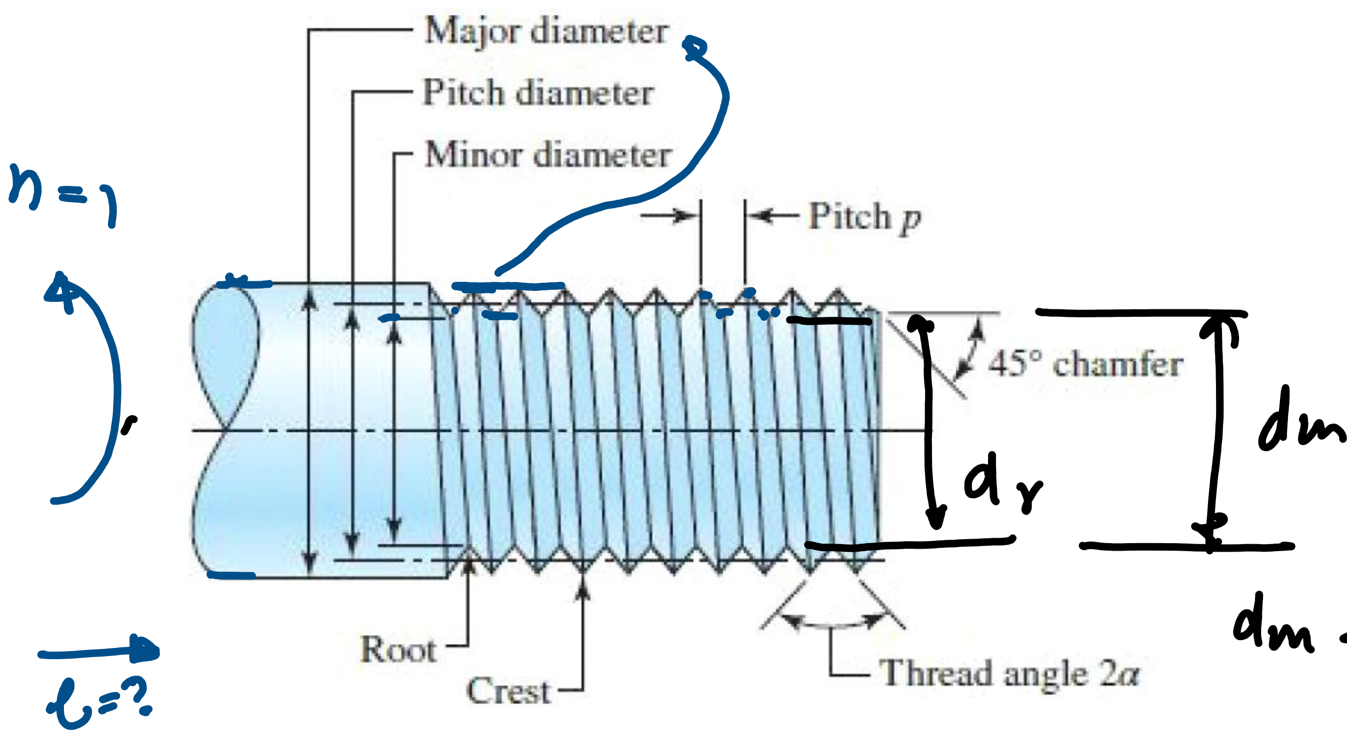
Definition

8-1 Thread Standards & Definitions



$d_m > d_r$

- Pitch: distance between adjacent threads measured parallel to the thread axis

- Minor (d_r) and Major diameter (d) smallest & largest diameter respectively

- Pich diameter (d_p). diameter midway between minor & major diameter

- lead (l) : distance the nut moves parallel to the screw axis when nut is given one turn.

- multiple threaded . 2 or more threads

- screw, bolts, nuts have single thread
  lead (one turn) leads to a travel
  from one pitch to the next.

- double & triple threads: lead equal
  to 2 & 3 times pitch respectively.

- Threads made per the right hand rule
  screw turned clockwise advances it to
  the nut thus tightening the screw.

