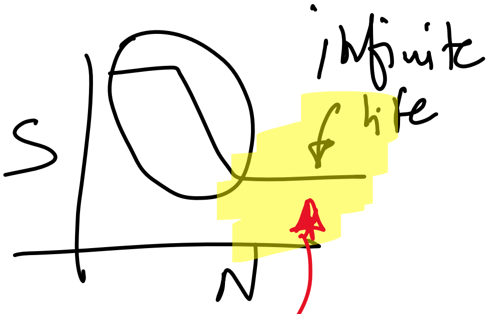
Fatigue Loading of Helical springs

Shot peening

- increase fatigue strength by 20%
- shots of size 1/64 in are sprayed high speeds on the spring wire

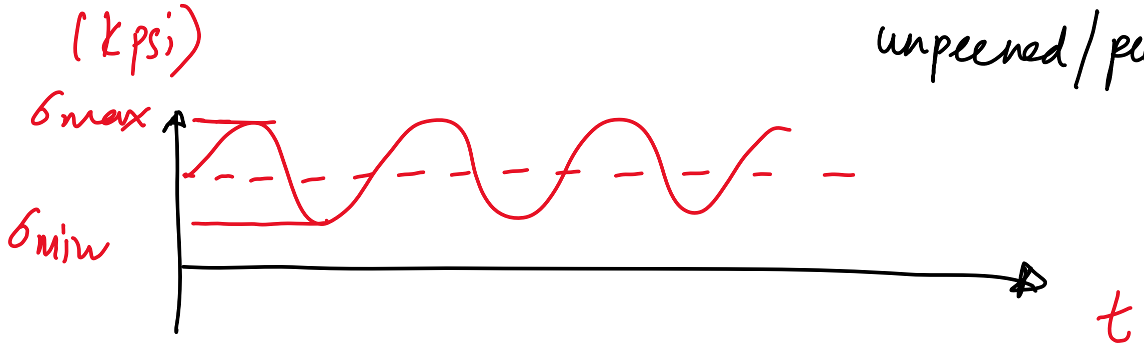
Spring for fatigue loading



spring are designed in this region

- infinite life
- Zimmerli endurance strength
- Found that size, material, tensile strength have no effect on the endurance for springs of size $\underbrace{3/8\text{ in}}_{\text{diameter.}}$ (10 mm)

unpeened/peened



unpeened: $\sigma_{min} = 20$ kpsi
$\sigma_{max} = 90$ kpsi

peened $\sigma_{min} = 20$ kpsi
$\sigma_{max} = 135$ kpsi

SPRINGS DONT FAIL for $N > 10^6$ cycles (INFINITE LIFE)

Compute endurance strength

$$S_{sm} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$S_{sa} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Unpeened:

$\underline{S_{sa}} = \frac{90-20}{2} = 35$ kpsi (241 MPa)

$\underline{S_{sm}} : \frac{90+20}{2} = \underline{55}$ kpsi (379 MPa)

Peened:

$S_{sa} = \underline{57.5}$ kpsi (398 MPa)

$S_{sm} : \underline{77.5}$ kpsi (534 MPa)

These can be used with the Goodman criteria as follows

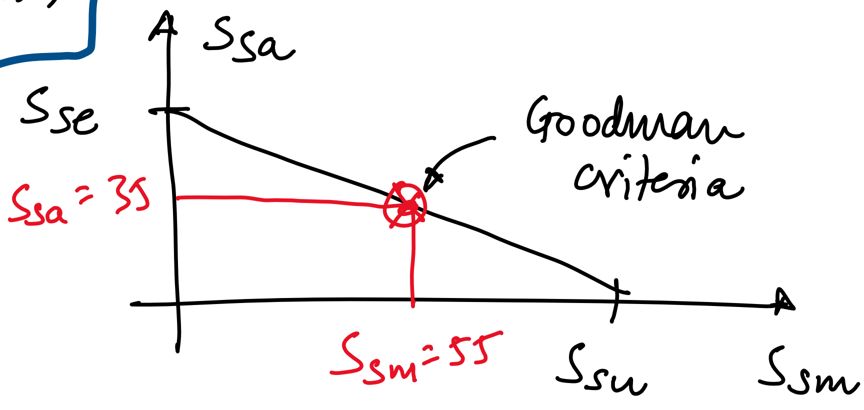
Goodman equation: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$

