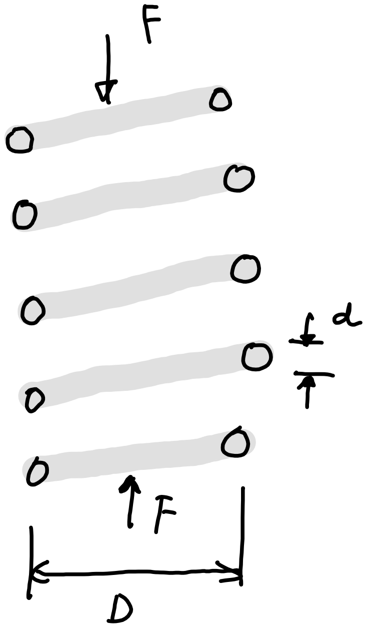
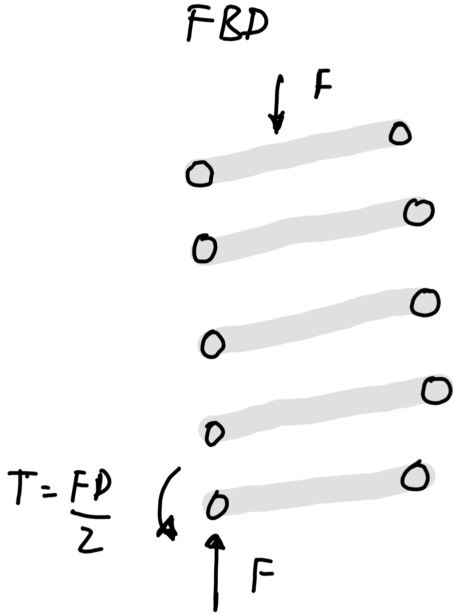
10.1 Stresses in Helical Springs





$$\tau_{max} = \frac{T\rho}{J} + \frac{F}{A}$$

$$\tau_{max} = \left(\frac{FD}{2}\right)\frac{d/2}{\pi d^4/32} + \frac{F}{\pi d^2/4}$$

$$\tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

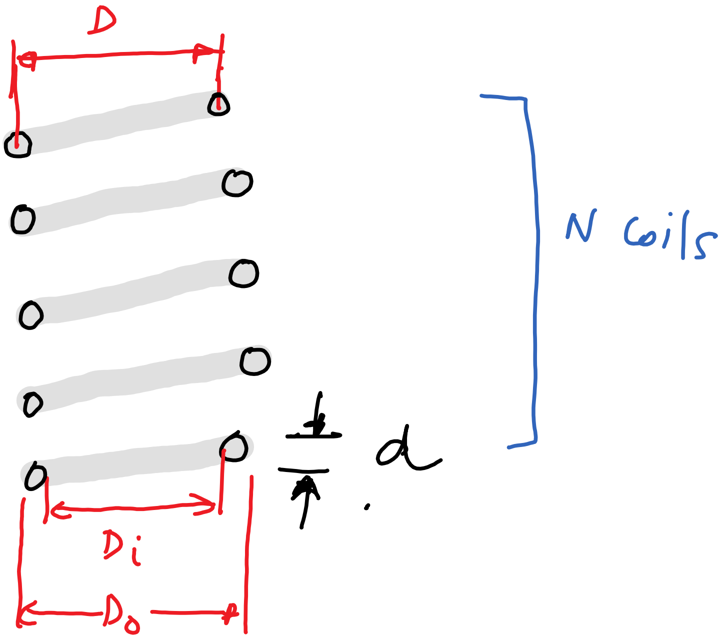
This can be written as

$$\tau = k_s \frac{8FD}{\pi d^3}$$

$C = \dfrac{D}{d}$ (spring index) $4 \le C \le 12$

$k_s = \dfrac{2C+1}{2C}$ (stress concentration factor)

10·2 Curvature effect



Inner diameter $D_i = D - d$
Outer diameter $D_o = D + d$

length of coil (inner) $N D_i = N(D-d)$
length of coil (outer) $N D_o = N(D+d)$

Since $N(D+d) > N(D-d)$ the length of the wire is not same at the 2 edges.

When the wire is bent, there are more stresses in the inner edge.

Static loading - can ignore curvature effect
Fatigue loading - cannot ignore curvature effect.

K_S - shear stress
K_C - curvature effect

There are 2 factors (by 2 people) for combined shear and curvature effect.