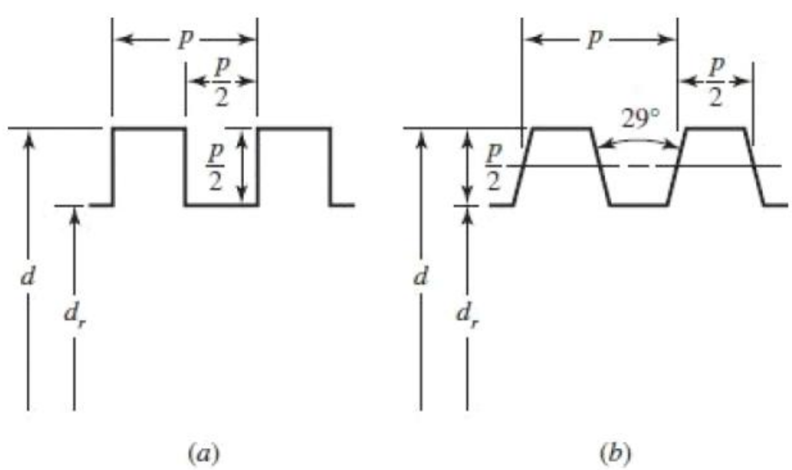
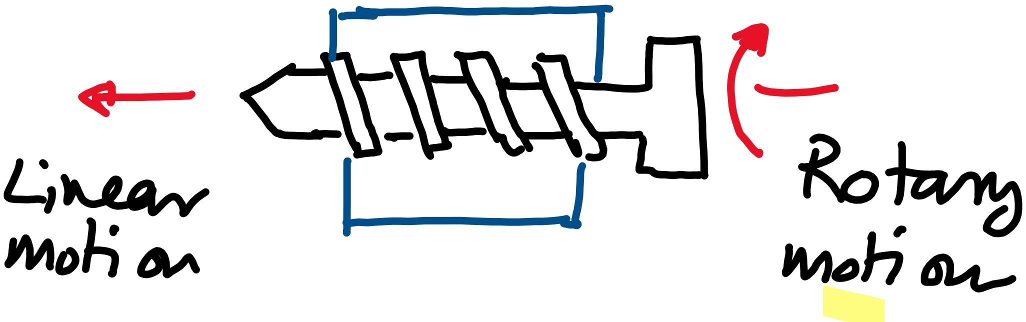


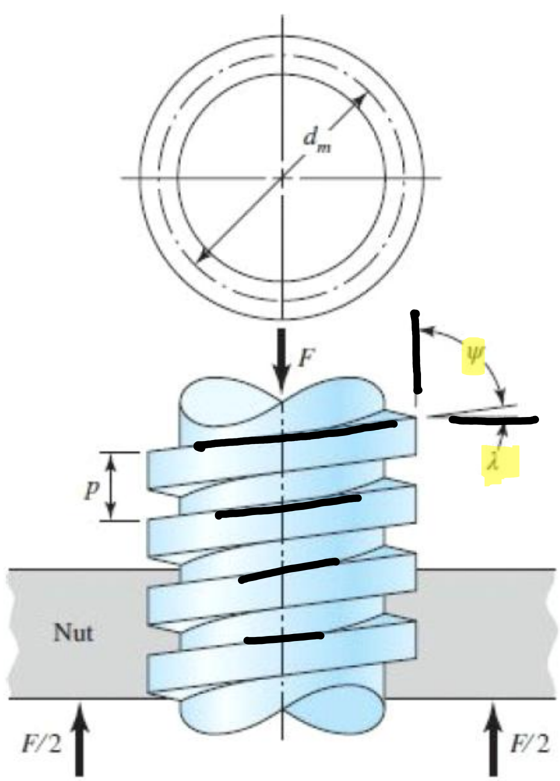
Figure 8-3
(*a*) Square thread; (*b*) Acme thread.

Square & Acme threads used when
we want to transmit power

8.2 Mechanics of power screws

- Power screw is used to change angular motion to linear motion





single threaded screw

d_m - mean diameter

p - pitch

λ - lead angle

ψ - helix angle

F - axial compressive load

Compute torque needed to raise/lower the load.

Lowering and Lifting a load

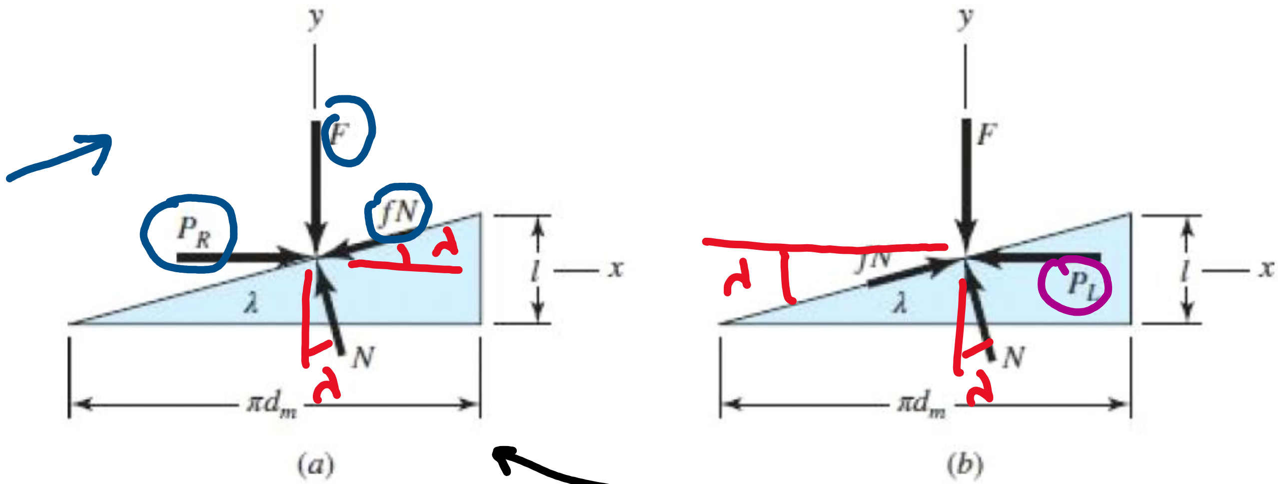


Figure 8-6
Force diagrams: (*a*) lifting the load; (*b*) lowering the load.

