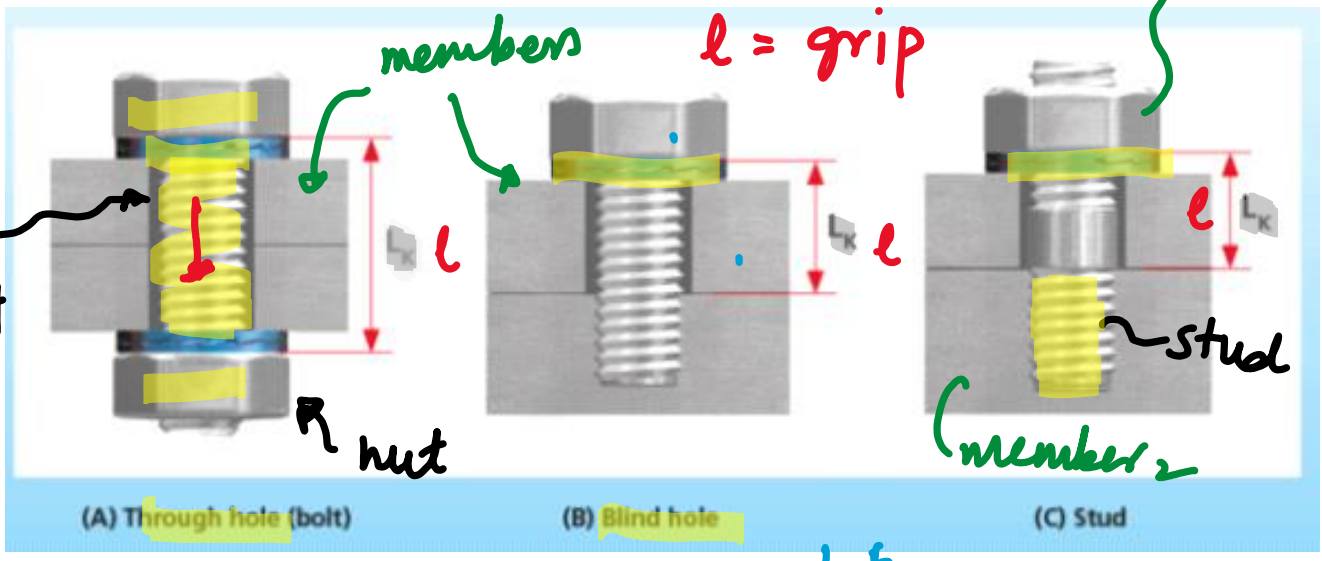
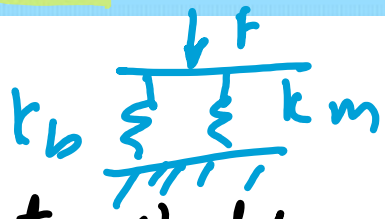
8-4 Fastener stiffness



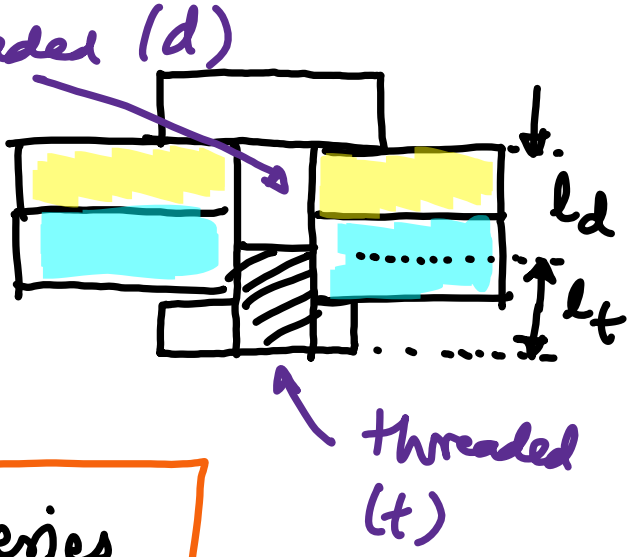


(A) Through hole
- twisting the nut stretches the bolt to produce a clamping force
- clamping force is known as pretension or bolt preload
- members are under compressive load

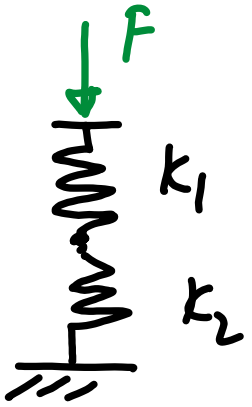
(B) Blind hole
- cap screw is stretched as it clamps

(C) Stud
- rod threaded on both ends
- is permanent
- nut / washer are non permanent

The threaded (t) and unthreaded (d) portion are in series.