K_W - Wahl factor
K_B - Bergsträsser factor

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$\boxed{K_B = \frac{4C+2}{4C-3}}$$

- They are 1% of each other
- K_B is preferred

K_C can now be obtained as

$$\boxed{K_C = \frac{K_B}{K_S} = \frac{2C(4C+2)}{(4C-3)(2C+1)}}$$

To predict combined shear and curvature use

$$\boxed{\tau = K_B \frac{8FD}{\pi d^3}}$$

10.3 Deflection and spring rate of helical springs

Deflection $$\boxed{y = \frac{8 F D^3 N}{d^4 G}}$$

Spring rate or spring constant or scale (k)

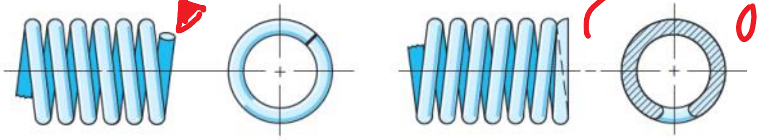
$$\boxed{k = \frac{d^4 G}{8 D^3 N}}$$

10.4

10.4 Compression springs

Figure 10–2

Types of ends for compression springs: (*a*) both ends plain; (*b*) both ends squared; (*c*) both ends squared and ground; (*d*) both ends plain and ground.



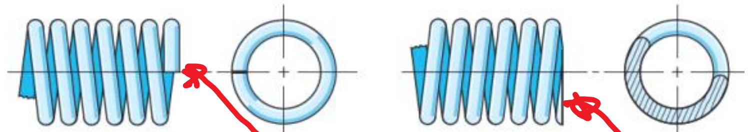
(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

plain end

squared 0° helix grounded

only right side shown



(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

plain & ground

squared, 0° helix (better attachment)

Table 10–1

Formulas for the Dimensional Characteristics of Compression-Springs. (N_a = Number of Active Coils)

Source: From *Design Handbook*, 1987, p. 32. Courtesy of Associated Spring.

	Type of Spring Ends			
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Set removal or presetting

- process to induce residual stresses

- manufacture spring longer than needed then compressing to solid length.

- this sets the spring to the final free length and induces residual stresses thus increasing the strength

- not recommended for fatigue applications

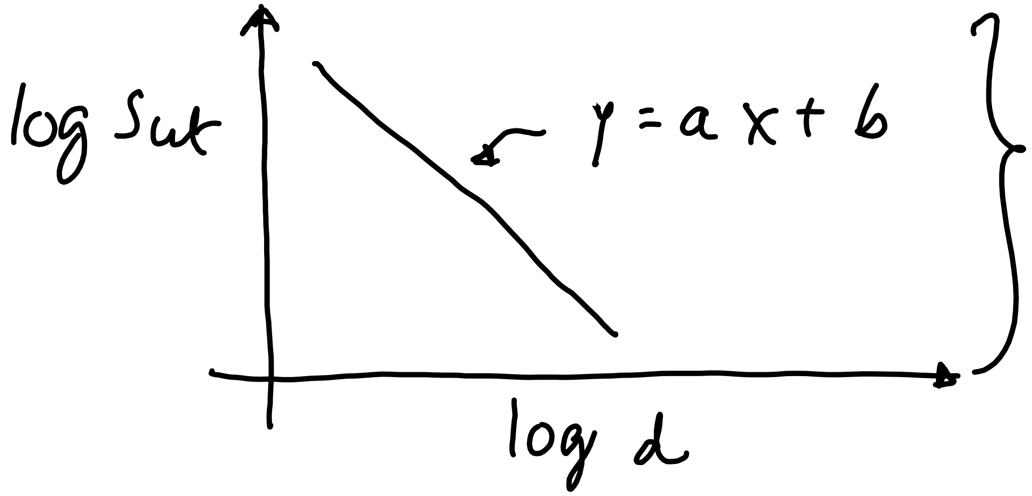
10·6 Spring material

Table 10–3

High-Carbon and Alloy Spring Steels

Source: From Harold C. R. Carlson, "Selection and Application of Spring Materials," *Mechanical Engineering,* vol. 78, 1956, pp. 331–334.

