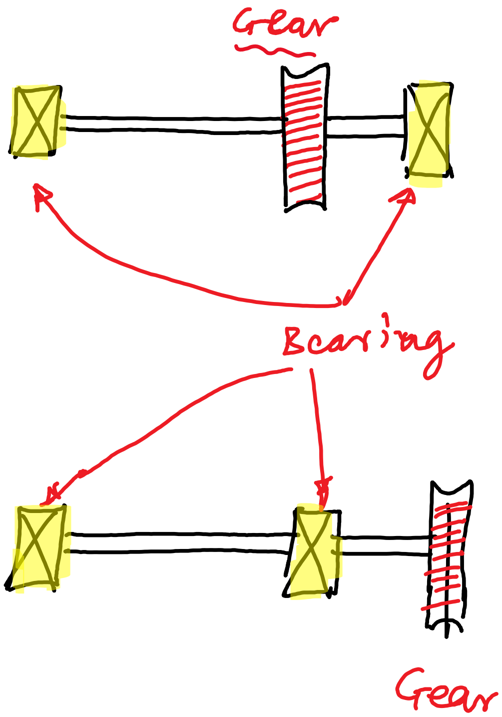
Shaft intro 1

7-03 Shaft layout

- The shaft is the most important element in a machine element
- Shafts hold gears, bearings, pulleys

Layout

- load carrying elements between bearings

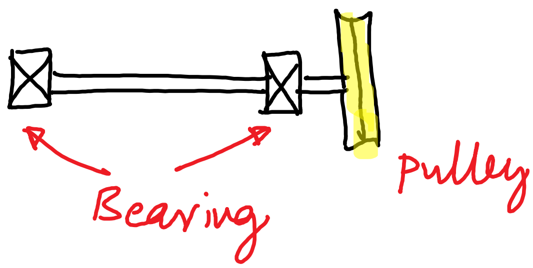


Good design / Simply supported

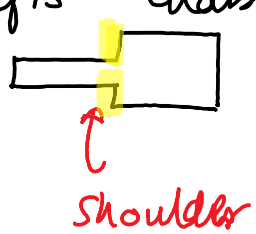
Bad design / cantilever

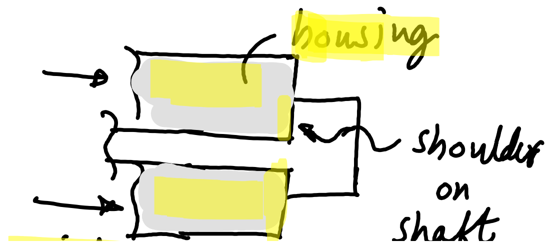
Shaft intro 2

- Transmission elements such as pulleys & sprockets (e.g. bicycle chains are on sprockets) can be mounted on the outboard end to enable mounting of chains / belts.

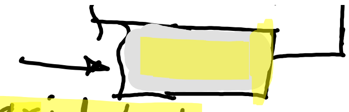


- Keep shaft lengths short, limit to 2 bearing
  - short length implies lower deflections and lower moments.
  - load carrying elements near the bearings to reduce the moment at the element and deflections
- If long shafts are needed, ≥ 2 bearings may be needed. Care needs to be taken to ensure bearing alignment.
- shoulders in shafts enables transmission of axial loads





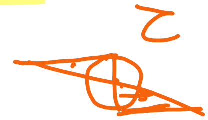
Shoulder



Shaft design 1

7-04 Shaft design for stresses

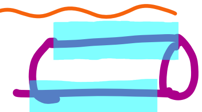
We want to find critical locations on the shaft that have high stresses and use these locations to design the shafts. This normally involves find an appropriate shaft diameter.

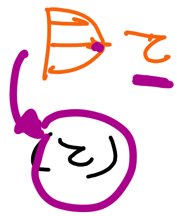


Critical locations

- outer surface where normal stress (σ) due to bending moment (M) and shear stress (τ) due to torque (T) are maximum

neutral axis





- center of the shaft where the shear stress (τ) due to shear loads (V) are maximum

- axial loads (P) lead to uniform normal stress along the cross-section and hence their effect is considered in conjunction with other stresses.

- stress concentration points are those where geometry changes abruptly

- draw shear force and bending moment diagrams. Sometimes these need to be drawn in 2 planes

Torque diagram

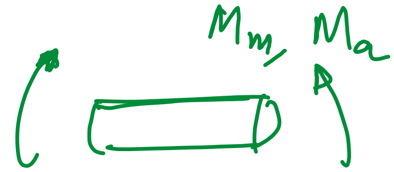
Shaft Design 2

Torque transmission

- keys
- splines
- set screws
- pins
- press or shrink fits
- taper fits

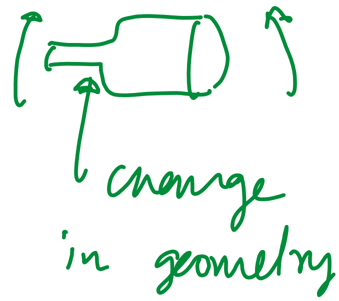
Read through ~~section 7-03~~

Shaft Stresses



Shaft stresses:

$$\sigma_a = K_f \frac{M_a c}{I} = K_f \frac{M_a \, d/2}{\pi d^4/64} = \frac{32 K_f M_a}{\pi d^3}$$



Similarly $\sigma_m = \dfrac{32 K_f M_m}{\pi d^3}$

$$\tau_a = K_{fs} \frac{T_a r}{J} = K_{fs} \frac{T_a \, d/2}{\pi d^4/32} = \frac{16 K_{fs} T_a}{\pi d^3}$$

Similarly $\tau_m = \dfrac{16 K_{fs} T_m}{\pi d^3}$

where

K_f, K_{fs} : stress concentration for normal and shear stress respectively.