Review: Springs in series



$$\delta = \delta_1 + \delta_2$$

$$\delta = \frac{F}{k_1} + \frac{F}{k_2} = F\left(\frac{k_1+k_2}{k_1 k_2}\right) = \frac{F}{k_{eq}}$$

Thus $$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

From stress-strain relation

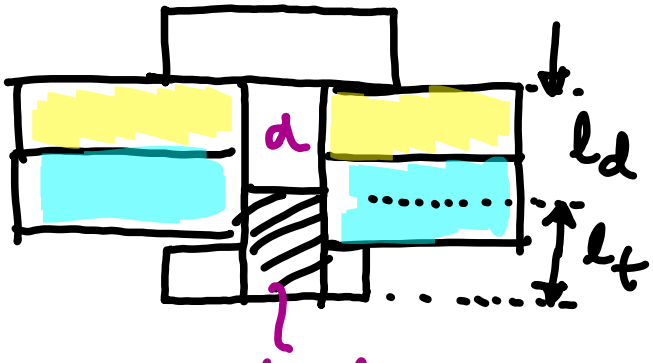
$$\sigma = E\,\varepsilon$$

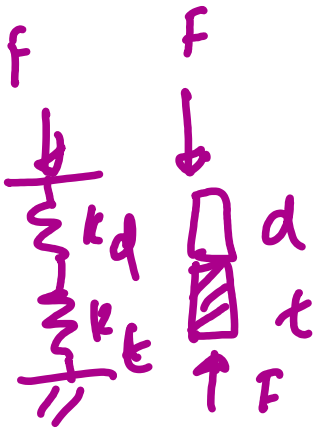
$$\frac{P}{A} = E\,\frac{\delta}{L} \Rightarrow \delta = \frac{P}{[AE/L]} = k$$

Thus $$k = \frac{AE}{L}$$

① Partial threaded / unthreaded

$k = \frac{AE}{l}$





$$k_{eff} = \frac{k_t k_d}{k_t + k_d} = \frac{\left(\frac{A_t E}{l_t}\right)\left(\frac{A_d E}{l_d}\right)}{\left(\frac{A_t E}{l_t} + \frac{A_d E}{l_d}\right)}$$

$$\boxed{k_{eff} = \frac{A_t A_d E}{A_t l_d + A_d l_t}}$$ corrected

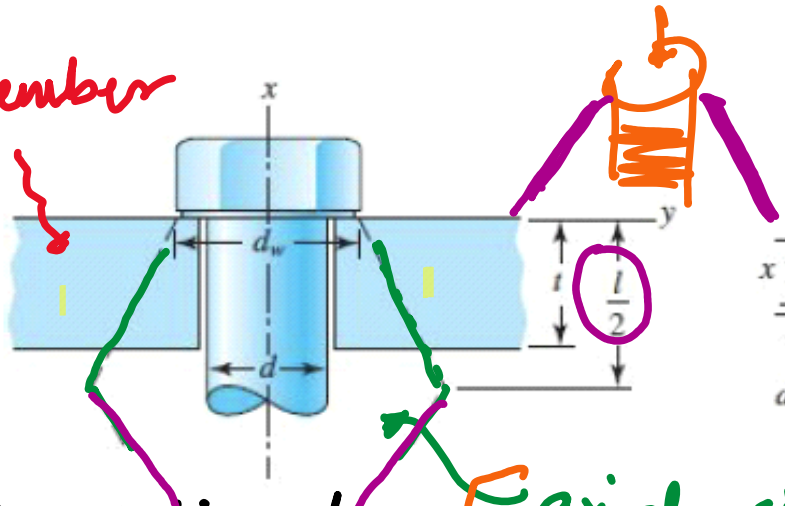
② Mostly unthreaded

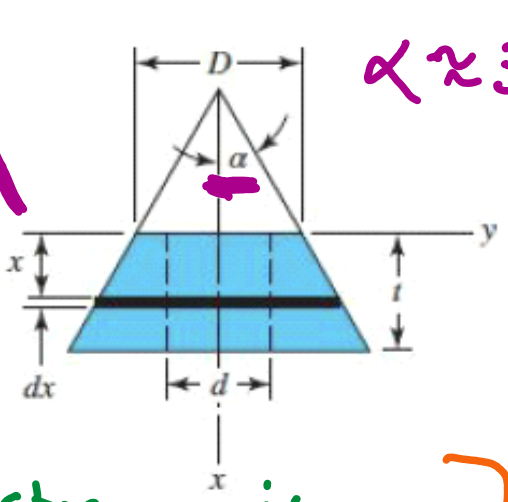
Neglect k_t $\quad k_{eff} = k_d = \frac{A_d E}{l_d}$

