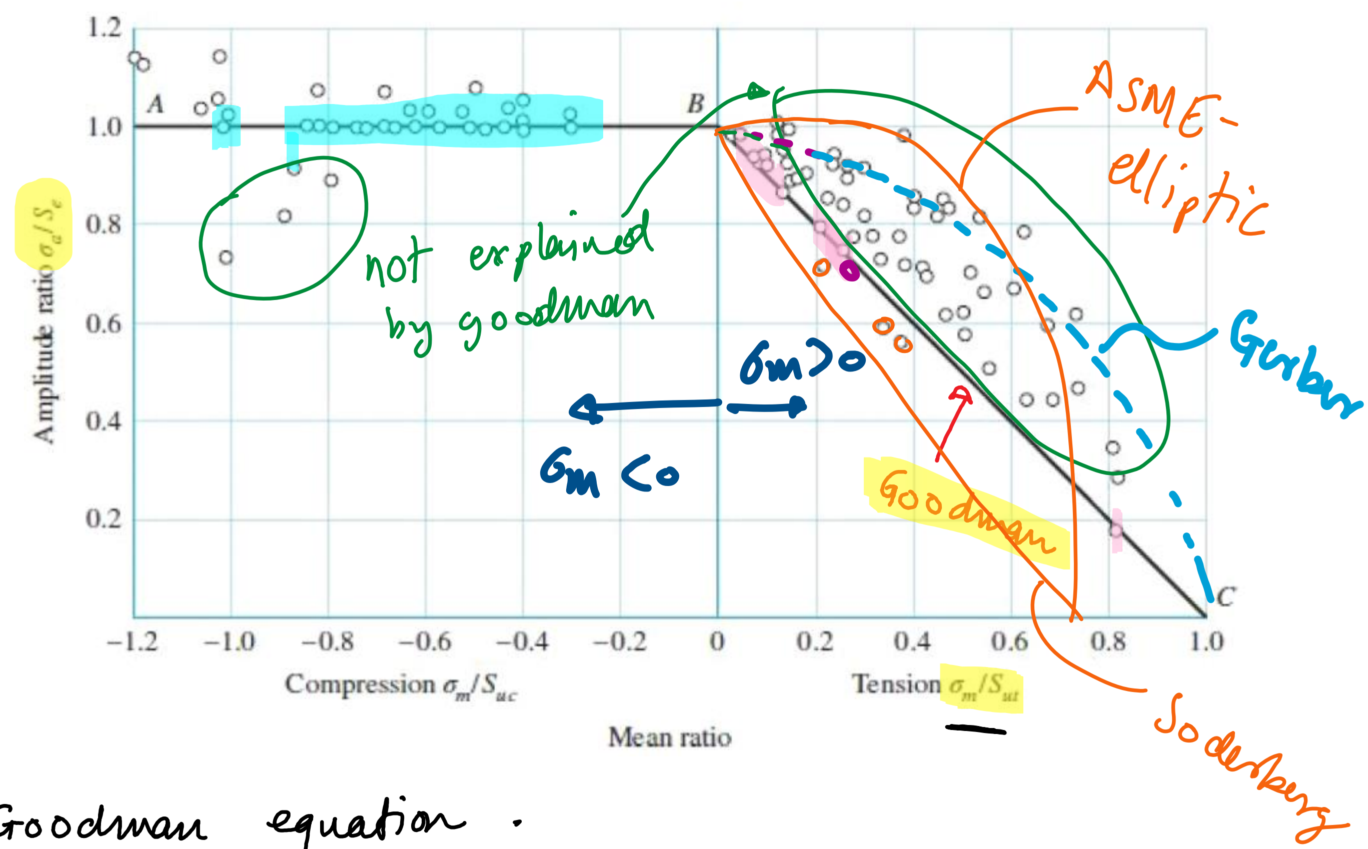
Fatigue Failure Criteria

6-13 Fatigue Failure Criteria

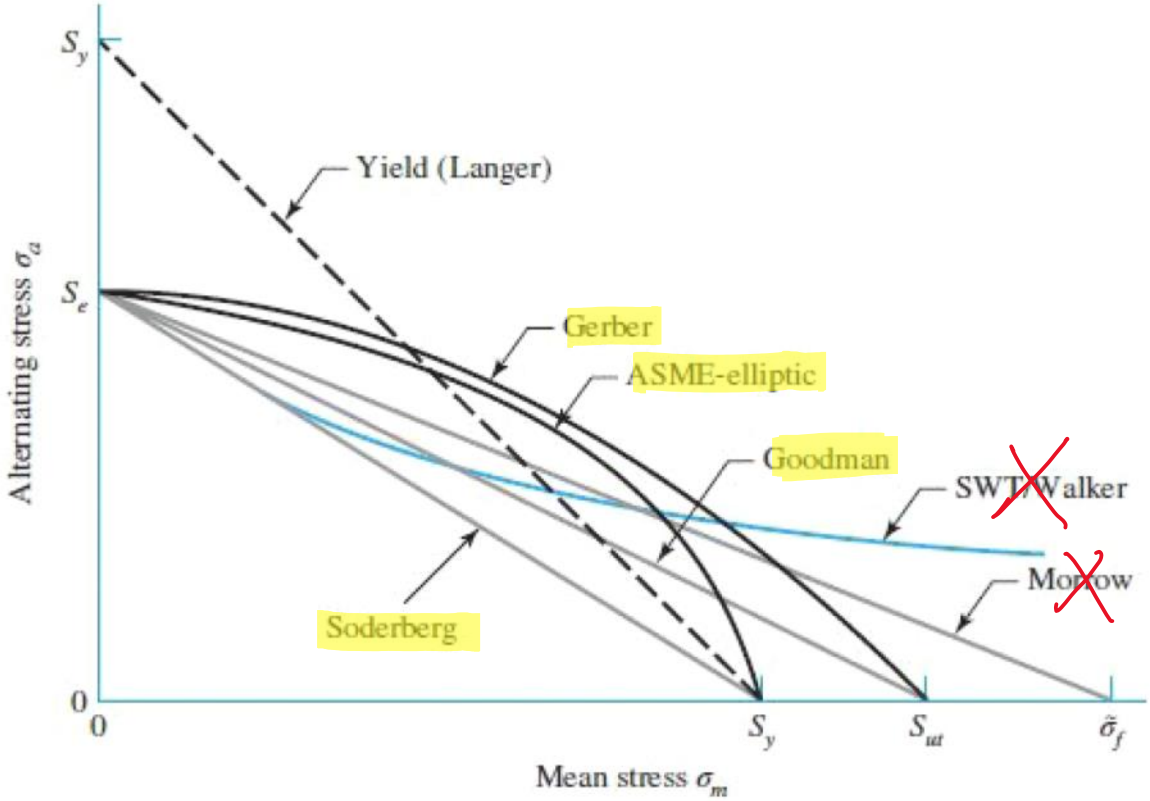


Goodman equation.

- → simple
- → non-conservative for compression
- → conservative for tension

→ other curves to explain the data in the tension region (next page onwards)

Goodman



(1) Goodman:

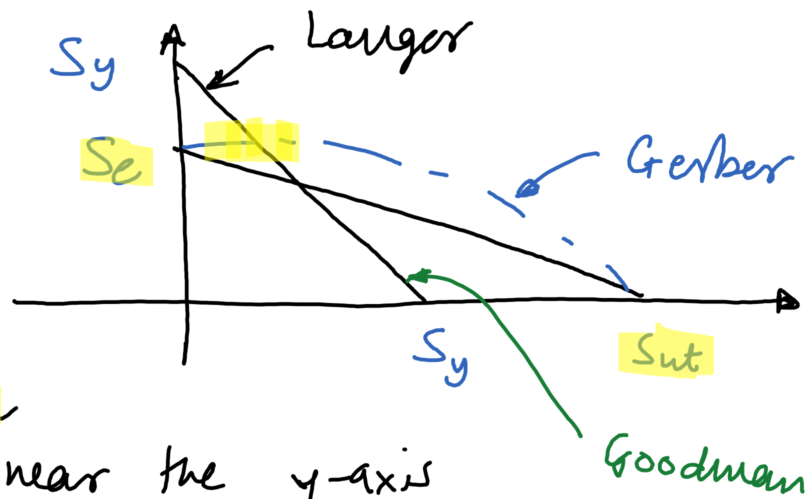
simple, conservative for positive stress. Not good for negative stress

Failure criterion: $\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1}$

Design equation: $\boxed{n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1}}$