M_a, M_m : alternating & mean bending moments

T_a, T_m : alternating & mean torques.

d : diameter

$n = ?$

Von Mises Stress

von Mises stress — Distortion energy theory

$$\sigma_a' = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(\frac{32\,K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16\,K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma_m' = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(\frac{32\,K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16\,K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$\boxed{\begin{aligned} \sigma_a' &= \frac{16}{\pi d^3}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} \\ \sigma_m' &= \frac{16}{\pi d^3}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2} \end{aligned}}$$ — I

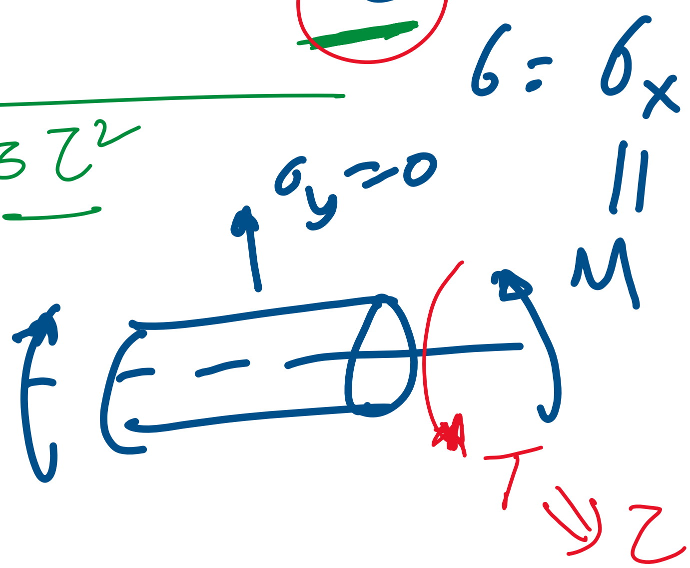
We can now use these into the failure criterions

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

$\sigma_x = \sigma$, $\sigma_y = 0$, $\sigma_x\sigma_y = 0$, $\tau_{xy} = \tau$

von Mises

$$\sigma' = \sqrt{\sigma^2 + 3\tau^2}$$



(a) DE - Goodman

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} \qquad \text{—} \; (1)$$

Substituting (I) in (1)

$$\boxed{\frac{1}{n} = \frac{16}{\pi d^3}\left[\frac{1}{S_e}\left\{4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right\}^{1/2} + \frac{1}{S_{ut}}\left\{4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right\}^{1/2}\right]} \qquad (\mathrm{I})$$

We can also write this equation in terms of d

$$\boxed{d = \left(\frac{16n}{\pi}\left[\frac{1}{S_e}\left\{4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right\}^{1/2} + \frac{1}{S_{ut}}\left\{4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right\}^{1/2}\right]\right)^{1/3}}$$

DE Gerber

② DE - Gerber

$$\frac{n\sigma_a'}{S_e} + \left(\frac{n\sigma_m'}{S_{ut}}\right)^2 = 1$$

Solving for n

$$n = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\,\sigma_m\,S_e}{\sigma_a\,S_{ut}}\right)^2}\,\right] \quad - ②$$

Substituting Ⓘ in ②

$$\boxed{\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{A\,S_{ut}}\right)^2\right]^{1/2}\right\}} \quad * \quad Ⅱ$$

and solving for d gives

$$\boxed{d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{A\,S_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3}} \quad *$$

where

$$A = \sqrt{4\,(K_f M_a)^2 + 3\,(K_{fs} T_a)^2}$$

$$B = \sqrt{4\,(K_f M_m)^2 + 3\,(K_{fs} T_m)^2}$$

DE Soderberg

(3) DE - Soderberg

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_y} \quad -(3)$$

Substituting (I) in (3)

III

$$\boxed{\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{\frac{1}{2}} + \frac{1}{S_y}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{\frac{1}{2}}\right\}}$$

$$\boxed{d = \left(\frac{16 n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{\frac{1}{2}} + \frac{1}{S_y}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{\frac{1}{2}}\right\}\right)^{\frac{1}{3}}}$$

DE ASME Elliptic

(4) DE - ASME Elliptic

$$\frac{1}{n^2} = \left(\frac{\sigma_a'}{S_e}\right)^2 + \left(\frac{\sigma_m'}{S_y}\right)^2 \quad - (4)$$

IV

Putting (I) in (4)

$$\boxed{\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_y}\right)^2 + 4\left(\frac{K_f M_m}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}}$$

$$\boxed{d = \left\{\frac{16 n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_y}\right)^2 + 4\left(\frac{K_f M_m}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}}$$

First Yielding

DE- Soderberg checks for yielding through its model

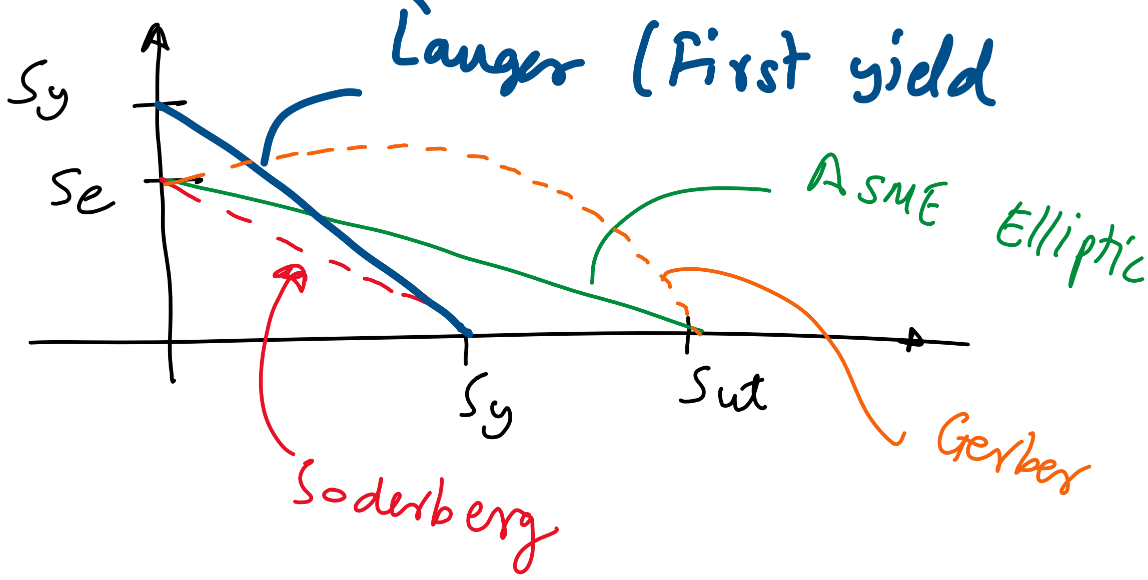
However for DE-Goodman, DE- Gerber, DE-ASME-elliptic one needs to check for yielding

For these criterion we compute von·Mises as follows

$$\sigma'_{max} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2 \right]^{1/2}$$

$$\boxed{\sigma'_{max} = \frac{16}{\pi d^3} \left[4\left[K_f (M_a + M_m)\right]^2 + 3\left[K_{fs}(T_a + T_m)\right]^2 \right]^{1/2} \qquad n_y = \frac{S_y}{\sigma'_{max}}}$$

V



Q1

For a machined shaft shoulder shown below, d = 1.1 in, D = 1.65 in, fillet radius r = 0.11 in. The shaft is subject to a combined load, a bending moment M = 1260 lbf-in and a steady torsional moment of T = 1100 lbf-in. The heat treated shaft has an ultimate strength of Sut = 105 kpsi and a yield strength Sy = 82 kpsi. The reliability goal for the endurance limit is 0.99.