③ Fully threaded

$$k_{eff} = k_t = \frac{A_t E}{l_t}$$

8.5 Member Stiffness





$\alpha \approx 30^\circ$

d : screw diameter

d_w, D = washer diameter

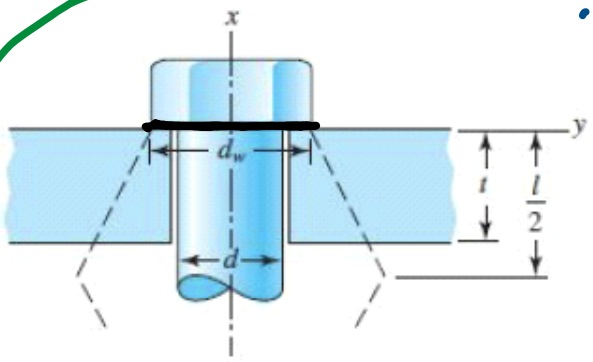
[axial stress is spread over area as a function of length.]

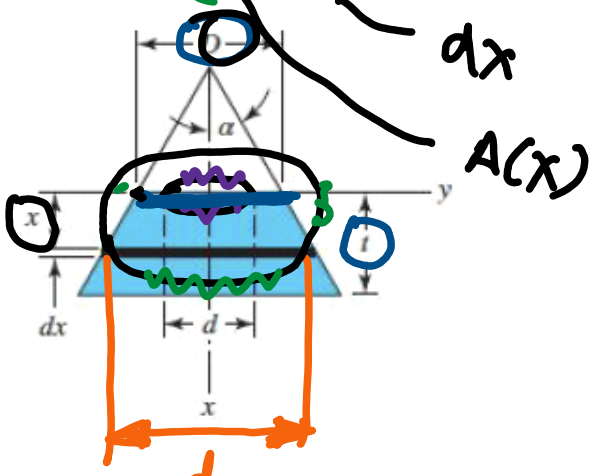
- Empirically found that stress area extends beyond the fastener diameter

- A pressure cone approach is used.
  Conical frusta
- The cross-section area for compressive stress is a function of length as shown on the figure on the right

- We will assume the cone angle α is fixed. $\alpha \approx 30^\circ$ gives a good approximation

$\delta = P/(AE/L) = P/k$





dx

A(x)

d_0

From the left Figure

$$d_0 = x\tan\alpha + \frac{D}{2}$$

$$d\delta = \frac{P\,dx}{E\,A(x)}$$

$$A(x) = \frac{\pi}{4}(d_0^2 - d_i^2) = \pi\left[\left(x\tan\alpha + \frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

$$\delta = \frac{P}{\pi E}\int_0^t \frac{dx}{\left[x\tan\alpha + \frac{D}{d}\right]^2 - \left(\frac{d}{2}\right)^2}$$

Integrating and solving for $k_i = \frac{P}{\delta}$

$$k_i = \frac{\pi E d \tan\alpha}{\ln\left[\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}\right]}$$

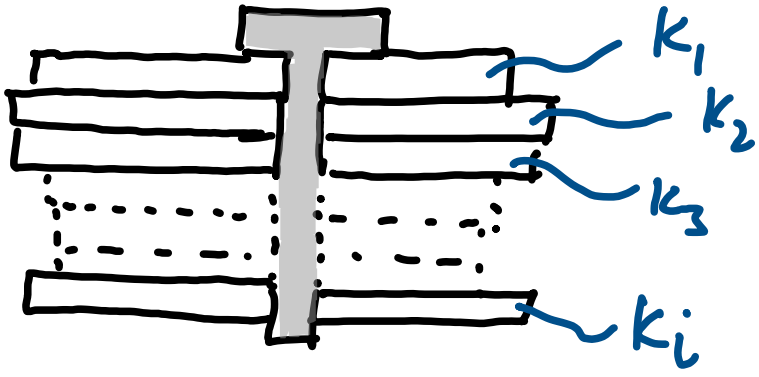
$\times \log_{10}$

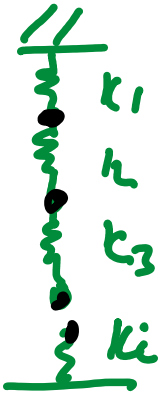
natural log. ln

Assuming $\alpha = 30^\circ$

$$k_i = \frac{0.5774\,\pi E d}{\ln\left[\dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}\right]}$$

