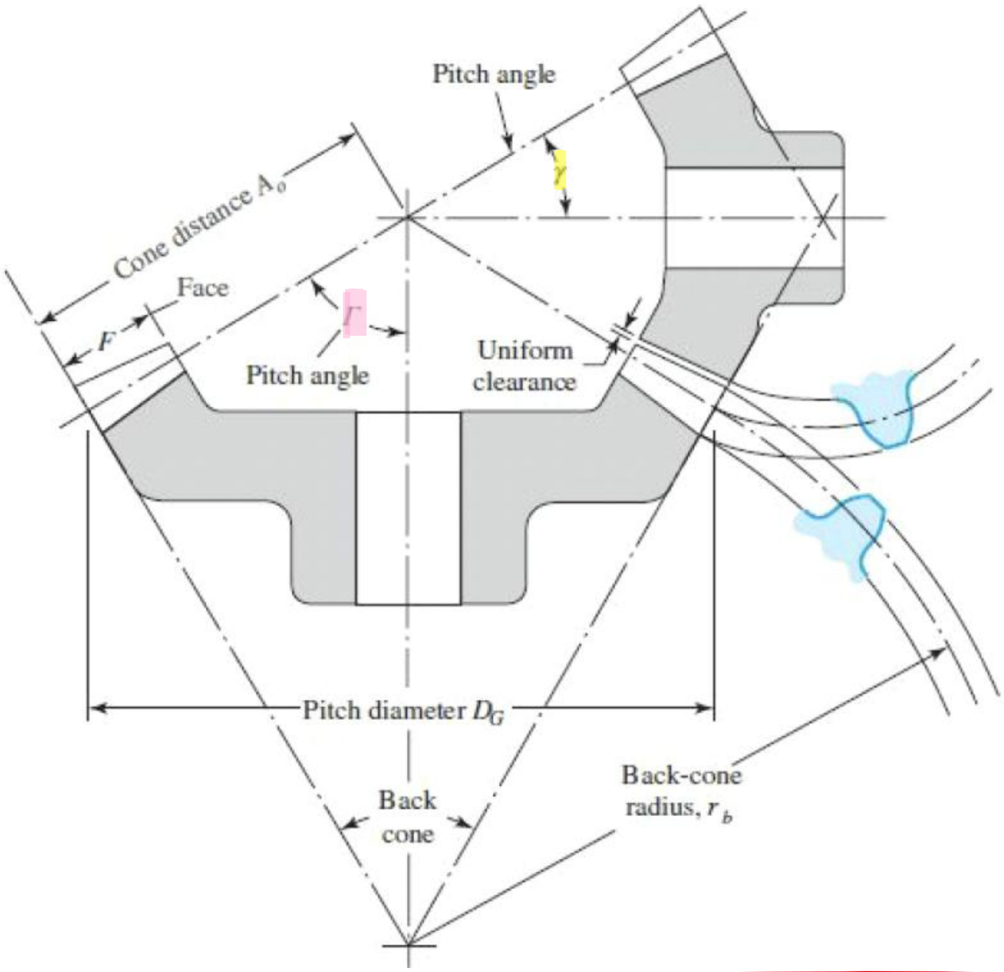
$$N_p = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$m = N_G / N_p$$

$$k = \begin{cases} 1 & \text{full depth} \\ 0.8 & \text{stub teeth} \end{cases}$$