Determine

n_f
(a) fatigue factor of safety using DE-Goodman criterion
(b) fatigue factor of safety using DE-Gerber criterion
(c) fatigue factor of safety using DE-Soderberg criterion
(d) fatigue factor of safety using DE-ASME-elliptic criterion

n_y
(e) yielding factor of safety

$S_{ut} = 105$ kpsi

$S_e =$

$S_y = 82$ kpsi

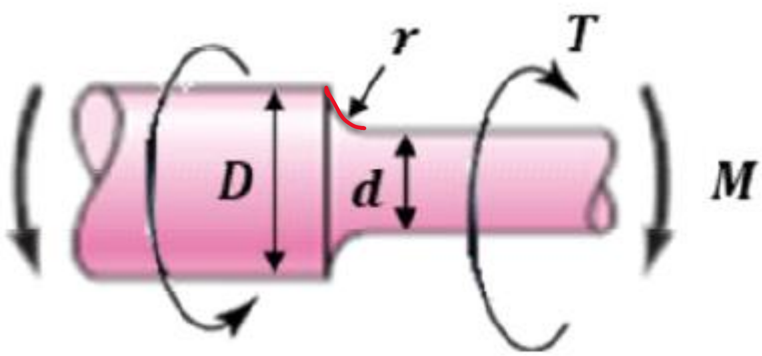
$d = 1.1$ in

$D = 1.65$ in

$r = 0.11$ in

$K_f =$

$K_{fs} =$



$T_a = 0$

$T_m = 1100$ lbf-in

$M_a = 1260$ lbf-in

$M_m = 0$

$n = ?$

$$S_e' = \begin{cases} 0.5\,S_{ut} & S_{ut} \le 200\text{ kpsi} \\ 100\text{ kpsi} & S_{ut} > 200\text{ kpsi} \end{cases}$$

ips

From Lec 17.

$$S_e' = 0.5(105) = 52.5\text{ kpsi}$$

$$k_a = a\,S_{ut}^{\,b} = 2(105)^{-0.217}$$

$$= 0.729$$

Lec 18

$$k_b = \begin{cases} 0.879\,d^{-0.107} & 0.11 < d \le 2\text{ in} \\ 0.91\,d^{-0.157} & 2 < d \le 10\text{ in} \end{cases}$$

Lec 18

$$k_b = (0.879)(1.1)^{-0.107} = 0.87$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

Lec 18

$S_e \propto k_c$ A large k_c will lead to a larger S_e. Since $n \propto S_e$, a larger S_e will lead to a less conservative design.

Use $k_c = 1$ (larger value, less conservative)

$K_d = 1$ (Done at room temperature)

$K_e = 0.814$ (99% reliability Lec 18)

$$S_e = k_a k_b k_c k_d k_e S_e'$$

$$S_e = (0.729)(0.87)(1)(1)(0.814)(52.5)$$

$$S_e = 27.1 \text{ kpsi}$$

How to compute K_f, K_{fs}

A-15

1) Compute K_t & K_{ts} from geometry (see Chap 5 stress concentration)

2) compute q, q_s from figure on the next page

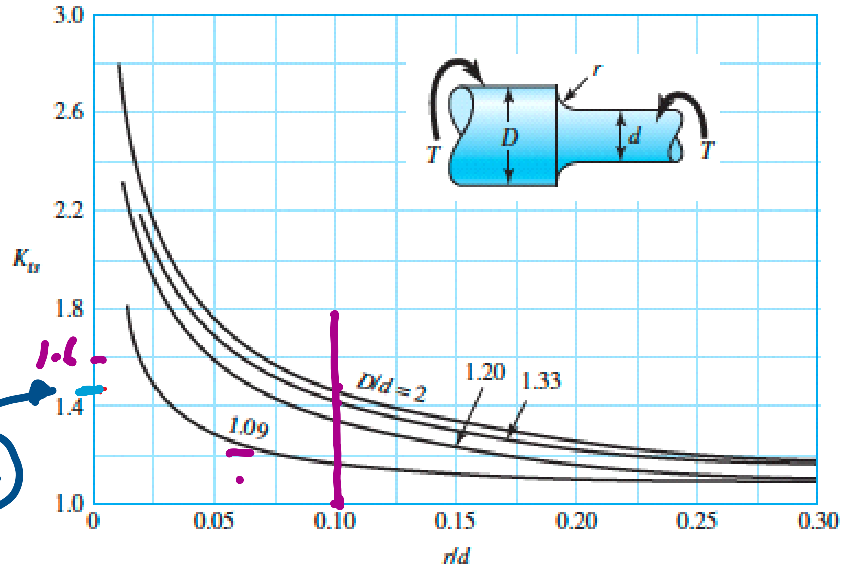
3) Compute K_f K_{fs} using the following

$$K_f = 1 + q(K_t - 1) \quad ; \quad K_{fs} = 1 + q_s(K_{ts} - 1) \quad \text{—(I)}$$