F - friction co-efficient
N - Normal force
$\boxed{P_R, P_L}$ - Force to lift / lower load
dm - mean diameter
F - compressive force (external)
dm - mean diameter.

Lifting

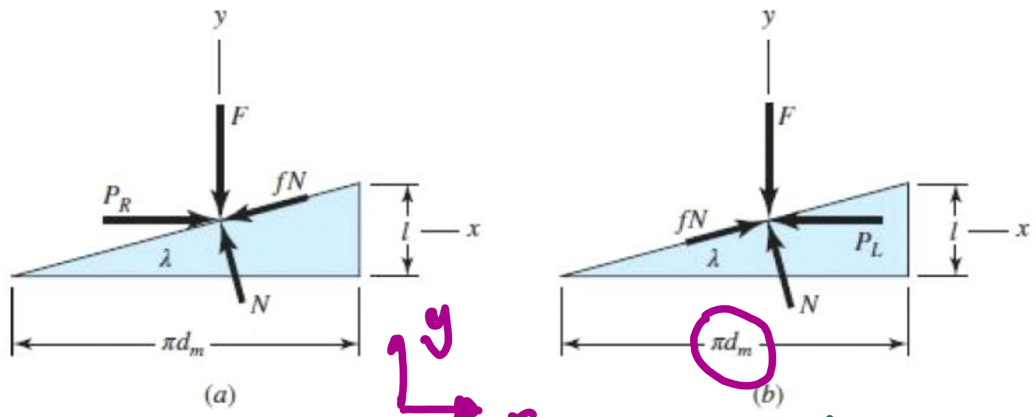


Figure 8-6
Force diagrams: (*a*) lifting the load; (*b*) lowering the load.

e.g. lifting jack for a car

Lifting a load

$$\Sigma F_x = P_R - N\sin\lambda - fN\cos\lambda = 0$$

$$\Rightarrow N = \frac{P_R}{\sin\lambda + f\cos\lambda} \qquad (1)$$

$$\Sigma F_y = -F - fN\sin\lambda + N\cos\lambda = 0 \qquad (2)$$

Substitute N from (1) into (2), solve for P_R

$$\rightarrow P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda}$$

But $\tan\lambda = l/\pi d_m$

$$\rightarrow P_R = \frac{F(l/\pi d_m + f)}{1 - fl/\pi d_m}$$

(Raise)

Lowering

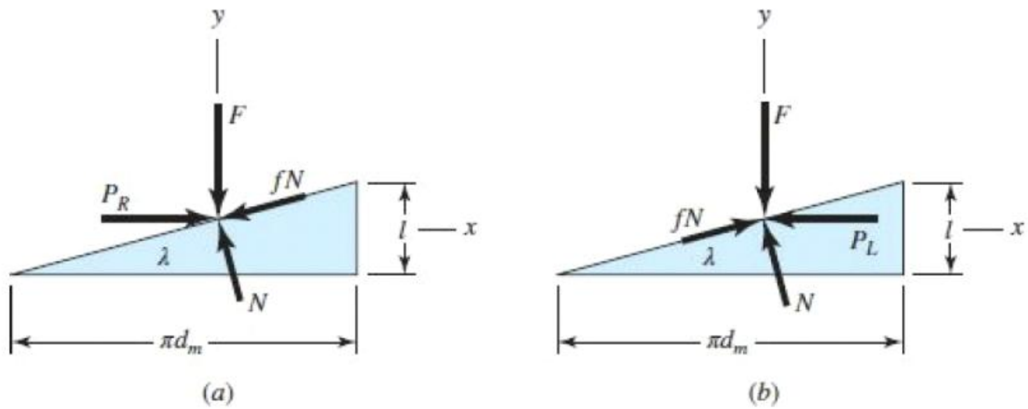


Figure 8-6
Force diagrams: (*a*) lifting the load; (*b*) lowering the load.