ϕ = pressure angle

Bevel Gears



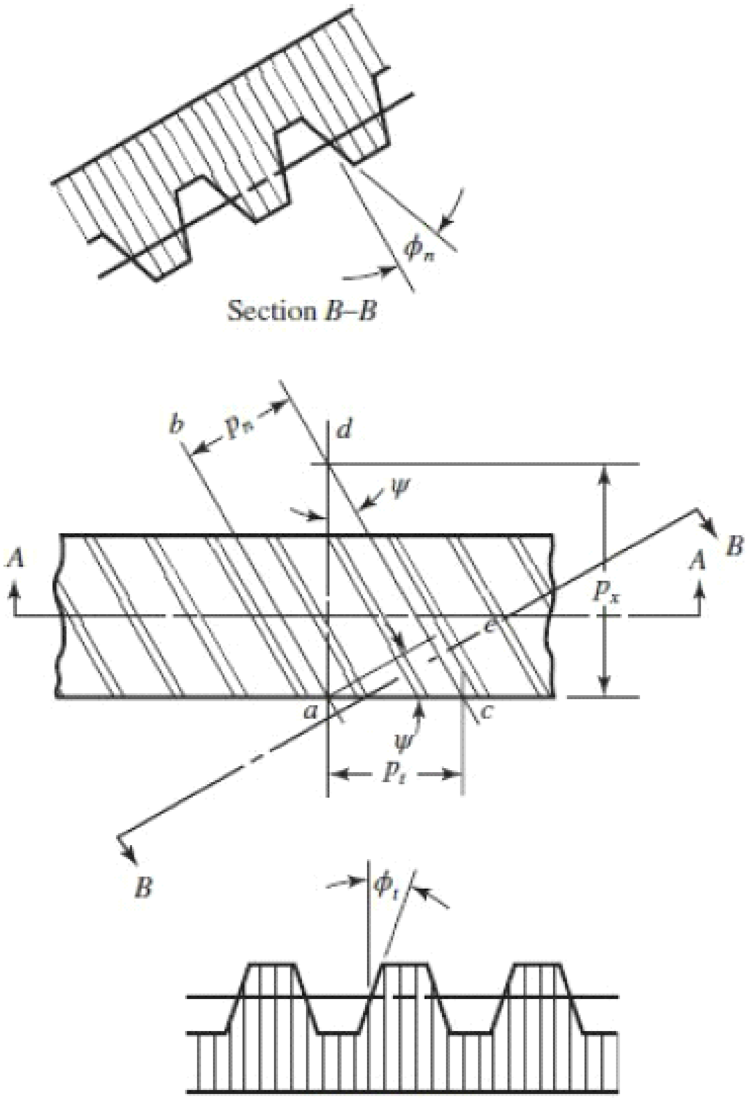
$$\boxed{\tan \gamma = \frac{N_p}{N_G} \quad \text{and} \quad \tan \Gamma = \frac{N_G}{N_p}}$$

N_p, N_G – number of teeth of pinion and gear

γ, Γ – pitch angle of pinion & gear

ttrr

Helical Gears



Section A–A

$$\boxed{\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}}$$

ϕ_n, ϕ_t pressure angle in the normal and tangential direction.

ψ – helix angle

$$P_t = \frac{N}{d} \qquad p_n P_n = p_t P_t = \pi$$

$$P_n = \frac{P_t}{\cos\psi} \qquad p_n = p_t \cos\psi$$

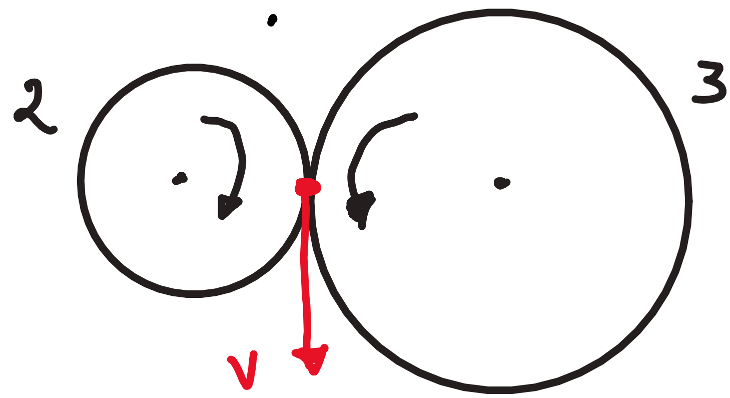
$$p_x = \frac{p_t}{\tan\psi}$$

p_t, p_n – transverse, normal circular pitch

p_x – axial pitch

P_t, P_n – transverse normal diametral pitch.