Modify for spring $\boxed{\frac{S_{sa}}{S_{se}\,?} + \frac{S_{sm}}{S_{su}} = 1}$

But $S_{su} = 0.67\, S_{ut}$ (Empirical)

$S_{su} = 0.67(315.67)$

$= 211.5$ kpsi

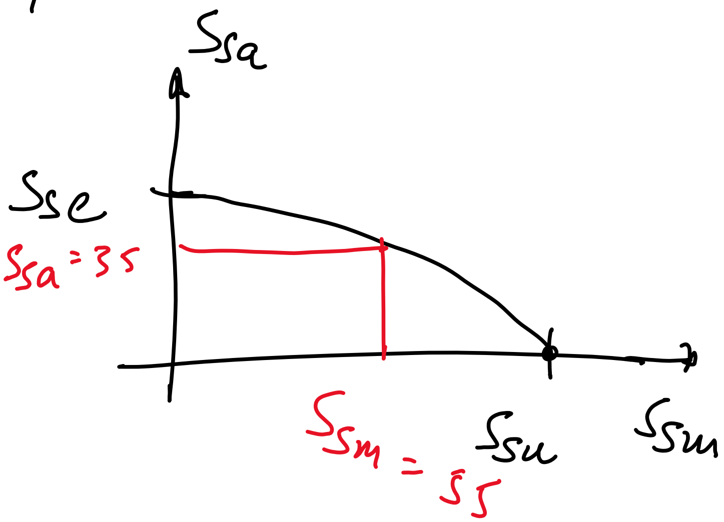


$$\frac{35}{S_{se}} + \frac{55}{211.5} = 1$$

Solve for Sse $\Rightarrow$ $\boxed{S_{se} = 47.5 \text{ kpsi}}$

Another common criteria is the Gerber criteria

$$\frac{S_{sa}}{S_{se}}? + \left(\frac{S_{sm}}{S_{su}}\right)^2 = 1$$



$$\frac{35}{S_{se}} + \left[\frac{55}{(211.5)}\right]^2 = 1$$

$$\boxed{S_{se} = 37.5 \text{ kpsi}}$$

Sine criteria for endurance strength

- applicable for polished, notch-free, cylindrical spring

- S_{sa} is constant and independent of S_{sm} provided S_{sm} is not greater than S_{su}

- DOES NOT HOLD for specimen with notches and abrupt change in section.

Formulae for Fatigue problems

$S_{ut} = A/d^m$ (Table 10-4)

$S_{su} = 0.67\, S_{ut}$

$$F_a = \frac{F_{max} - F_{min}}{2} \quad ; \quad F_m = \frac{F_{max} + F_{min}}{2}$$

$$\tau_a = k_b \frac{8 F_a D}{\pi d^3} \quad ; \quad \tau_m = k_b \frac{8 F_m D}{\pi d^3}$$

Goodman :

✓ $$\boxed{\frac{S_{sa}}{S_{se}} + \frac{S_{sm}}{S_{su}} = 1}$$

✓ $$\boxed{n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)^{-1}}$$

Gerber :

✓ $$\boxed{\frac{S_{sa}}{S_{se}} + \left(\frac{S_{sm}}{S_{su}}\right)^2 = 1}$$

✓ $$\boxed{n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \left(\frac{\tau_a}{S_{se}}\right)\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{\tau_a S_{su}}\right)^2}\,\right]}$$

Sine

✓ $$\boxed{n_f = \frac{S_{sa}}{\tau_a}}$$

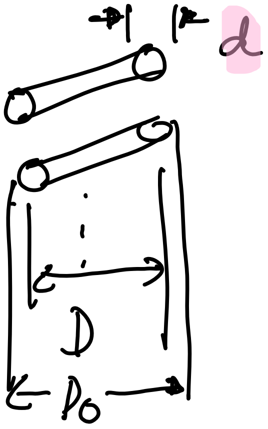
Q2

A helical compression spring made of music wire has a wire size of 0.092 in, an outside diameter of (9/16) in, a free length of 4 (3/8) in. The spring is unpeened. The spring is assembled with a pre-load of 5 lbf and will operate with a maximum load of 35 lbf during use. Estimate

(a) Factor of safety assuming torsional Gerber failure and Zimmerli data

(b) Factor of safety assuming Sines torsional failure.