Gerber

(2) Gerber



- parabolic equation
- non-conservative near the y-axis
- only good for tensile stress.

Failure criteria

$$\boxed{\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1}$$

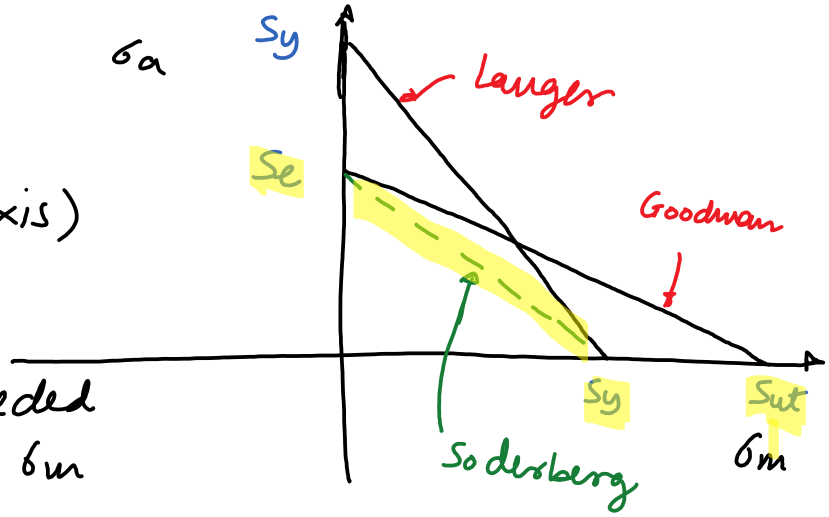
Design equation

Put $\sigma_a = n_f \sigma_a$ & $\sigma_m = n_f \sigma_m$ in the failure criteria. This will give a quadratic equation in n_f. This is solved for the positive root to get

$$\boxed{n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \left(\frac{\sigma_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{\sigma_a S_{ut}}\right)^2}\,\right]}$$