Gear Trains Formula



$$\boxed{\frac{N_2}{N_3} = \frac{d_2}{d_3} = -\frac{n_3}{n_2}}$$

N – number of teeth
d – pitch diameter
n – speed in rev/min

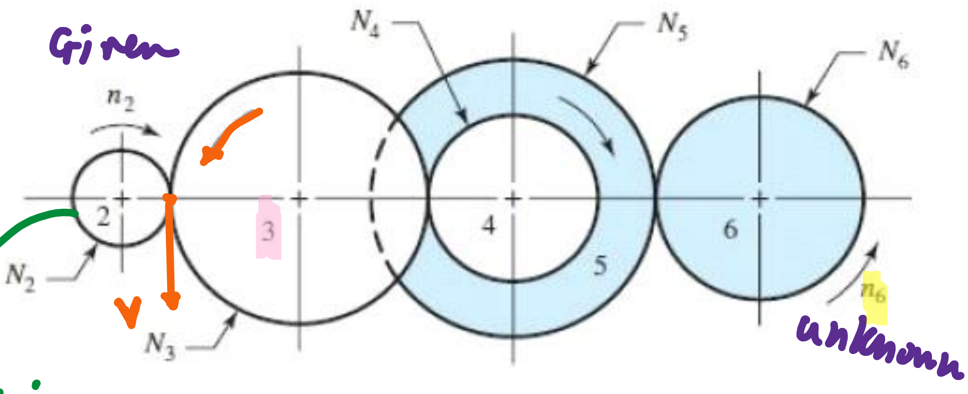
NOTE

$$\boxed{N \propto d \propto \frac{1}{n} \propto \frac{1}{\omega}}$$

$P = N/d$

↑ pitch

Gear Train Example



driving (pinion)

always start numbering with 2 because 1 is reserved for the shaft for gear 2.

$$\frac{n_3}{n_2} = -\frac{N_2}{N_3}, \quad \frac{n_4}{n_3} = -\frac{N_3}{N_4}$$

same shaft

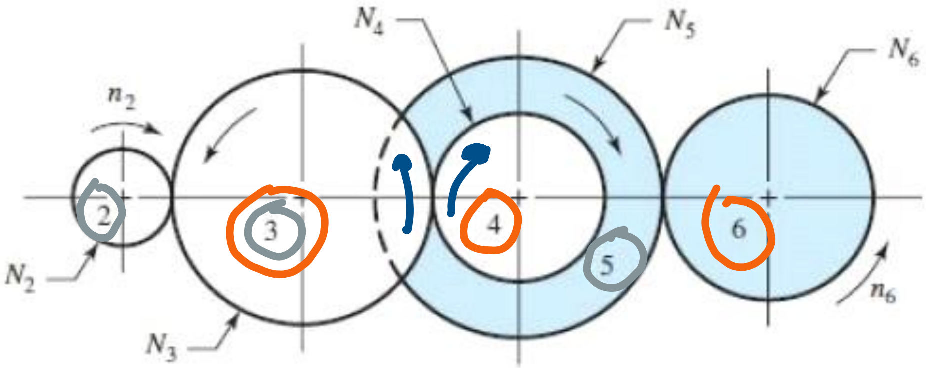
$$\left(\frac{n_5}{n_4} = 1\right), \quad \frac{n_6}{n_5} = -\frac{N_5}{N_6}$$

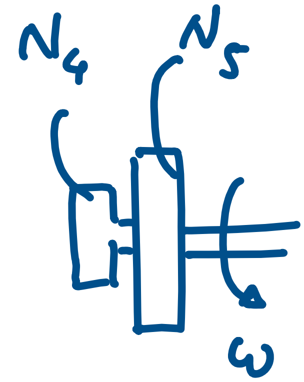
$$n_6 = \left(-\frac{N_5}{N_6}\right)(1)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_2}{N_3}\right) n_2$$

$$\boxed{n_6 = -\left(\frac{N_5}{N_6}\right)\left(\frac{N_2}{N_4}\right) n_2}$$

Note. N_3 does not appear in this expression. It acts as an idler ensuring that 2 and 4 spin in the same direction.