Lowering the load

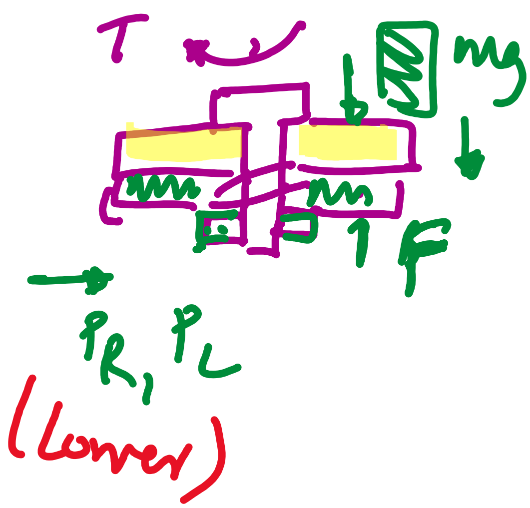
$$\Sigma F_x = -P_L - N\sin\lambda + fN\cos\lambda = 0$$

$$\Rightarrow \quad N = \frac{P_L}{f\cos\lambda - \sin\lambda} \qquad - (3)$$

$$\Sigma F_y = -F + fN\sin\lambda + N\cos\lambda = 0 \qquad - (4)$$

Substitute N from (3) into (4) & solve for P_L

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda}$$



But $\tan\lambda = l/\pi d_m$

$$P_L = \frac{F\left[f - l/\pi d_m\right]}{1 + (fl/\pi d_m)}$$

Torque need to lift / lower the load

$$\boxed{\begin{aligned} T_R &= P_R \frac{d_m}{2} = \frac{F d_m}{2}\left[\frac{l + \pi f d_m}{\pi d_m - f l}\right] \\ T_L &= P_L \frac{d_m}{2} = \frac{F d_m}{2}\left[\frac{\pi f d_m - l}{\pi d_m + f l}\right] \end{aligned}} \quad (6)$$

when $T_L > 0$, the screw is self-locking. That is, the friction is large enough to hold the power screw stationary

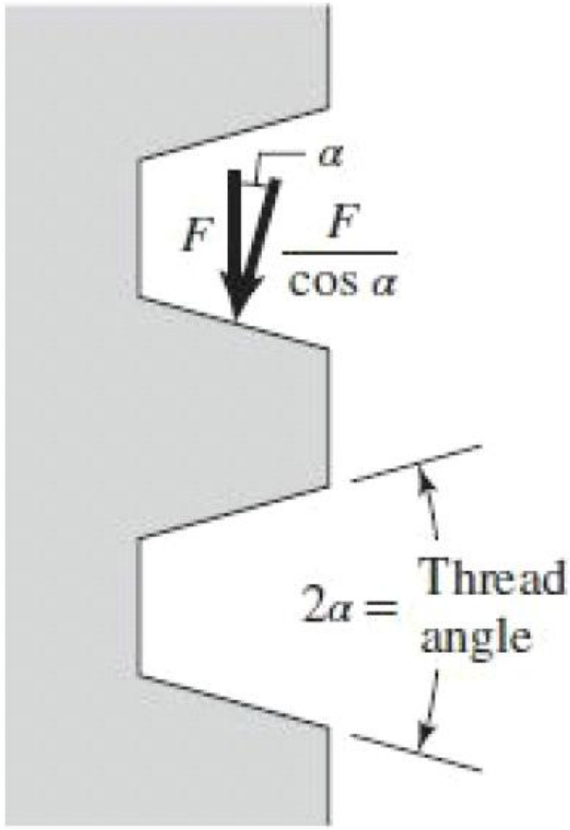
$$T_L > 0 \qquad f \pi d_m - l > 0$$

$$f > \frac{\pi d_m}{l} = \tan\lambda$$

$$\boxed{f > \tan\lambda} \quad \text{Self locking condition}$$

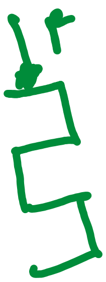
Efficiency (e): Put $f = 0$ in T_R equation in (6)

$$T_0 = \frac{F l}{2\pi}, \qquad \boxed{e = \frac{T_0}{T_R} = \frac{F l}{2\pi T_R}}$$



For ACME threads the normal force is inclined at an angle α as shown

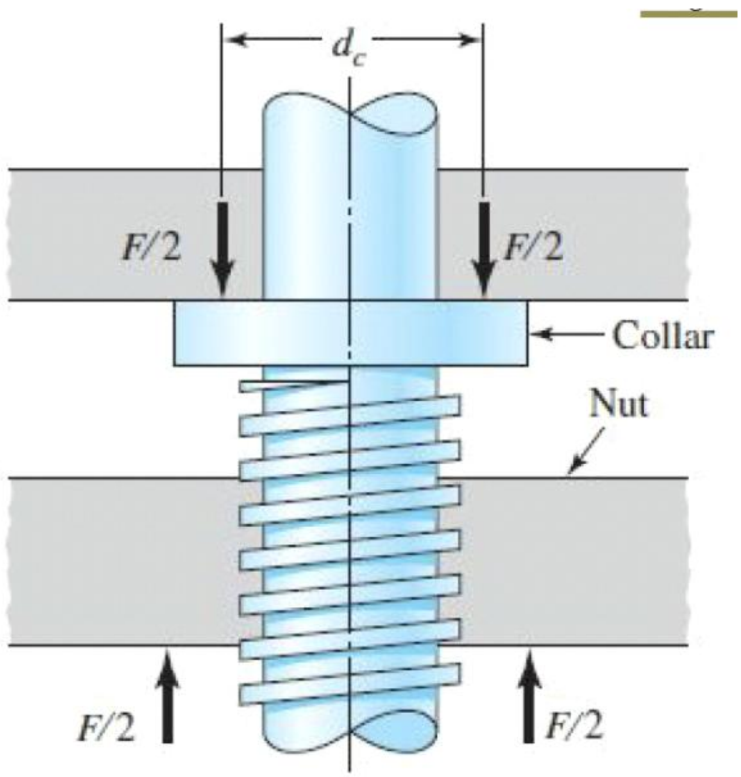
Replace F with $F/\cos\alpha$ in square thread derivation to compute equivalent load/torque