The net spring constant k_m for multiple members

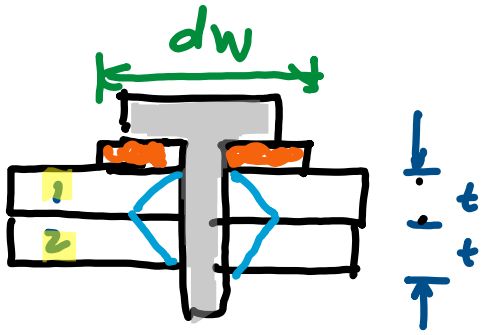




The equation for k_m is used for each cone and the net member stiffness is computed using the springs in series formula

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_i}$$

Members 1, 2 have the same Young's modulus E. Assume they have identical back-to-back cones:



$l = 2t$ and washer diameter is d_w

The 2 members are in series, each with identical stiffness k.

$$k_m = \frac{k}{2} \in \frac{k \cdot k}{k+k}$$

$$k_m = \frac{\pi E d \tan\alpha}{2 \ln\left[\frac{(l\tan\alpha + d_w - d)(d_w + d)}{(l\tan\alpha + d_w + d)(d_w - d)}\right]}$$

For standard hexagon-head bolts & cap screws $d_w = 1.5\,d$ and assume $\alpha = 30°$. Using above formula

d_w – washer diameter

$$k_m = \frac{0.5774\,\pi E d}{2 \ln\left[5\,\frac{0.5774\,l + 0.5d}{0.5774\,l + 2.5d}\right]}$$

Q1

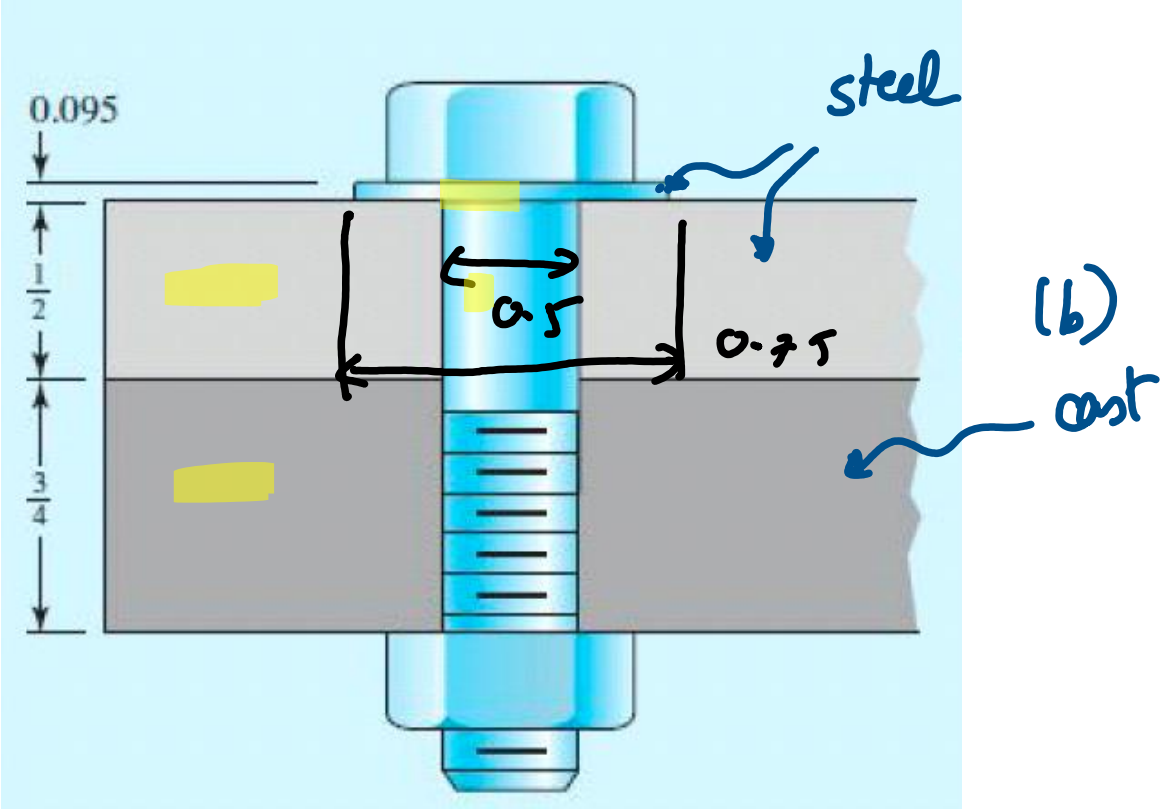
The fastener - member dimension shown below is for the untightened configuration. Compute the following