Gear Trains, Generlization





Train value (e)

$$\boxed{e = \pm \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}}$$

If last gear rotates in the same direction as the first the sign of e is positive else it is negative

$$\boxed{n_L = e\, n_F}$$

n_L, n_F is speed of last & first gear

$\boxed{2, 3, 5}$ are drivers, $\boxed{3, 4, 6}$ are driven

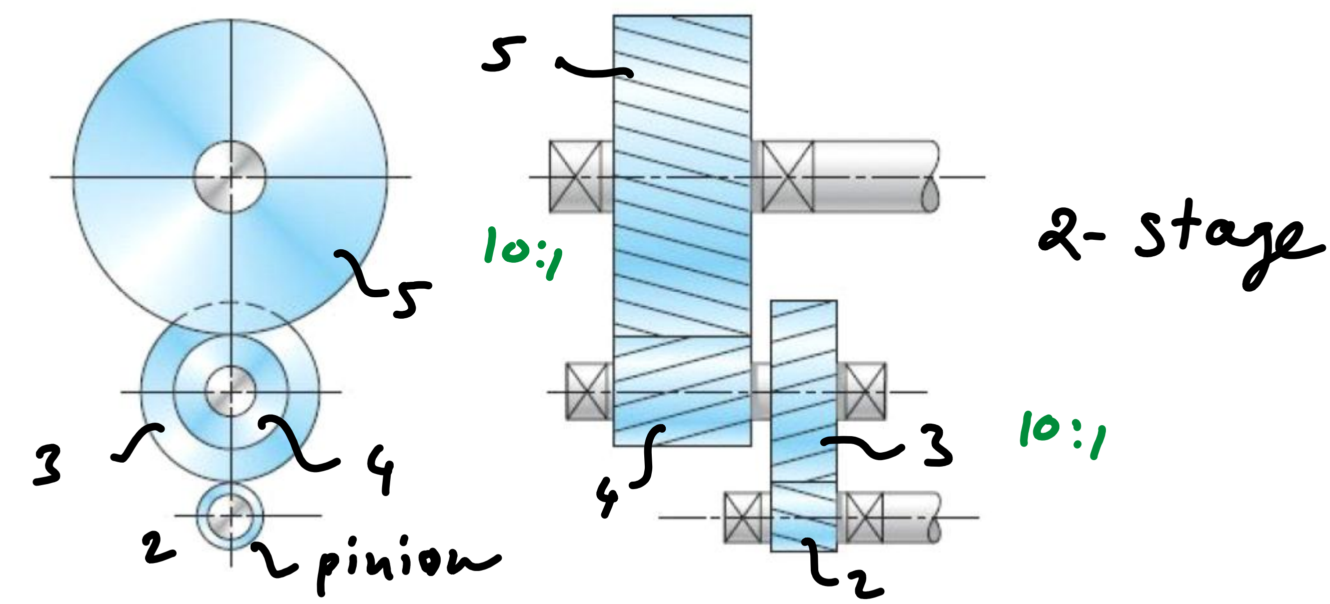
last gear rotates in opposite direction of first

$$e = -\frac{N_2 \cancel{N_3} N_5}{\cancel{N_3} N_4 N_6} = -\frac{N_2}{N_4}\frac{N_5}{N_6}$$

$$n_6 = e\, n_2 \Rightarrow \boxed{n_6 = -\frac{N_2}{N_4}\frac{N_5}{N_6} n_2}$$

$$n_6 = e\, n_2 \Rightarrow \boxed{n_6 = -\frac{N_2}{N_4}\,\frac{N_5}{N_6}\,n_2}$$

Compounding gear train



A gear reduction of 10:1 can be obtained with a pair of gears.

To obtain a larger reduction within a limited space compound gears as shown above are used.

In the above compound gear train a reduction of 100:1 is obtained using 2-stage gear reduction.

To choose gear in a n-stage compound gear train with a gear ratio of m, use $\boxed{m^{1/n}}$ for reduction at each stage. Then tune them up for other constraints.