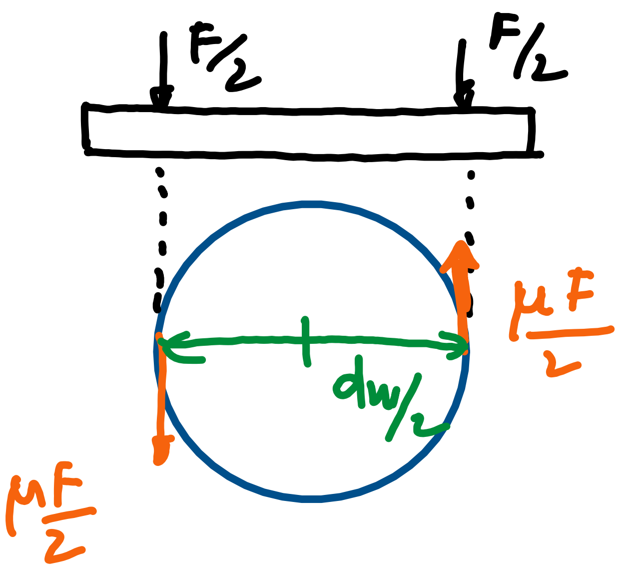


$$\boxed{\begin{aligned} T_R &= \frac{F d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - l f \sec\alpha}\right) \\ T_L &= \frac{F d_m}{2}\left(\frac{\pi f d_m \sec\alpha - l}{\pi d_m + l f \sec\alpha}\right) \end{aligned}}$$

Square thread is more efficient than ACME but ACME are easier to machine.