a) The bolt spring rate, kb. Assume the bolt is made of steel and in the clamped state lt = 1.095 in, ld = 0.25 in, and At = 0.1599 in^2.

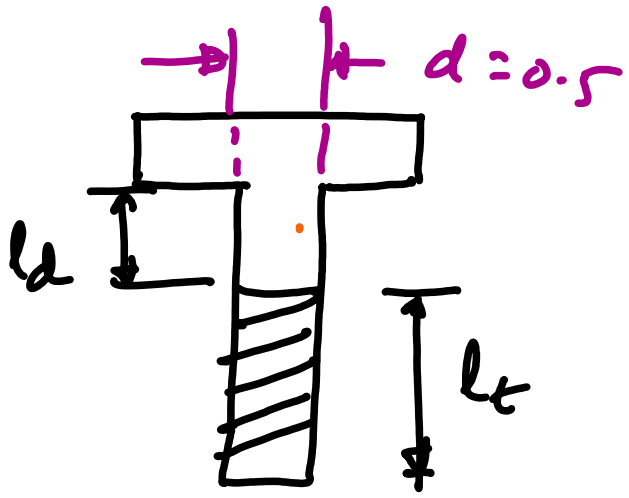
b) Member spring rate, km. Assume the washer and top member plate are made of steel and bottom member plate is made of gray cast iron. Use the pyramid cone method (also known as the conical frusta method)

c) Member spring rate, km. Assume the washer, top and bottom members plate are all made of steel. Use the conical frusta method.

Assume for steel, E = 30 (10^6), gray cast iron E = 14.5 (10^6). The diameter of the bolt is 0.5 in, diameter of the washer is 0.75 in.



(a)