Name of Material	Similar Specifications	Description
Music wire, 0.80–0.95*C*	UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures.
Oil-tempered wire, 0.60–0.70*C*	UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.5000 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures.
Hard-drawn wire, 0.60–0.70*C*	UNS G10660 AISI 1066 ASTM A227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures.
Chrome-vanadium	UNS G61500 AISI 6150 ASTM 231-41	This is the most popular alloy spring steel for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue resistance and long endurance are needed. Also good for shock and impact loads. Widely used for aircraft-engine valve springs and for temperatures to 220°C (425°F). Available in annealed or pretempered sizes 0.8 to 12 mm (0.031 to 0.500 in) in diameter.
Chrome-silicon	UNS G92540 AISI 9254	This alloy is an excellent material for highly stressed springs that require long life and are subjected to shock loading. Rockwell hardnesses of C50 to C53 are quite common, and the material may be used up to 250°C (475°F). Available from 0.8 to 12 mm (0.031 to 0.500 in) in diameter.



Relation found empirically

$$y = \log S_{ut} \qquad \log S_{ut} = a \log d + b$$

$$x = \log d \qquad S_{ut} = e^b d^a$$

Put $a = -1/m$ $\quad e^b = A$

$$\boxed{S_{ut} = \frac{A}{d^m}}$$

See table 10-4 (next page)

Yield strength in tension

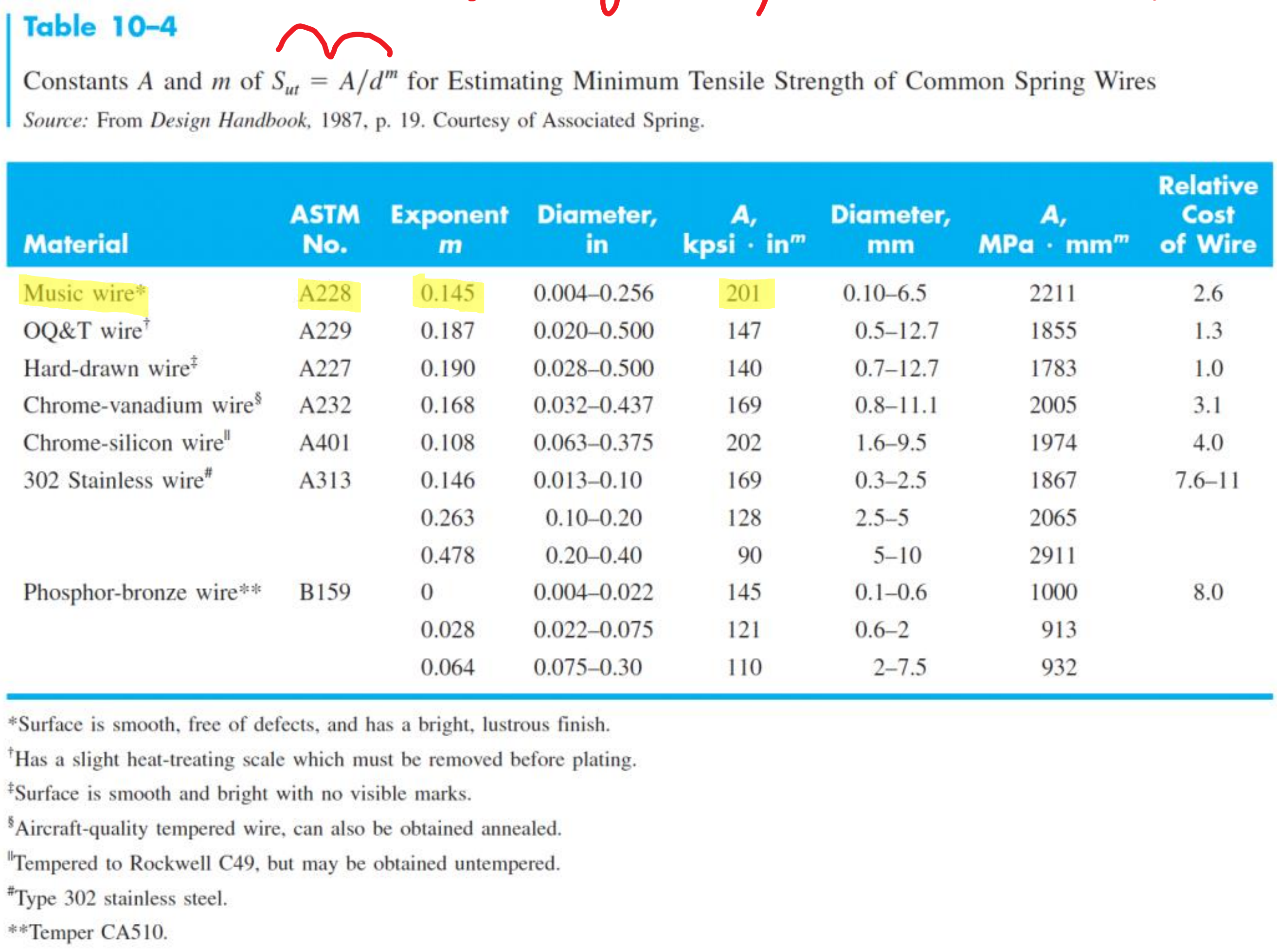
$$\boxed{0.6\, S_{ut} \leq S_y \leq 0.9\, S_{ut}}$$