From Lec 19

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



$r = 0.11$ in

$d = 1.1$ in

$D = 1.65$ in

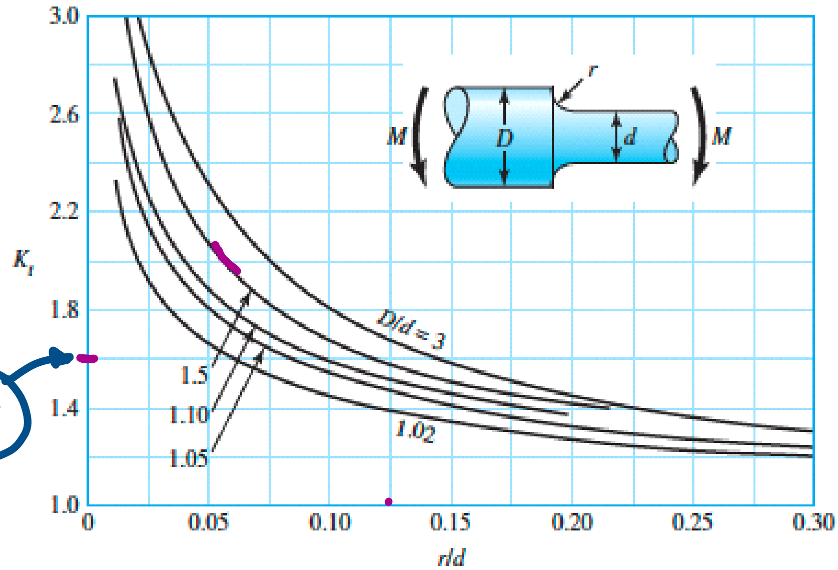
K_{ts}

1.6

1.42

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



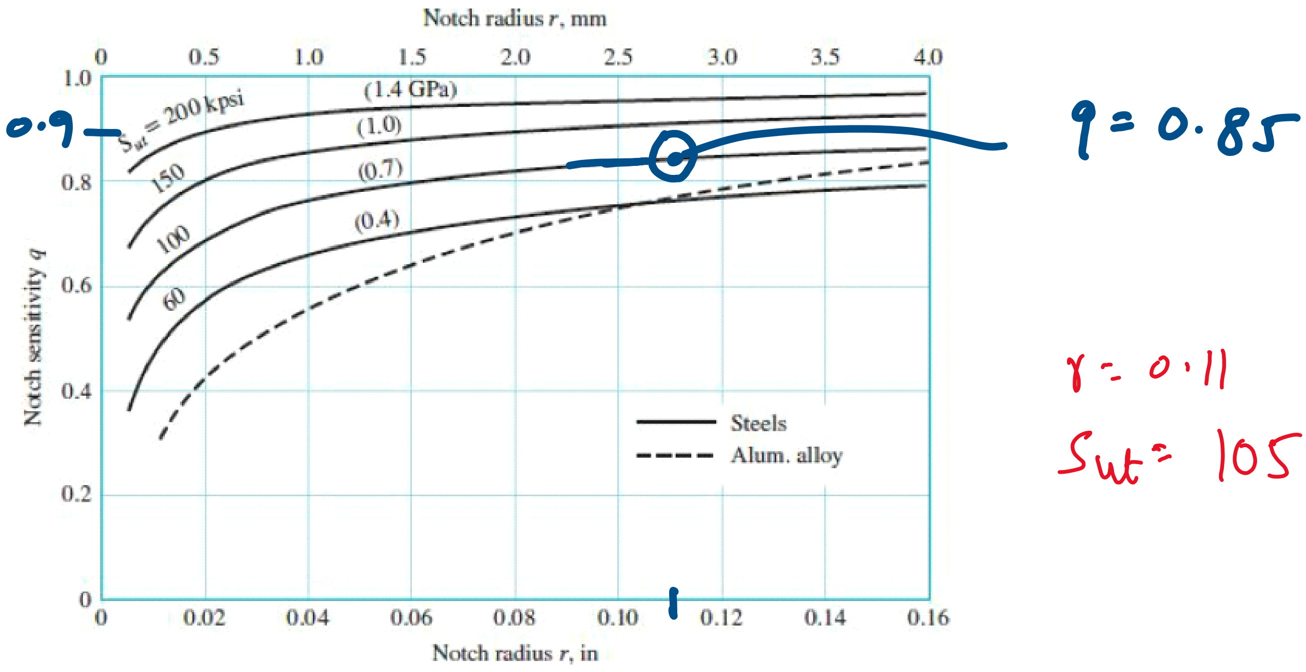
$\frac{D}{d} = \frac{1.65}{1.1} = 1.5$

K_t

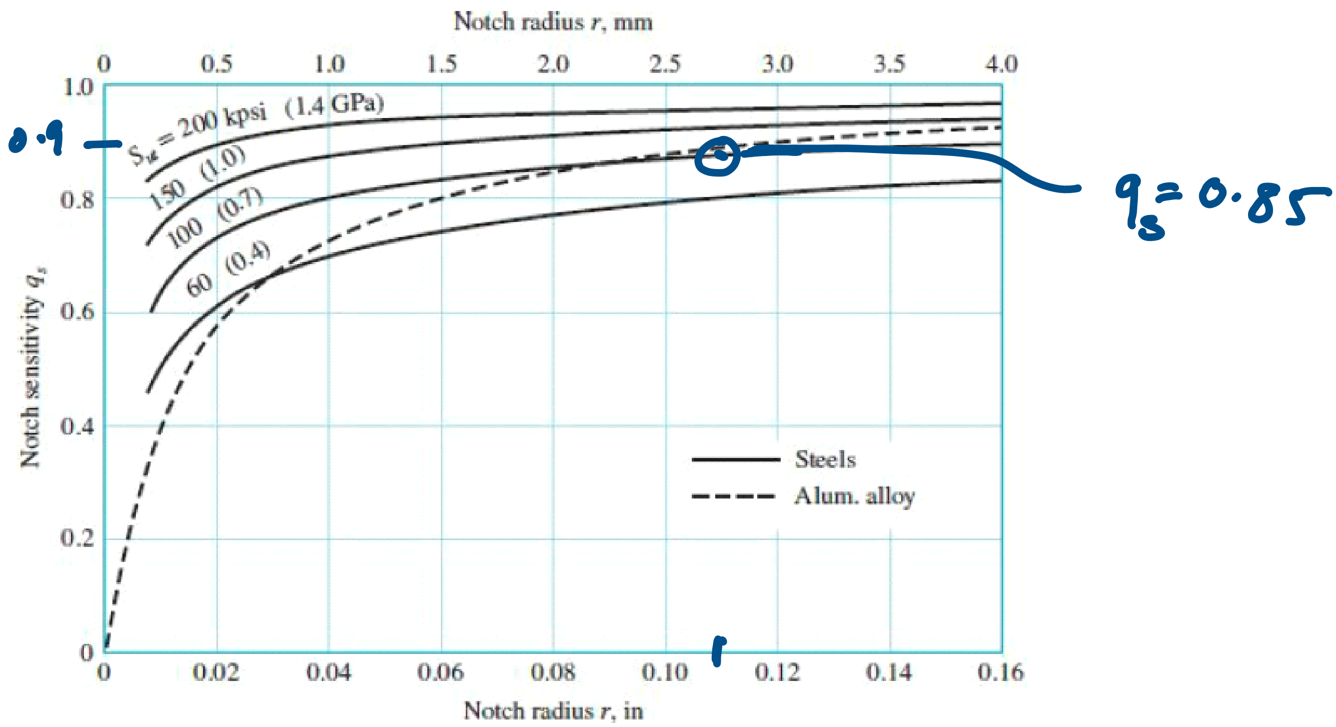
$\frac{r}{d} = \frac{0.11}{1.1} = 0.1$

1.68

$$\left.\begin{array}{l} K_{ts} = 1.42 \\ K_t = 1.68 \end{array}\right\} \quad 1 \checkmark$$



r = 0.11
S_{ut} = 105 kpsi



$q = q_s = 0.85$ — 2

(3) $K_f = 1 + q\,(K_t - 1)$

$= 1 + 0.85(1.68 - 1)$

$K_f = 1.58$

$K_{fs} = 1 + q_s\,(K_{ts} - 1)$

$= 1 + 0.85(1.42 - 1)$

$K_{fs} = 1.36$

(a) From I