$(10^6)^{1/n} \approx 10 \quad n = 6$

then take them up for other constraints;

$$(10^6)^{1/n} \approx 10 \qquad n = 6$$

Q1

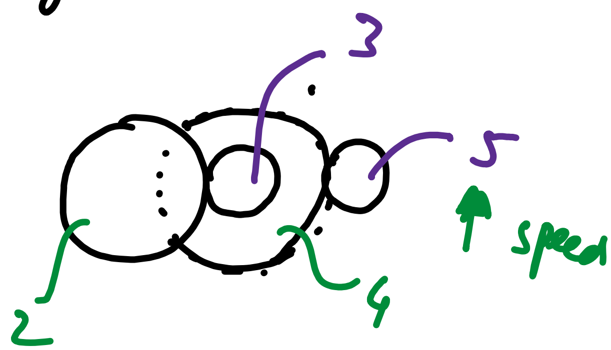
A gear box is needed to provide 30:1 increase in speed while minimizing the overall gearbox size. Specify appropriate tooth numbers

Since the reduction is between 10:1 and $10^2:1$, we need 2-stages.

Each stage we want

$$(M)^{1/n} = (30)^{1/2}$$

$$= 5.4472$$



Lets compute the minimum number of gears.

$$N_p = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$m = 5.4472$, $\phi = 20^\circ$, $k = 1$ (full depth)

Solve $N_p = 15.86 \simeq 16$ (minimum number of teeth to avoid interference

$N_3 = N_5 = N_p = 16$ (driven / gear)] Design

$N_2 = N_4 = 16(5.4472) = 87.64 \approx 88$

(driving gear/pinion)

Check

$$\underline{e} = \frac{\text{driving gear}}{\text{driven gear}} = \frac{N_2 \cdot N_4}{N_3 \cdot N_5} = \frac{88}{16} \cdot \frac{88}{16}$$

$$e = 30.25 > 30$$

$$N \propto d \propto \frac{1}{n} \propto \text{Torque}$$

$$\text{Torque} \times n \; \cancel{\propto} \; \text{Power}$$

Q2

A gear box is needed to provide exact 30:1 increase in speed while minimizing the overall gearbox size. Specify appropriate tooth numbers

Previous example, we got a gear reduction of 30.25.

Choose minimum teeth for N_3, N_5

$$N_3 = N_5 = 16$$

$$e = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = 30$$

$$= (6)(5)$$

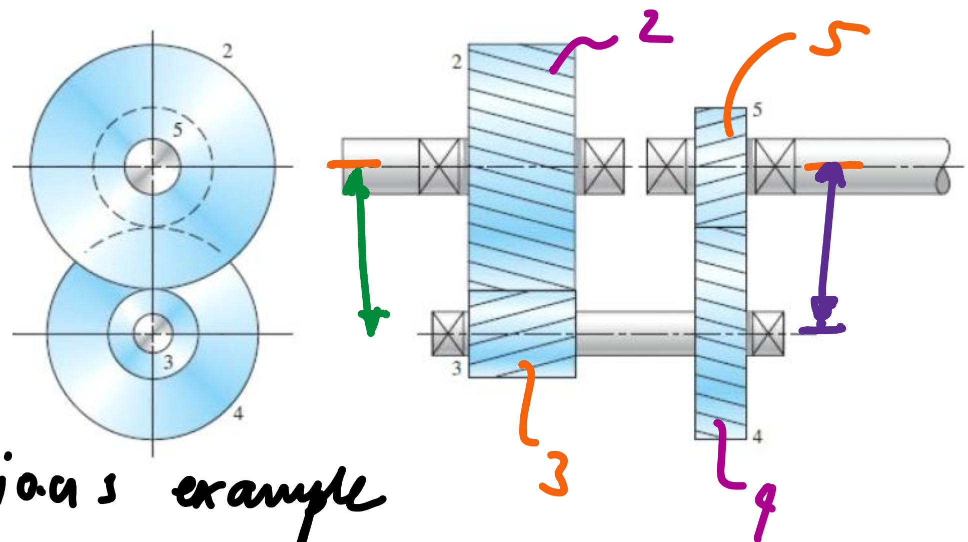
$$N_2 = 6(16) = 96$$

$$N_4 = 5(16) = 80$$

Design

Q3

The compound reverted gear train needs to provide exact 30:1 increase in speed. Specify appropriate tooth numbers