Thrust collar



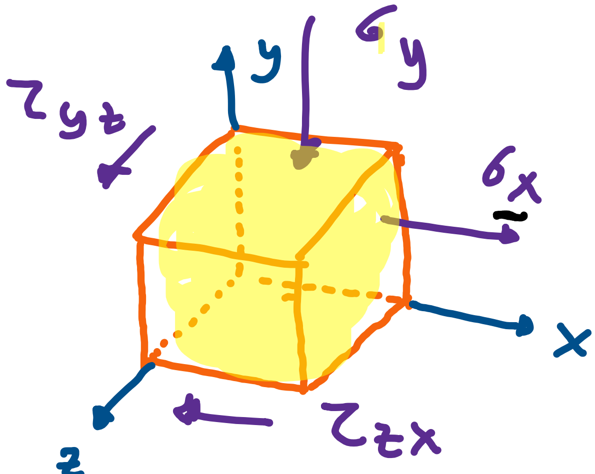


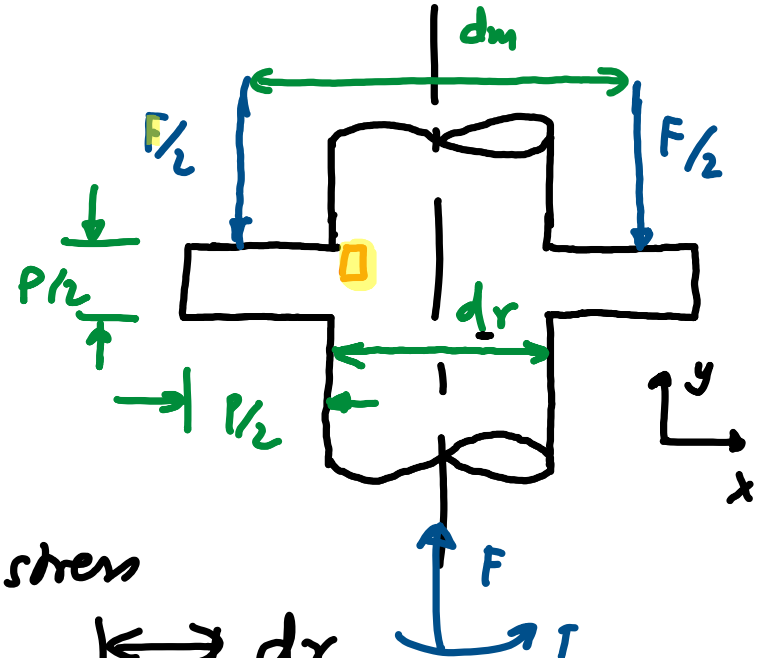
due to the couple

$$T_c = 2\left(\frac{\mu F}{2}\right)\frac{d_m}{2}$$

$$\boxed{T_c = \frac{\mu F d_m}{2}}$$

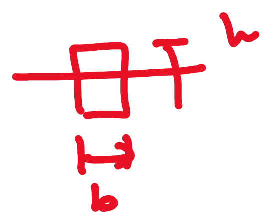
Stresses in a power screw



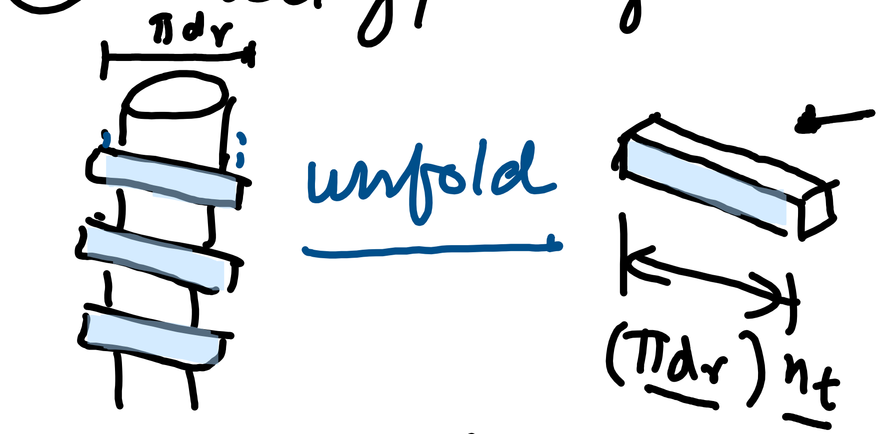


① Axial/Compressive stress

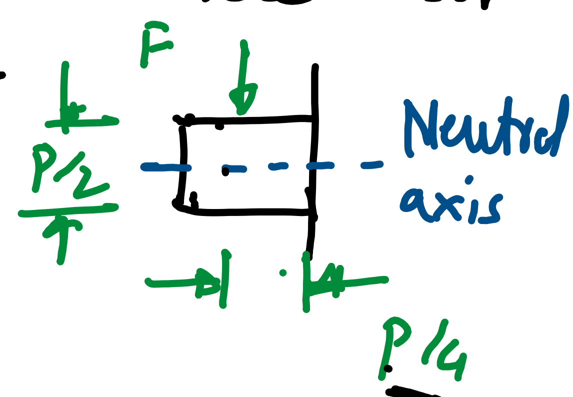
$$\sigma_y = \frac{-F}{\pi d_r^2/4}$$



② Bending/Bearing stress

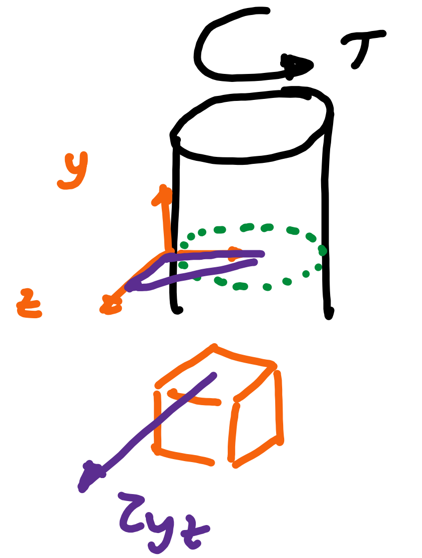


Side view



$$\sigma_x = \frac{My}{I} = \frac{\underbrace{\left(F\frac{p}{4}\right)}_{M}\underbrace{\left(\frac{p}{4}\right)}_{y}}{\frac{1}{12}\underbrace{(\pi d_r n_t)}_{b}\underbrace{\left(\frac{p}{2}\right)^3}_{h^3}} \quad = I$$