$$K_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

$E = 30\,(10^6)$

$l_t = 1.095$ in

$l_d = 0.25$ in

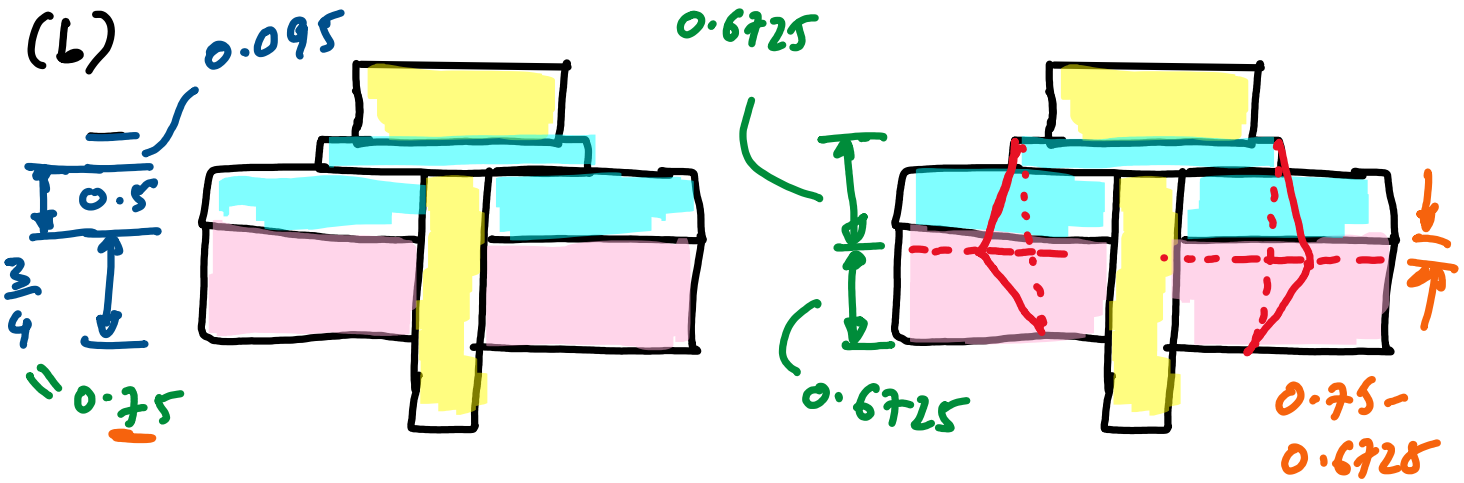
$A_t = 0.1599$ in^2

$$A_d = \frac{\pi d^2}{4} = \frac{\pi\,(0.5^2}{4} = 0.1963$$

$$k_b = \frac{(0.1963)(0.1599)(30)(10^6)}{(0.1963)(1.095) + (0.1599)(0.25)}$$

$$K_b = 3.69\,(10^6)\ \text{lbf/in}$$

(L)



$l = 0.095 + 0.5 + 3/4 = 1.345$

$t = \frac{1}{2} l = 0.6725$

steel

gray cast iron

---

3 spring constants

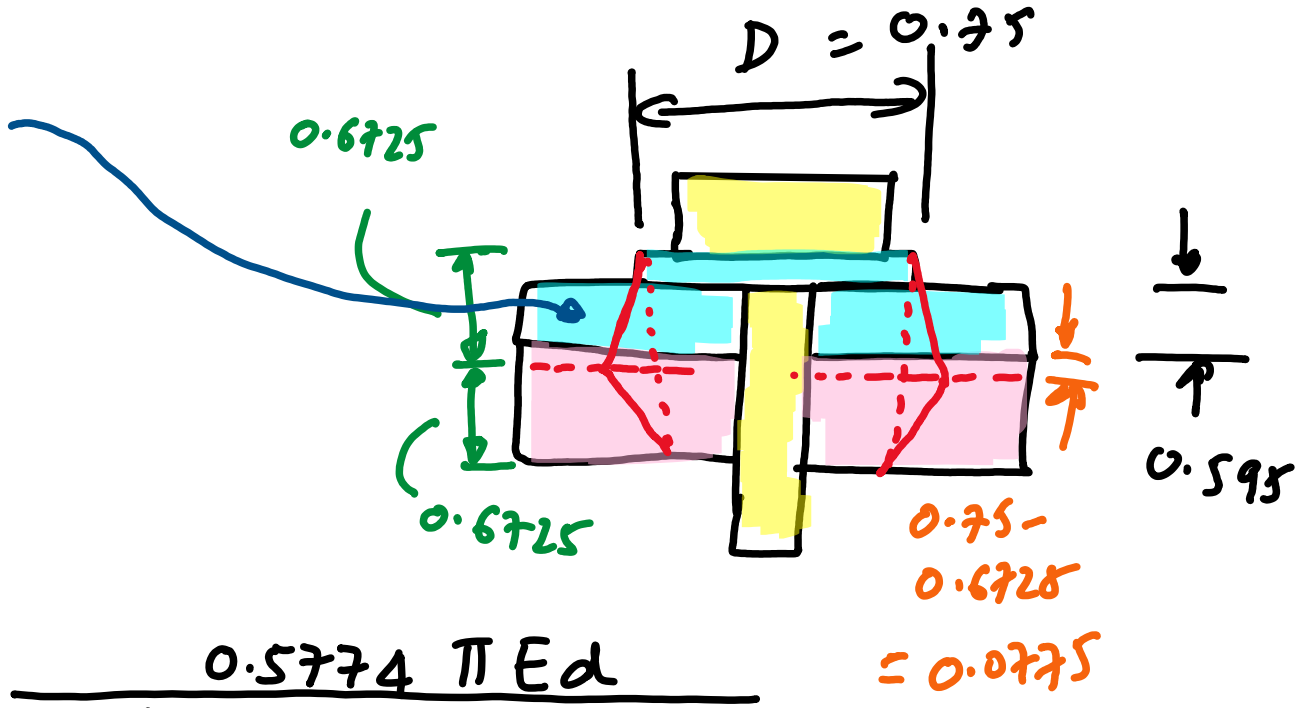
K_1 — steel washer + member 1

K_2 — cast iron member 2 of $t = 0.0775$] Cone 1

K_3 — cast iron member 2 of $t = 0.6725$] Cone 2

$\frac{1}{k_m} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$ (parallel springs)