Yield strength in shear

Since $S_{sy} = 0.577\, S_y$ (distortion energy)

$$\boxed{0.35\, S_{ut} \leq S_{sy} \leq 0.52\, S_{ut}}$$

Table 10-4



Values for A, m are below

Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook,* 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire[†]	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire[‡]	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire[§]	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire[‖]	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire[#]	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

*Surface is smooth, free of defects, and has a bright, lustrous finish.

[†] Has a slight heat-treating scale which must be removed before plating.

[‡] Surface is smooth and bright with no visible marks.

[§] Aircraft-quality tempered wire, can also be obtained annealed.

[‖] Tempered to Rockwell C49, but may be obtained untempered.

[#] Type 302 stainless steel.

**Temper CA510.

Table 10-5

$0.65 \le S_y \le 0.75$

$0.45 \le S_{sy} \le 0.6$

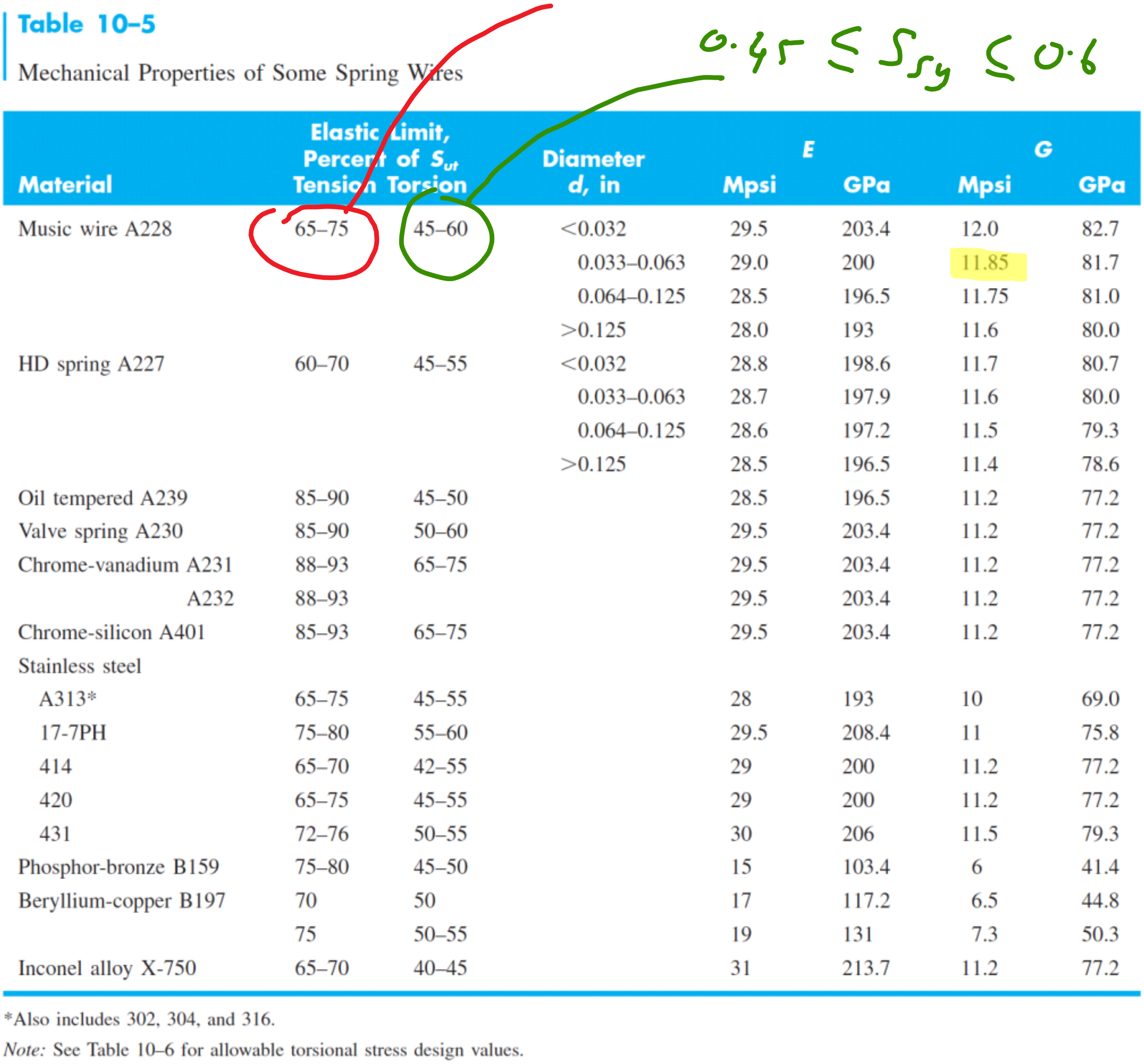


Table 10–5

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Torsion	Diameter d, in	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2
Valve spring A230	85–90	50–60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