(c) Factor of safety assuming torsional Goodman failure and Zimmerli data



$d = 0.092$

$D_o = 9/16$ in

$$D_o = D + d \Rightarrow D = D_o - d$$

$$= 9/16 - 0.092$$

$$= 0.4705 \text{ in}$$

$F_{min} = 5$ lbf

$F_{max} = 35$ lbf

$\underline{\tau_a}, \underline{\tau_m}, \underline{S_{su}}, \boxed{\underline{S_{se}}} = ?$

✓ ✓ ✓ ✓

$$\tau_a = \overset{\checkmark}{K_b}\,\frac{\overset{\checkmark}{8 F_a} \overset{\checkmark}{D}}{\pi\, \underset{\checkmark}{d^3}} \quad ; \quad \tau_m = K_b\,\frac{8 \overset{\checkmark}{F_m} \overset{\checkmark}{D}}{\pi\, \underset{\checkmark}{d^3}}$$

$$K_b = \frac{4C+2}{4C-3} \quad ; \quad C = \frac{D}{d} = \frac{0.4705}{0.092} = 5.11$$

$$K_b = \frac{4(5.11)+2}{4(5.11)-3} \Rightarrow K_b = 1.287$$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{35-5}{2} = 15 \text{ lbf}$$

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{35+5}{2} = 20 \text{ lbf}$$

$$\tau_a = (1.287)\frac{(8)(15)(0.4705)}{\pi (0.092)^3} = \underline{29.7} \text{ kpsi}$$

$$\tau_m = (1.287)\frac{(8)(20)(0.4705)}{\pi (0.092)^3} = \underline{39.6} \text{ kpsi}$$

From Table 10-4 (Music wire)

$A = 201 \qquad m = 0.145$

$d = 0.092$

$$S_{ut} = A/d^m = 201/(0.092)^{0.145} = 284.1 \text{ kpsi}$$

$$\underline{S_{su}} = 0.67\, S_{ut} = 0.67\,(284.1) = \underline{190.3} \text{ kpsi}$$

(a) Gerber

$$\left(\frac{S_{sa}}{S_{se}}\right) + \left(\frac{S_{sm}}{S_{su}}\right)^2 = 1$$

Zimmerli $\quad S_{sa} = \underline{35}$ kpsi ; $S_{sm} = 55$ kpsi

$S_{su} = 190.3$ kpsi

$$\left(\frac{35}{S_{se}}\right) + \left(\frac{55}{190.3}\right)^2 = 1 \Rightarrow \underline{S_{se}} = 38.2 \text{ kpsi}$$

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2\left(\frac{\tau_a}{S_{se}}\right)\left[-1 + \sqrt{1 + \left(\frac{2\,\tau_m\,S_{se}}{\tau_a\,S_{su}}\right)^2}\right]$$

$$n_f = \frac{1}{2}\left(\frac{190.3}{39.6}\right)\left(\frac{29.7}{38.2}\right)\left[-1 + \sqrt{1 + \left(\frac{2\,(39.6)\,(38.6)}{(29.7)(190.3)}\right)^2}\right]$$

$$\boxed{n_f = 1.23}$$

(b) $$n_f = \frac{S_{sa}}{\tau_a} = \frac{35}{29.7} \Rightarrow \boxed{n_f = 1.18}$$

(c) Goodman: $$\frac{S_{sa}}{S_{se}} + \frac{S_{sm}}{S_{su}} = 1$$

✓

$$\frac{35}{S_{se}} + \frac{55}{190.3} = 1 \Rightarrow S_{se} = \underline{49.23}\ \text{kpsi}$$