$$\boxed{\frac{1}{n} = \frac{16}{\pi d^3}\left[\frac{1}{S_e}\left\{4(k_f M_a)^2 + 3(k_{fs} T_a)^2\right\}^{1/2} + \frac{1}{S_{ut}}\left\{4(k_f M_m)^2 + 3(k_{fs} T_m)^2\right\}^{1/2}\right]}$$

I

$d = 1.1$ in

$S_e = 27.1$ kpsi

$S_{ut} = 105$ kpsi

$K_f = 1.58$

$K_{fs} = 1.36$

$M_m = 0$

$T_a = 0$

$M_a = 1260$ lbf·in

$T_m = 1100$ lbf·in

(a) $\boxed{n = 1.52}$ — I

(b) $\boxed{n = 1.73}$ — II

(c) $\boxed{n = 1.46}$ — III

(d) $\boxed{n = 1.74}$ — IV

(e) $\boxed{n = 4.5}$ — V

DE - Goodman

DE - Gerber

DE - Soderberg (most conservative)

DE ASME Elliptic

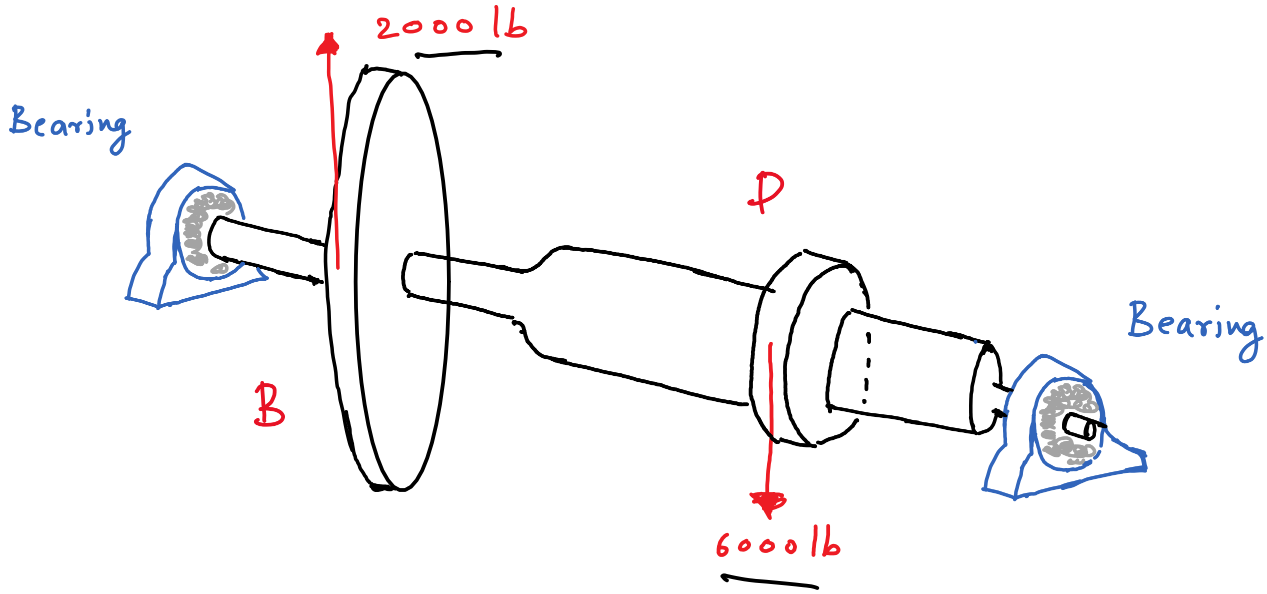
First Yield

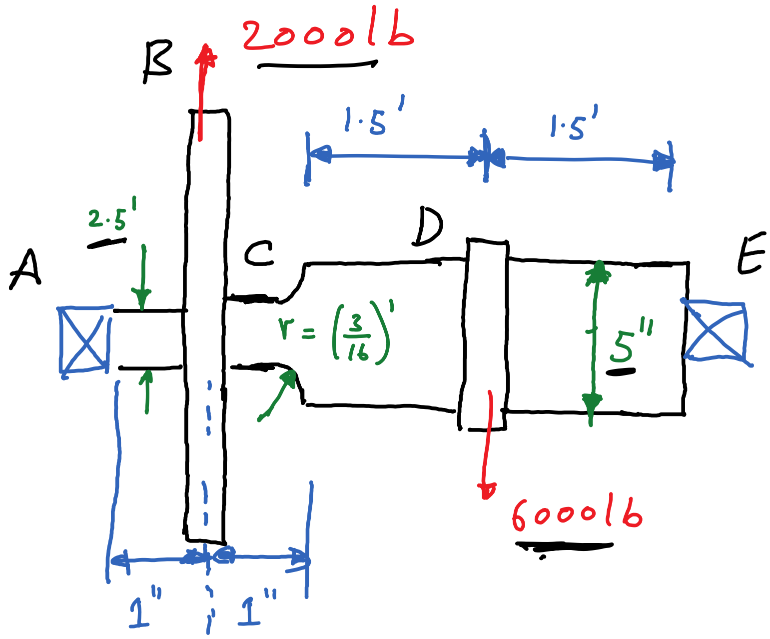
Q2

For the shaft shown below, the mating gears (not shown) drive the gears at B and D through forces in the vertical directions as shown. Assume Sut = 100 kpsi, Sy = 80 kpsi and Se = 24 kpsi. Compute