420	65–75	45–55		29	200	11.2	77.2
431	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

*Also includes 302, 304, and 316.

Note: See Table 10–6 for allowable torsional stress design values.

Table 10-6

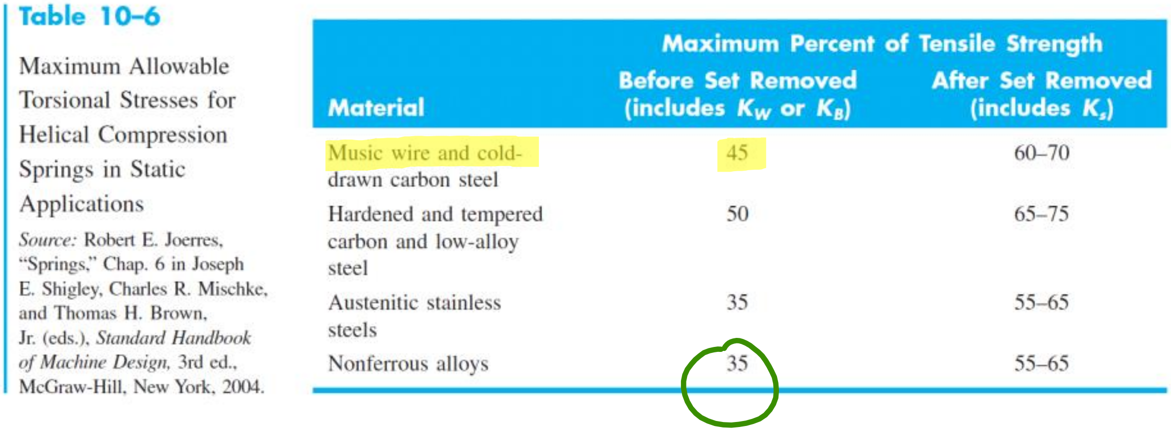


Table 10–6

Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Source: Robert E. Joerres, "Springs," Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

	Maximum Percent of Tensile Strength	
Material	**Before Set Removed (includes K_W or K_B)**	**After Set Removed (includes K_s)**
Music wire and cold-drawn carbon steel	45	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

$S_{sy} = 0.35 S_{ut}$

Q1

A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is (7/16) in. The ends are squared and there are 12 (1/2) total turns.

(a) Estimate the torsional yield strength of the wire

(b) Estimate the static load corresponding to the yield strength

(c) Estimate the scale of the spring

(d) Estimate the deflection that would be caused by the load in (b)

(e) Estimate the solid length of the spring

(f) What length should the spring be to ensure that when it is compressed solid and then released, there will be no permanent change in the free length?

(g) What is the pitch of the body coil?