$$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)^{-1}$$

$$n_f = \left(\frac{29.7}{49.23} + \frac{39.6}{190.3}\right)^{-1}$$

$$\boxed{n_f = 1.23}$$

Table 10-4

Values for A, m are below

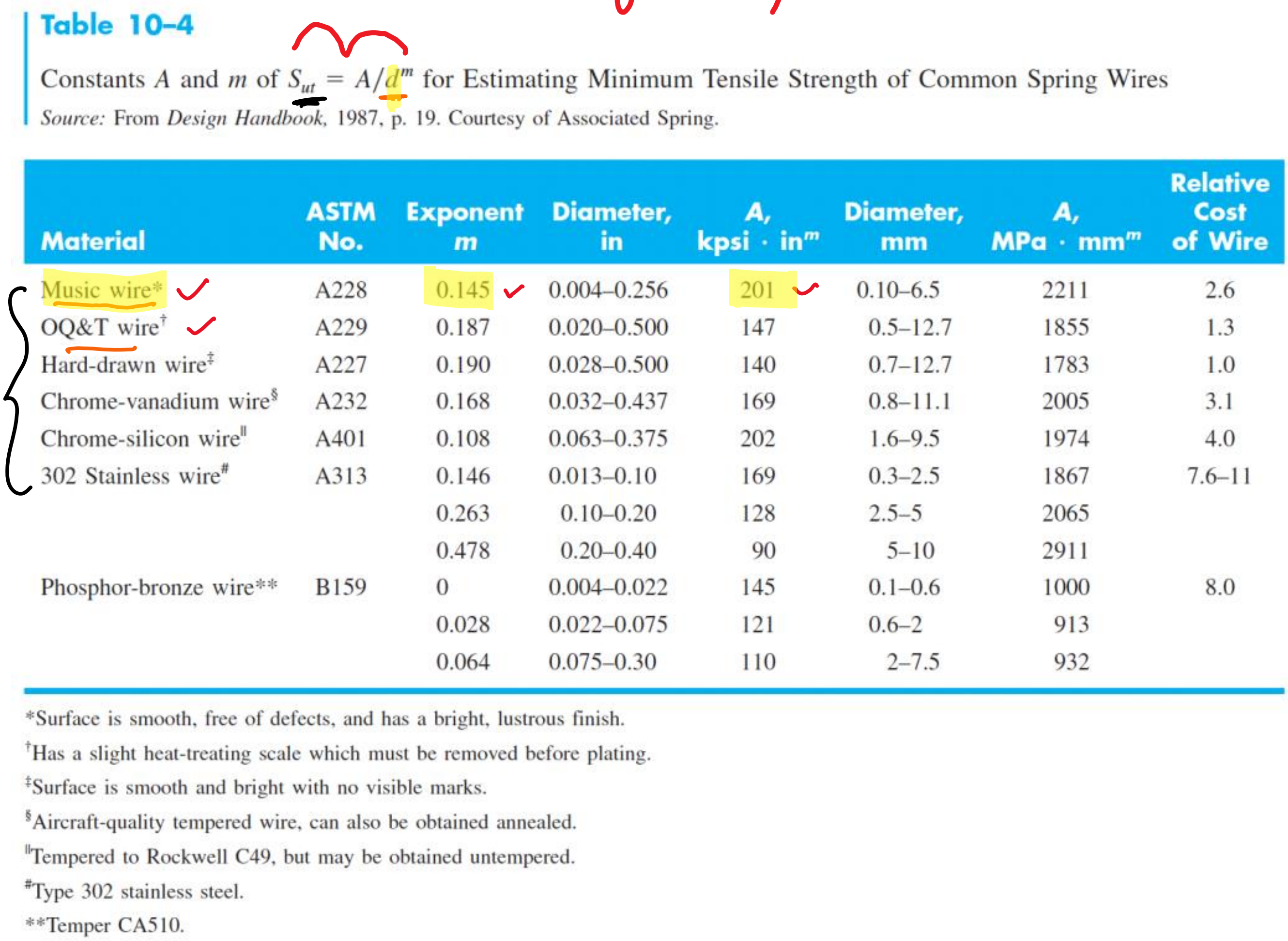


Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook,* 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire[*] ✓	A228	0.145 ✓	0.004–0.256	201 ✓	0.10–6.5	2211	2.6
OQ&T wire[†] ✓	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire[‡]	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire[§]	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire[‖]	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire[#]	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire[**]	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

[*] Surface is smooth, free of defects, and has a bright, lustrous finish.

[†] Has a slight heat-treating scale which must be removed before plating.

[‡] Surface is smooth and bright with no visible marks.

[§] Aircraft-quality tempered wire, can also be obtained annealed.

[‖] Tempered to Rockwell C49, but may be obtained untempered.

[#] Type 302 stainless steel.

[**] Temper CA510.