(a) factor of safety assuming static yielding using Distortion Energy (DE)

(b) factor of safety assuming fatigue failure and DE-ASME Elliptic criteria





$r_B = 9\,in$; $r_D = 3\,in$

$T_B = F_B r_B = (2000)(9) = 18000$ || $T_D = F_D r_D = 6000(3) = 18000$

$T_B = F_B r_B = (2000)(9) = 18000$ lbf·in $\quad || \quad T_D = F_D r_D = 18000$ lbf·in

(a) First cycle yielding

$$\boxed{\sigma'_{max} = \frac{16}{\pi d^3}\left[4\left[K_f\,(M_a + M_m)\right]^2 + 3\left[K_{fs}\,(T_a + T_m)\right]^2\right]^{1/2} \qquad n_y = \frac{S_y}{\sigma'_{max}}}$$

V

IV

(b) DE ASME- Elliptic

$$\boxed{\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_y}\right)^2 + 4\left(\frac{K_f M_m}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}}$$

$S_e = 24$ kpsi

$S_y = 80$ kpsi

$d = ?$

$K_f = ?$

$K_{fs} = ?$

$T_a = ?$

$T_m = ?$

$M_a = ?$

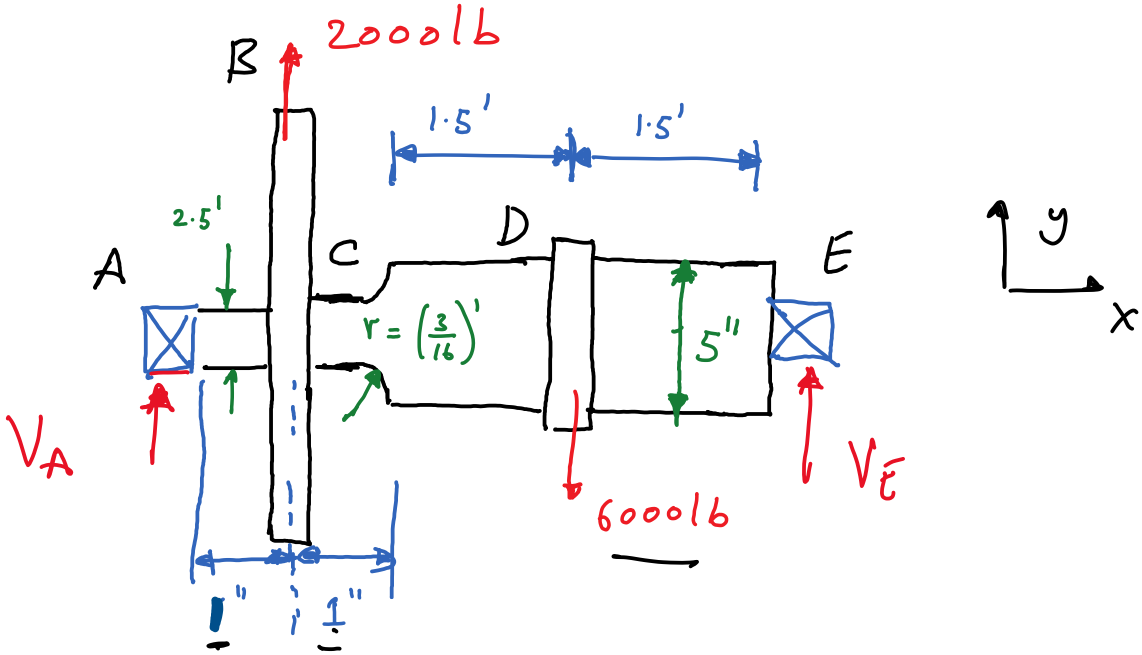
$M_m = ?$

as a function of distance

↓

Figure out the critical location

FBD



$$\Sigma F_y = V_A + 2000 - 6000 + V_E = 0$$

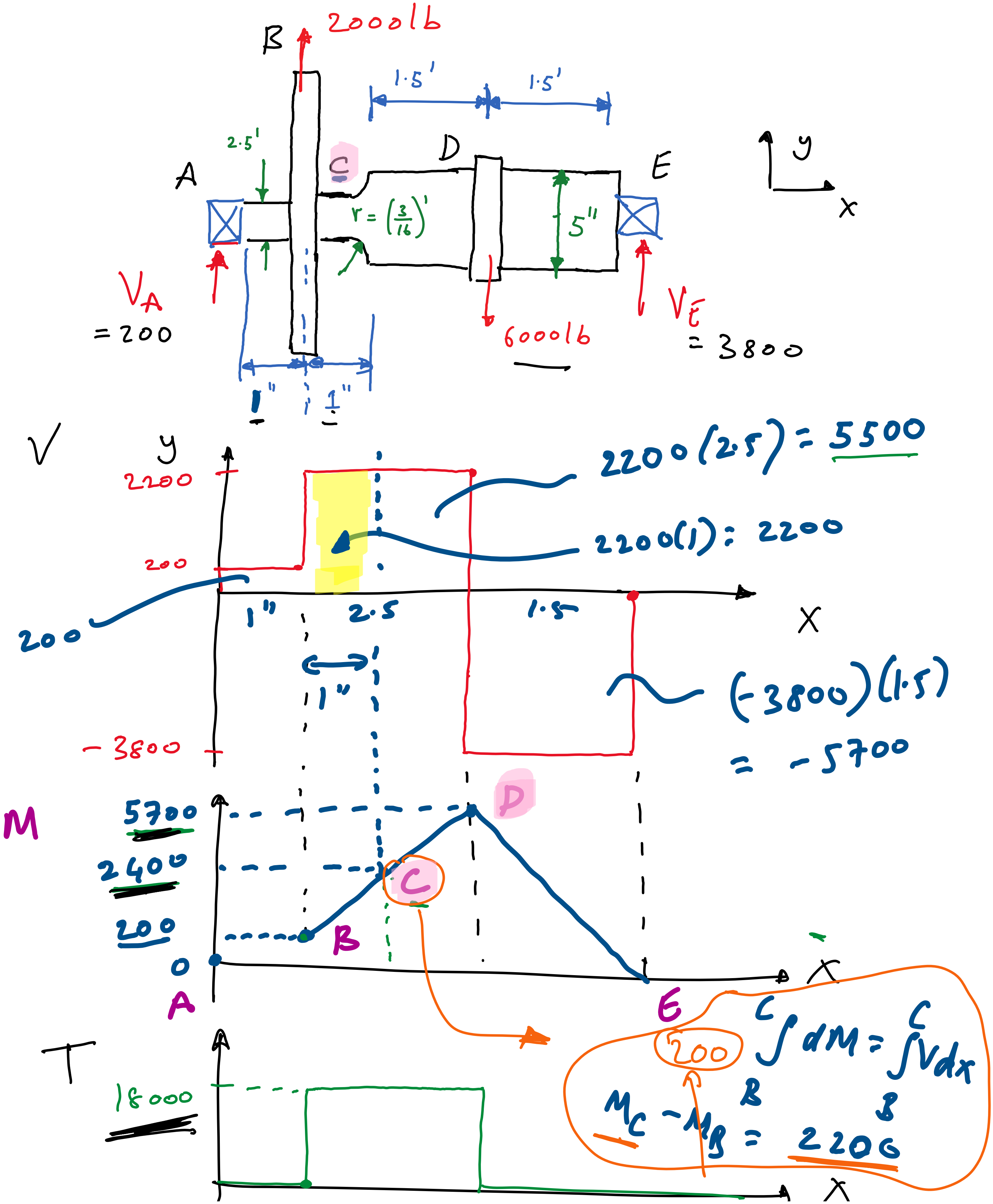
$$V_A + V_E = 4000$$

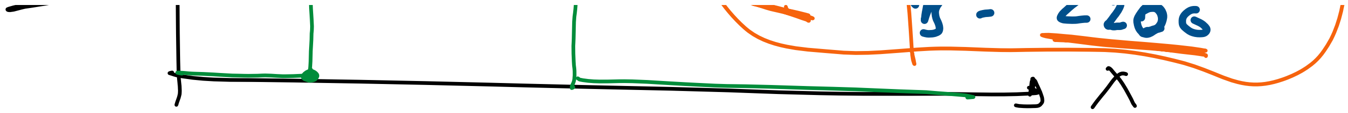
$$\Sigma M_A = +2000(1) - 6000(1+1+1.5) + V_E(1+1+1.5+1.5) = 0$$

$$V_E = \frac{6000(3.5) - 2000}{5} = 3800 \text{ lbf}$$

$$V_A = 4000 - 3800 = 200 \text{ lbf}$$

SFD/BMD/Torque diagram





Critical location

2 possible locations: C and D

Both C and D have the same torque
Although C has a lower M than D as shown in the BMD, there is a fillet at C which leads to a stress concentration at C. The actual moment at C would be $K_f M_C$