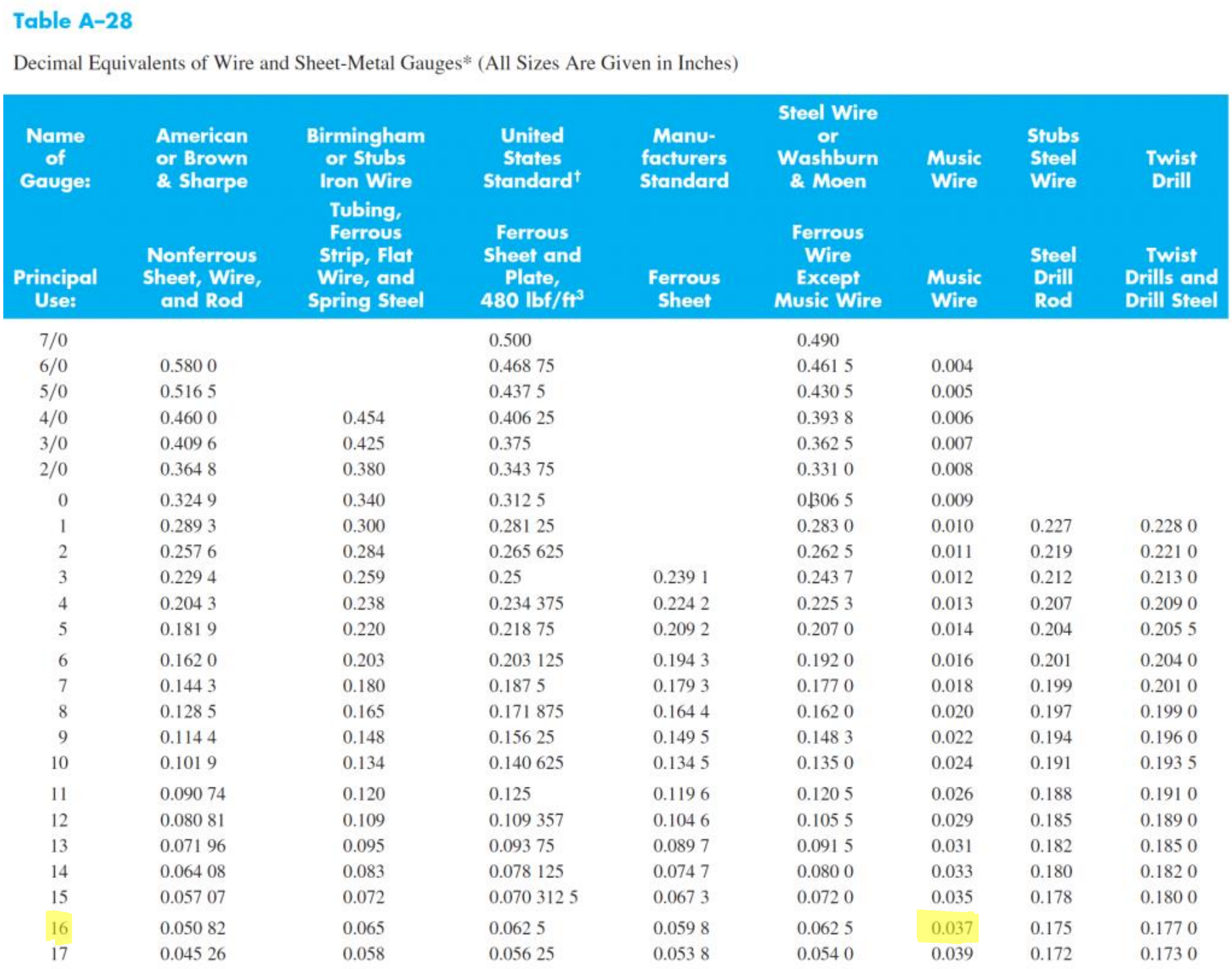


Table A–28

Decimal Equivalents of Wire and Sheet-Metal Gauges* (All Sizes Are Given in Inches)