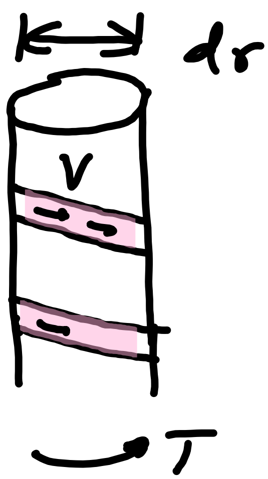
$$\sigma_x = \frac{6F}{\pi d_r n_t p}$$



③ Torsion stress

$$\tau_{yz} = \frac{T\rho}{J} = \frac{T\,d_r/2}{\pi d_r^4/32}$$

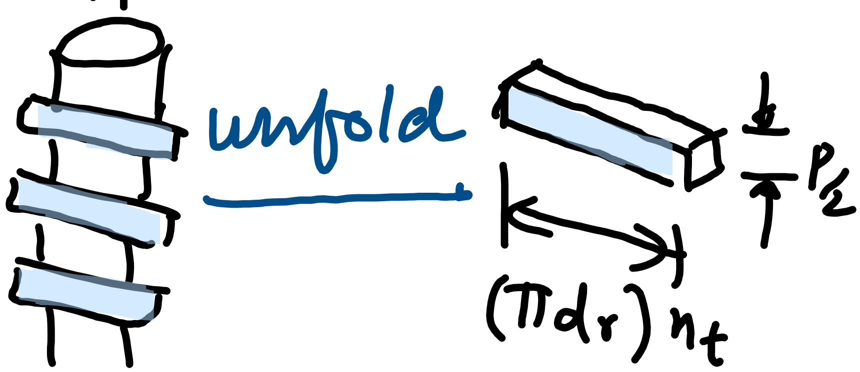
$$\tau_{yz} = \frac{16T}{\pi d_r^3}$$



④ Transverse shear

$$T = V\frac{d_r}{2} \Rightarrow V = \frac{2T}{d_r}$$

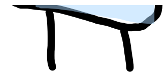
$$A = (\pi d_r n_t)(p/2)$$



$$\tau_{zx} = V/A$$

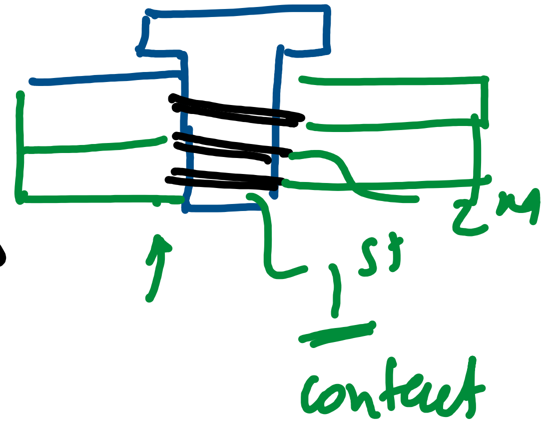
$$\tau_{zx} = \frac{4T}{\pi d_r^2 n_t p}$$

$(\pi d_r) n_t$



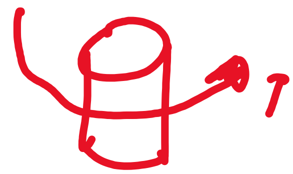
$$\tau_{tx} = \frac{4T}{\pi d_r^2 n_t p}$$

Summary



$$\sigma_x = \frac{6F}{\pi d_r n_t p} \qquad \tau_{xy} = 0$$

$$\sigma_y = \frac{-4F}{\pi d_r^2} \qquad \tau_{yz} = \frac{16T}{\pi d_r^3}$$



$$\sigma_z = 0 \qquad \tau_{zx} = \frac{4T}{\pi d_r^2 n_t p}$$

carry

This analysis assumes all threads equal loads. But this is not true.

Experiments show that
first thread shares 38% load
second thread shares 25% load
third thread shares 18% load
seventh thread is free.

Thus, in estimating stresses, setting 0.38F to F and n_t to 1 will give the largest stress.

stress.

von - Mises stress

$$\sigma' = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + \ldots \right.$$

$$\left. 6 (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right]^{1/2}$$

Q1

ed

A square-thread power screw has a major diameter of 32 mm and a pitch of 4mm with double threads. Assume friction coefficient to be f = 0.08, d = 40 mm, and F = 6.4kN per screw. Compute the following

a) Thread depth, thread width, pitch diameter, minor diameter, and lead

collar diameter

b) Torque needed to raise and lower the load
c) Efficiency for lifting the load
d) Body stresses, namely the torsional and compressive
e) Bearing stresses on the first thread
f) Von-Mises stress at the critical stress element where the root of the first thread interfaces with the screw body