From previous example

$$\frac{N_2}{N_3} = 6 \quad -(1), \qquad \frac{N_4}{N_5} = 5 \quad -(2)$$

We also need to have

$$\frac{d_2}{2} + \frac{d_3}{2} = \frac{d_4}{2} + \frac{d_5}{2}$$

Since $p = N/d$

$$\frac{N_2}{\cancel{2P}} + \frac{N_3}{\cancel{2P}} = \frac{N_4}{\cancel{2P}} + \frac{N_5}{\cancel{2P}} \quad (3)$$

We have 3 equations and 4 unknowns.

(i) Assume N_3 based on minimum gear formula and calculate the rest using the equations

X $\underset{\sim\sim\sim}{N_3 = 16}$; $N_2 = 6N_3 = 96$

$N_2 + N_3 = \underline{N_4} + N_5$ X

$96 + 16 = \underline{5N_5} + N_5$

$112 = 6N_5$

$\Rightarrow N_5 = \frac{112}{6} = \boxed{18.67} \approx 19$

round

But the issue is that

✓ $e = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \left(\frac{96}{16}\right)\left(\frac{5 \cdot 19}{19}\right) = 30$ ✓

X $96 + 16 = 5(19)$

$112 \neq 114$ $[N_2 + N_3 \neq N_4 + N_5]$

We need to solve with

$N_3 = 17, 18, \ldots$

till both the condition e & distance equations are satisfied

(ii) Assume $N_3 = N$ (parameter)

$$N_2 = 6 N_3 = 6N$$

$$N_4 = 5 N_5$$

$$N_2 + N_3 = N_4 + N_5$$

$$6N + N = 5N_5 + N_5$$

$$7\underline{N} = 6 N_5$$

Find integer solutions to this equation

$$N = 6 \,;\; N_5 = 7 \quad \text{X}$$

$N_{min} = 16$

$$7N = 6N_5$$

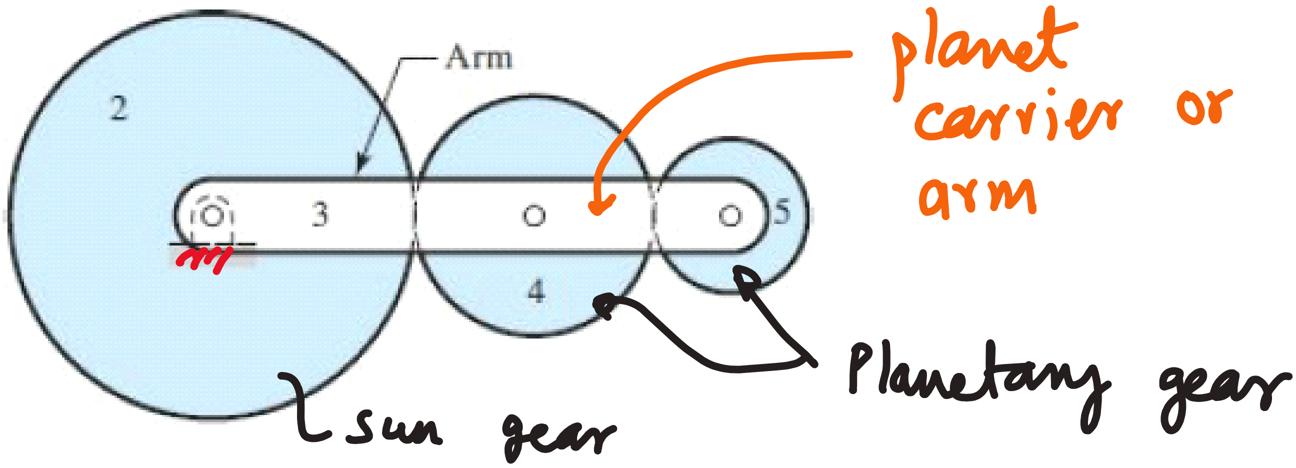
a) $7 \cdot 16 = 6 \cdot N_5$ $\qquad N_5 = 18.67$ ✗