K_1:



$$K_i = \frac{0.5774\,\pi E d}{\ln\left[\frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}\right]}$$

$E = 30\,(10^6)$; steel

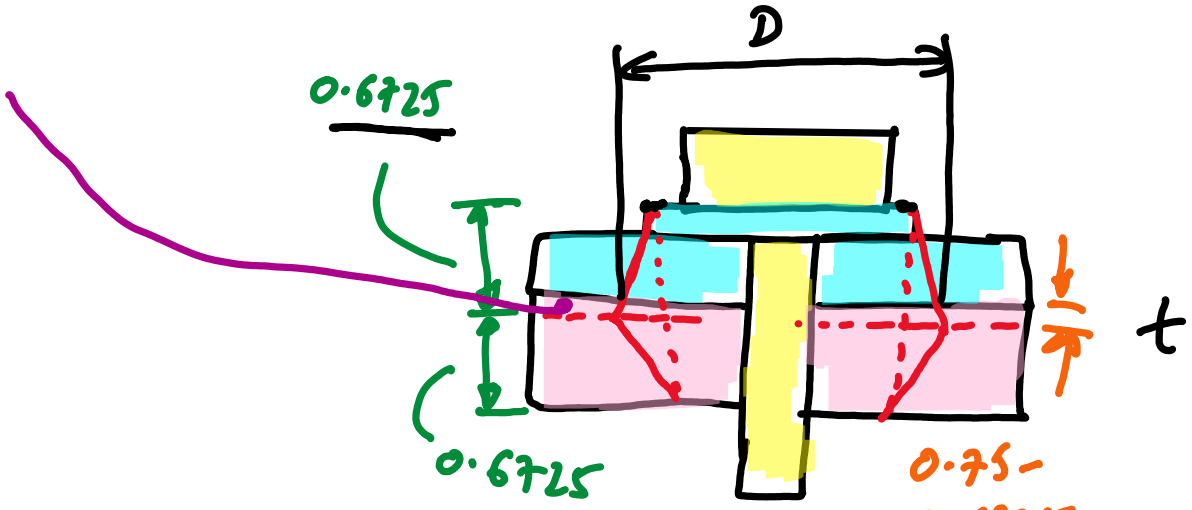
$t = 0.6725 - 0.0775 = 0.595$

$d = 0.5$ (diameter of bolt)

$D = 0.75$ (diameter of washer)

$$K_1 = 30.8\,(10^6)\ \text{lbf/in}$$

K_2:



$$K_i = \frac{0.5774\,\pi E d}{\ln\left[\frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}\right]}$$

$E = 14.5(10^6)$ cast iron

$t = 0.0775$

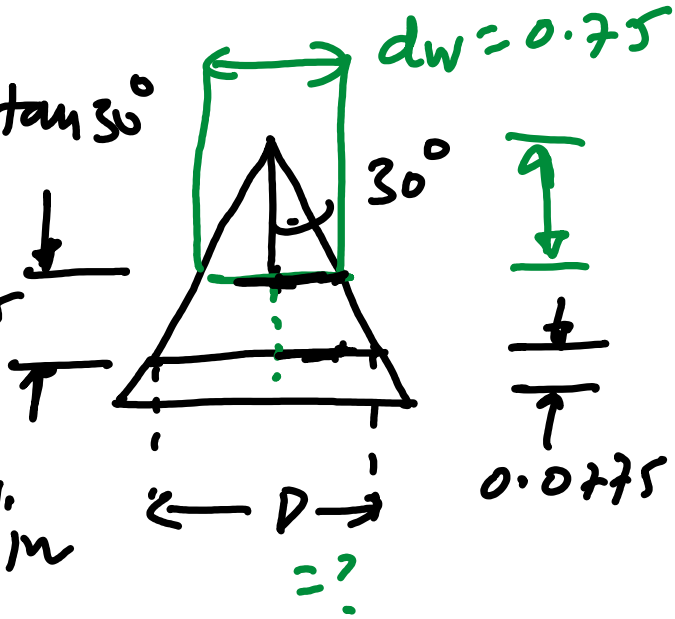
$d = 0.5$ (diameter of the bolt)