$d = 32\,mm$

$p = 4\,mm$

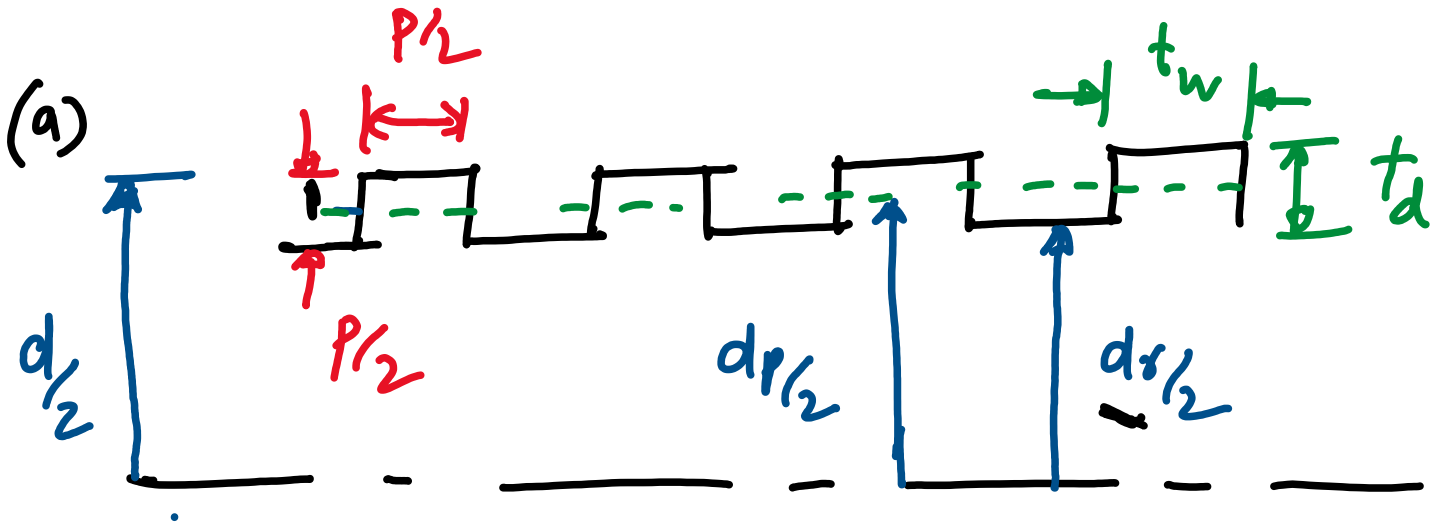
double threads

$f = 0.08$

$d_c = 40\,mm$

$F = 6.4\,kN / screw$

(a)



$\boxed{t_w = \text{thread width} = p/2 = 2\,mm}$

$\boxed{t_d = \text{thread depth} = p/2 = 2\,mm}$

pitch diameter $= d_p = d - p/2$

$$\boxed{d_p = 32 - 4/2 = 30\,mm}$$

minor diameter $: d_r = d - p$

$$= 32 - 4$$

$$\boxed{d_r = 28\,mm}$$

lead $= 2p = 2(4)$ $\boxed{l = 8\,mm}$

double thread

(b)

$$T_R = P_R \frac{d_m}{2} = \frac{F d_m}{2}\left[\frac{l + \pi f d_m}{\pi d_m - f l}\right]$$

Raise

$$T_L = P_L \frac{d_m}{2} = \frac{F d_m}{2}\left[\frac{\pi f d_m - l}{\pi d_m + f l}\right]$$

lower

$$T_c = \frac{\mu F d_m}{2}$$

$d_m = 30mm$ (same as d_p)

$F = 6.4 kN$

$d_c = 40mm$

$l = 8mm$

$f = \mu = 0.08$

(Screw) (collar)

$T_L = -0.466$ * not self locking

$T_R = 15.94$

$T_c = 10.24$

$$T_{total\ lift} = T_R + T_c = 15.94 + 10.24 = 26.18\ Nm$$

$$T_{total\ lower} = T_L + T_c = -0.466 + 10.24 = 9.77\ Nm$$

(c) $e = \frac{FL}{2\pi T_{total\ lift}} = \frac{(6.4)(8)}{2\pi(26.18)}$

$$\boxed{e = 0.311}$$

(d) Torsion stress

$$\boxed{\tau_{yz} = \frac{16T}{\pi d_r^3}}$$

Since $T_{lift} > T_{lower}$ we use T_{lift}

$\tau_{yz} = \frac{16(26.18)(10^3)}{\pi(28)^3}$ $\boxed{\tau_{yz} = 6.07 \text{ MPa}}$

Compressive stress

$\sigma_y = \frac{-F}{\pi d_r^2/4}$; $\sigma_y = -\frac{6.4(10^3)}{\pi\, 28^2/4}$

$$\boxed{\sigma_y = -10.39 \text{ MPa}}$$

(e) Bearing stress on the first thread

$$\boxed{\sigma_x = \frac{6F}{\pi d_r n_t p}}$$

$F = 0.38(6.4) \rightarrow$ 1st thread takes 38% of F

$n_t = 1 \rightarrow$ since we are considering only the first thread

$$\sigma_x = \frac{6(0.38)(6.4)}{\pi(28)(1)(4)} \qquad \boxed{\sigma_x = 41.5\ \text{MPa}}$$

(f)

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \underset{=0}{\sigma_z})^2 + (\underset{=0}{\sigma_z} - \sigma_x)^2 + \ldots \right.$$
$$\left. 6(\underset{=0}{\tau_{xy}^2} + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2}$$

$$\boxed{\tau_{zx} = \frac{4T}{\pi d_r^2 n_t p}}$$

$$\boxed{\tau_{zx} = \frac{4T}{\pi d_r^2 \eta_t p}}$$

$$T = T_{lift} = 26.18 \text{ Nm}$$

Since its the first thread which is critical

$$T = 0.38\, T_{lift} = 0.38\,(26.18)$$

$$\tau_{zx} = \frac{4\,(0.38)(26.18)}{\pi\,(28)^2(1)(4)} = 4.04 \text{ MPa}$$

$$\sigma' = \frac{1}{\sqrt{2}}\left[\left[41.5 - (-10.39)\right]^2 + (-10.39 - 0) + (0 - 41.5)^2 + 6\left[0^2 + 6.07^2 + 4.04^2\right]\right]^{1/2}$$

$$\boxed{\sigma' = 49.2 \text{ MPa}}$$