Name of Gauge:	American or Brown & Sharpe	Birmingham or Stubs Iron Wire	United States Standard†	Manu-facturers Standard	Steel Wire or Washburn & Moen	Music Wire	Stubs Steel Wire	Twist Drill
Principal Use:	Nonferrous Sheet, Wire, and Rod	Tubing, Ferrous Strip, Flat Wire, and Spring Steel	Ferrous Sheet and Plate, 480 lbf/ft³	Ferrous Sheet	Ferrous Wire Except Music Wire	Music Wire	Steel Drill Rod	Twist Drills and Drill Steel
7/0			0.500		0.490			
6/0	0.580 0		0.468 75		0.461 5	0.004		
5/0	0.516 5		0.437 5		0.430 5	0.005		
4/0	0.460 0	0.454	0.406 25		0.393 8	0.006		
3/0	0.409 6	0.425	0.375		0.362 5	0.007		
2/0	0.364 8	0.380	0.343 75		0.331 0	0.008		
0	0.324 9	0.340	0.312 5		0.306 5	0.009		
1	0.289 3	0.300	0.281 25		0.283 0	0.010	0.227	0.228 0
2	0.257 6	0.284	0.265 625		0.262 5	0.011	0.219	0.221 0
3	0.229 4	0.259	0.25	0.239 1	0.243 7	0.012	0.212	0.213 0
4	0.204 3	0.238	0.234 375	0.224 2	0.225 3	0.013	0.207	0.209 0
5	0.181 9	0.220	0.218 75	0.209 2	0.207 0	0.014	0.204	0.205 5
6	0.162 0	0.203	0.203 125	0.194 3	0.192 0	0.016	0.201	0.204 0
7	0.144 3	0.180	0.187 5	0.179 3	0.177 0	0.018	0.199	0.201 0
8	0.128 5	0.165	0.171 875	0.164 4	0.162 0	0.020	0.197	0.199 0
9	0.114 4	0.148	0.156 25	0.149 5	0.148 3	0.022	0.194	0.196 0
10	0.101 9	0.134	0.140 625	0.134 5	0.135 0	0.024	0.191	0.193 5
11	0.090 74	0.120	0.125	0.119 6	0.120 5	0.026	0.188	0.191 0
12	0.080 81	0.109	0.109 357	0.104 6	0.105 5	0.029	0.185	0.189 0
13	0.071 96	0.095	0.093 75	0.089 7	0.091 5	0.031	0.182	0.185 0
14	0.064 08	0.083	0.078 125	0.074 7	0.080 0	0.033	0.180	0.182 0
15	0.057 07	0.072	0.070 312 5	0.067 3	0.072 0	0.035	0.178	0.180 0
16	0.050 82	0.065	0.062 5	0.059 8	0.062 5	0.037	0.175	0.177 0
17	0.045 26	0.058	0.056 25	0.053 8	0.054 0	0.039	0.172	0.173 0

(a) A-28

Wire 16 $\Rightarrow$ $d = 0.037$ in

Table 10-4 Music wire; $A = 201$ kpsi$\cdot$in^m

$m = 0.145$

$S_{ut} = A/d^m = 201/(0.037)^{0.145} = 324$ kpsi

Table 10.6 $S_{sy} = 0.45\, S_{ut} = 0.45(324)$

$$\boxed{S_{sy} = 146 \text{ kpsi}}$$

(b) $\tau = K_B \frac{8FD}{\pi d^3}$ — ①

$D = D_o - d = 7/16 - 0.037 = 0.4$ in

$C = \frac{D}{d} = \frac{0.4}{0.037} = 10.8$

$$K_B = \frac{4C+2}{4C-2} = \frac{4(10.8)+2}{4(10.8)-2} = 1.128$$

$\tau = S_{sy} = 146$ kpsi

Substitute in ①

$$146 = 1.128 \frac{(8\ F\ 0.4)}{\pi\ (0.037)^3} \Rightarrow \boxed{F = 6.46\ \text{lbf}}$$

(c) Scale $k = \frac{d^4 G}{8 D^3 N_a}$

Table 10-1 $N_t = 12.5 = N_a + 2 \Rightarrow N_a = 10.5$

Table 10-5 $G = 11.85$ Mpsi

$$k = \frac{0.037^4\ (11.85\ (10^6))}{8\ (0.4)^3\ (10.5)} \Rightarrow \boxed{k = 4.13\ \text{lbf/in}}$$

(d) Deflection $y = \frac{F}{k} = \frac{6.46}{4.13}$

$$\boxed{y = 1.56 \text{ in}}$$

(e) Table 10.1

$$L_s = (N_t + 1)d = (12.5+1)(0.037)$$

$$\boxed{L_s = 0.5 \text{ in}}$$

(f) $L_o = y + l_s = 1.56 + 0.5$

$$\boxed{L_o = 2.06 \text{ in}}$$

(g) pitch $p = \frac{L_o - 3d}{N_a} = \frac{2.06 - 3(0.037)}{10.5}$

$$\boxed{p = 0.186 \text{ in}}$$