The stress at D: $\sigma_D = \dfrac{32 M_D}{\pi d_D^3}$

The stress at C: $\sigma_C = \dfrac{32 M_C K_f}{\pi d_C^3}$

$d_C = 2.5$ in $\quad d_D = 5$ in

C is the critical location because of lower diameter at C and $K_f > 1$

At C: $M_m = 0$

$M_a = 2400$ lbf-in $\qquad d = d_C = 2.5$ in

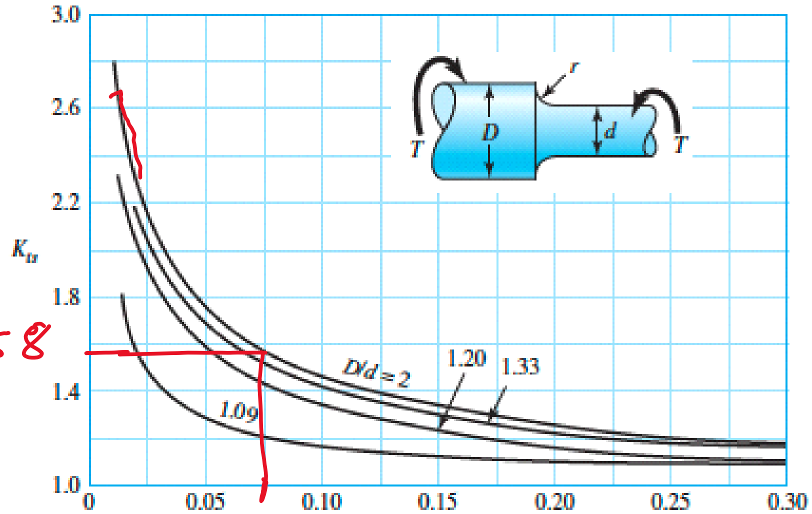
$T_m = 18000$ lbf·in

$T_a = 0$

Kt, Kts

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

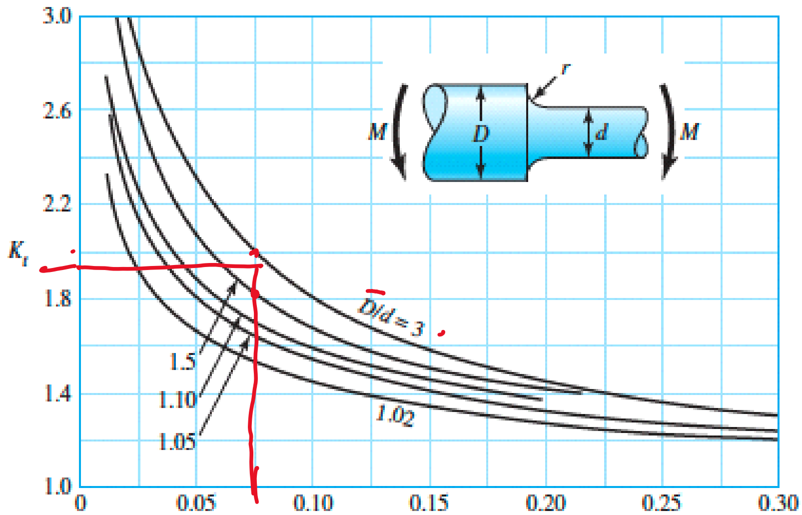


$K_{ts} = 1.58$

0.075

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

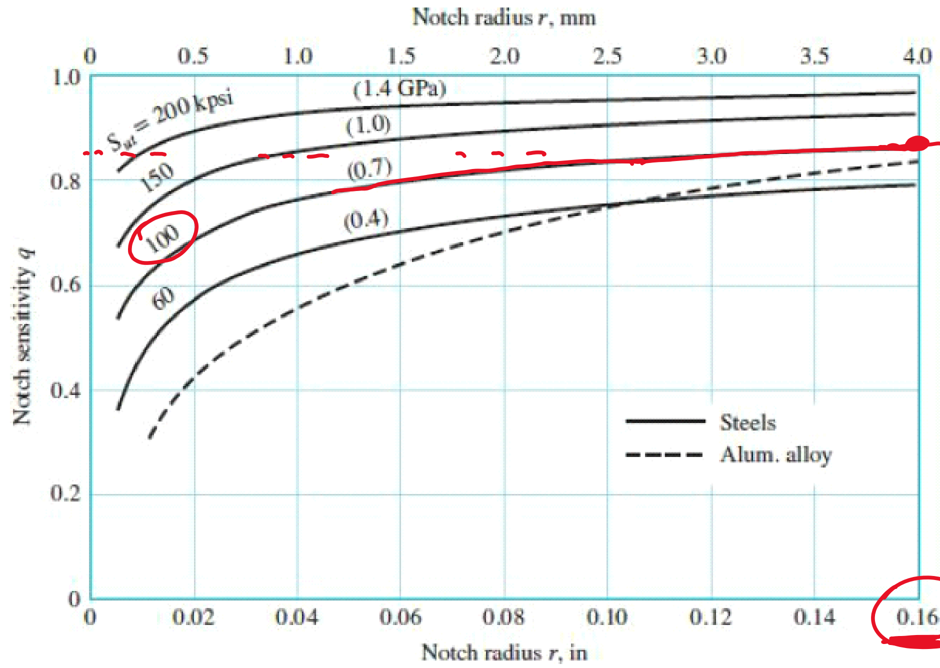


$K_t = 1.9$

0.075

$$\frac{D}{d} = \frac{5}{2.5} = 2 \qquad \frac{r}{d} = \frac{3/16}{2.5} = 0.075$$

q, qs



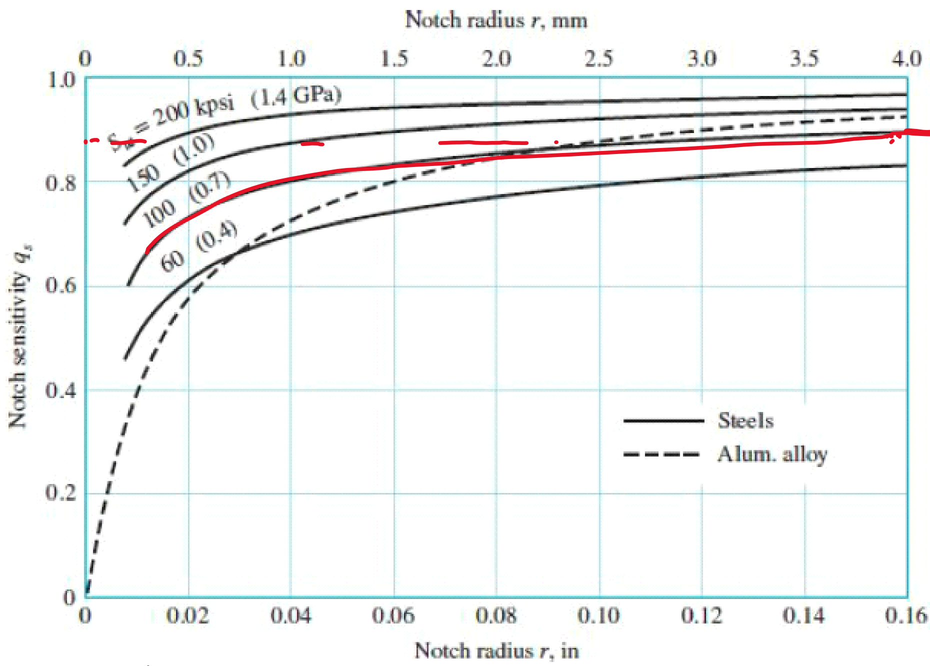
$0.85 = q$

$r = \left(3/16\right)''$

$= 0.1875$

$S_{ut} = 100 \text{ kpsi}$

0.1875



$0.9 = q_s$

0.1875

✓ $K_f = 1 + q\,(K_t - 1) = 1 + 0.85(1.9 - 1)$

$= 1.765$

✓ $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.9\,(1.58 - 1)$