b) $7 \cdot 17 = 6 \cdot N_5$ $\qquad N_5 = 19.8$ ✗

c) $7 \cdot 18 = 6 N_5$ $\qquad N_5 = 21$ ✓

$$\boxed{\begin{array}{l} N_5 = 21 \; ; \; N_4 = 5N_5 = 105 \\ N = N_3 = 18 \; ; \; N_2 = 6N = 108 \end{array}}$$

Planetary or Epicylic Gear Trains



Planetary gears include (i) sun gear (ii) planet gears (iii) planet carrier or arm

We need to specify 2 inputs to be able to analyze motion. However, most often the arm is fixed to the frame and has zero speed

$$\boxed{\frac{n_5 - n_3}{n_2 - n_3} = e}$$

More generally,

$$\boxed{\frac{n_L - n_A}{n_F - n_A} = e}$$

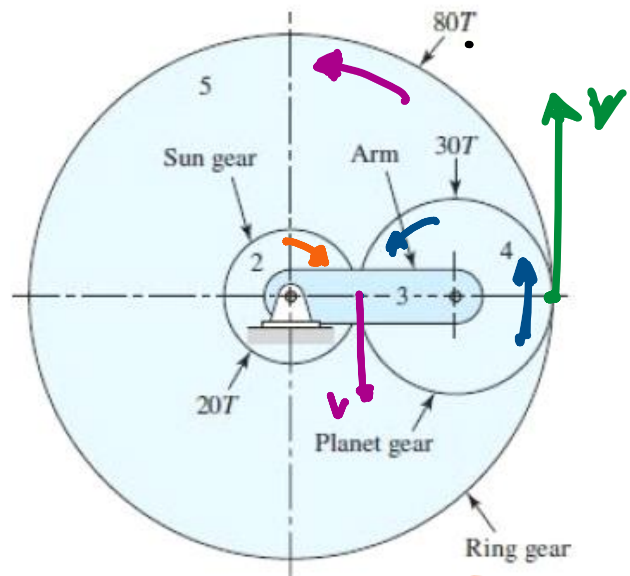
L, F, A are last gear first gear and arm

Q4

The sun gear is the input and it is driven clockwise at 100 rev/min. The ring gear is held stationary as it is fastened to the frame. Find the rev/min and direction of rotation of the arm and the gear 4

$n_3 = ?$

$n_4 = ?$



fixed

- unlock ring gear
- lock arm 3
- planet / ring

$$e = -\frac{\text{teeth on driving}}{\text{teeth on driven}} = -\frac{N_2}{\cancel{N_4}} \cdot \frac{\cancel{N_4}}{N_5} = -\frac{20}{80}$$

$e = -0.25$

$$\frac{n_L - n_A}{n_F - n_A} = e \Rightarrow \frac{0 - n_A}{-100 - n_A} = -0.25$$

$$\Rightarrow \boxed{\begin{array}{l} n_A = -20 \text{ rev/min} \\ n_A = 20 \text{ rev/min} \quad \text{CW} \end{array}}$$

$$\frac{n_{43}}{n_{23}} = \frac{n_4 - n_3}{n_2 - n_3} = - \frac{\text{teeth driving}}{\text{teeth driven}}$$

$$= - \frac{N_2}{N_4} = \frac{-20}{30}$$

$$\frac{n_4 - (-20)}{-100 - (-20)} = \frac{-20}{30}$$

Solving

$$\boxed{\begin{array}{l} n_4 = +33.5 \text{ rev/min} \\ n_4 = 33.5 \text{ rev/min CCW} \end{array}}$$