$D = d_w + 2x \tan\alpha$

$= 0.75 + 2(0.595)\tan 30°$

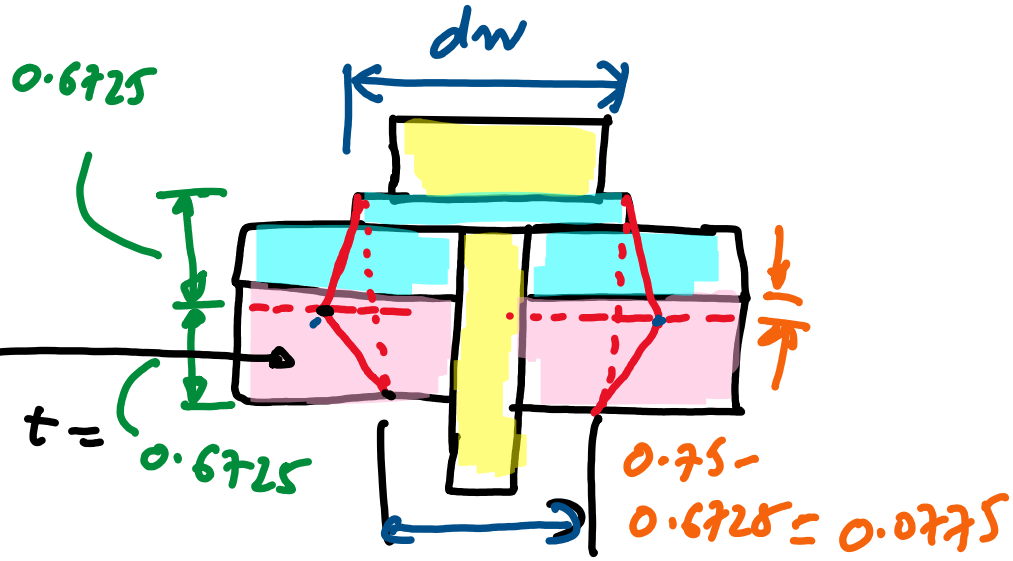
$= 1.437$

$x = 0.595$



$K_2 = 285.5(10^6)$ lbf/in

K_3:



$$K_i = \frac{0.5774\,\pi E d}{\ln\left[\frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}\right]} \qquad D = 0.75$$

$E = 14.5\,(10^6)$ psi gray cast iron

$d = 0.5$ in (bolt diameter)

$t = 0.6725$ in

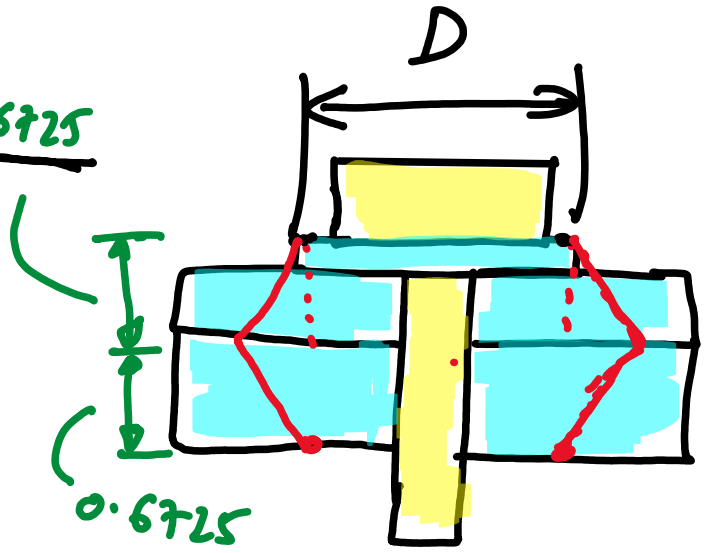
$D = 0.75$ in

$k_3 = 14.15$ lbf/in

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} = \left(\frac{1}{30.8} + \frac{1}{285.5} + \frac{1}{14.15}\right)10^{-6}$$

$$k_m = 9.738\,(10^6) \text{ lbf/in}$$

(c) $t = 0.6725$



steel

Each cone has stiffness K.

Net stiffness is $k_m = \frac{K}{2}$

$$K = \frac{0.5774\,\pi E d}{\ln\left[\frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}\right]}$$

$E = 30(10^6)$ steel

$d = 0.5$ diameter of bolt

$t = 0.6725$

$D = d_w = 0.75$

$k_m = \frac{K}{2} \Rightarrow$ **$k_m = 14.64$ lbf/in**