$= 1.5222$

(a) $$n_y = \frac{S_y}{\sigma'_{max}}$$

$$\sigma'_{max} = \frac{16}{\pi d^3}\left\{4\left[K_f\left(M_a + \cancelto{0}{M_m}\right)\right]^2 + 3\left[K_{fs}\left(\cancelto{0}{T_a} + T_m\right)^2\right]\right\}^{1/2}$$

$$= \frac{16}{\pi (2.5)^3}\left\{4\left[(1.765)(2400)\right]^2 + 3\left[1.522\,(18000)\right]^2\right\}^{1/2}$$

$$= 1.57\,(10^4)\ \text{psi}$$

$$n_y = \frac{80\,(10^3)}{1.57\,(10^4)} \Rightarrow \boxed{n_y = 5.09}$$

(b) $$\frac{1}{n_f} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\cancelto{0}{\left(\frac{K_{fs} T_a}{S_y}\right)^2} + \cdots\right.$$

$$\left. 4\cancelto{0}{\left(\frac{K_f M_m}{S_e}\right)^2} + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}$$

$$\frac{1}{n_f} = \frac{16}{\pi (2.5)^3}\left[4\left(\frac{(1.765)(2400)}{24\,(10^3)}\right)^2 + 3\left(\frac{(1.522)(18000)}{(80)(10^3)}\right)^2\right]^{1/2}$$

$$\frac{1}{n_f} = 0.2257$$

$$\boxed{n_f = 4.43}$$