Soderberg

(3) Soderberg



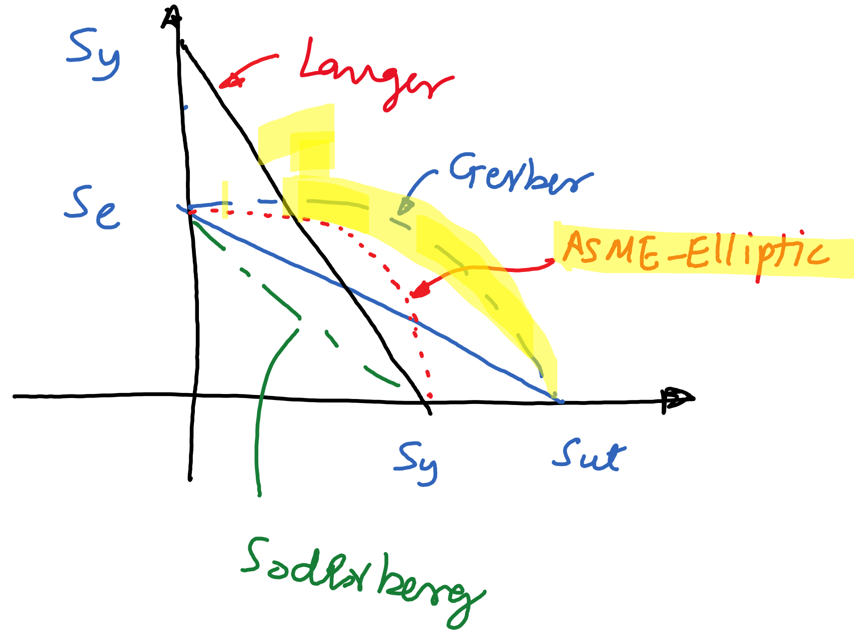
- Line connecting S_e (y-axis) with S_y (x-axis)
- Yield check is not needed for high σ_a & low σ_m
- Ultra conservative

Failure criterion: $\boxed{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_y} = 1}$

Design equation: $\boxed{n_f = \left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_y}\right)^{-1}}$

ASME-Elliptic

(4) ASME-Elliptic



- Considers yielding and fatigue without being ultra-conservative like Soderberg.
- no need to consider yielding curve

Failure criteria: $\boxed{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2 = 1}$

Design equation:

$$\boxed{n_f = \left[\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2\right]^{-1/2}}$$

Q1

A steel bar undergoes cyclic loading with nominal stress at the notch location given as Sigma_max = 60 kpsi, sigma_min = -20 kpsi, stress concentration factor Kf = 1.2. The material ultimate strength is S_ut = 100 kpsi, yield strength is S_y = 85 kpsi, and fully corrected endurance limit is S_e = 40 kpsi. Estimate the infinite life factor of safety using

(a) Goodman
(b) Gerber
(c) Soderberg
(d) ASME-Elliptical

n_f

$\sigma_{max} = 60$ kpsi

$\sigma_{min} = -20$ kpsi

$K_f = 1.2$

$S_{ut} = 100$ kpsi

$S_y = 85$ kpsi

$S_e = 40$ kpsi

$$\sigma_m^o = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{60 + (-20)}{2} = 20 \text{ kpsi}$$

$$\sigma_a^o = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{60 - (-20)}{2} = 40 \text{ kpsi}$$

$$\sigma_m = K_f \sigma_m^o = 1.2(20) = 24 \text{ kpsi}$$

$$\sigma_a = K_f \sigma_a^o = 1.2(40) = 48 \text{ kpsi}$$

(a) Goodman

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{48}{40} + \frac{24}{100} \right)^{-1}$$

$$\boxed{n_f = 0.69}$$

(b) Gerber

$$n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m} \right)^2 \left(\frac{\sigma_a}{S_e} \right) \left[-1 + \sqrt{1 + \left(\frac{2 \sigma_m S_e}{\sigma_a S_{ut}} \right)^2} \right]$$

$$= \frac{1}{2} \left(\frac{100}{24} \right)^2 \left(\frac{48}{40} \right) \left[-1 + \sqrt{1 + \left(\frac{2(24)(40)}{48(100)} \right)^2} \right]$$

$$\boxed{n_f = 0.8}$$

(c) Soderberg

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} \right)^{-1} = \left(\frac{48}{40} + \frac{24}{85} \right)^{-1}$$

$$\boxed{n_f = 0.67}$$

(d) ASME- Elliptic

$$n_f = \left[\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2\right]^{-1/2}$$

$$= \left[\left(\frac{48}{40}\right)^2 + \left(\frac{24}{88}\right)^2\right]^{-1/2}$$

$$\boxed{n_f = 0.81}$$

Summary

a) Goodman: 0.69

b) Gerber : 0.8

c) Soderberg: 0.67 Most conservative

d) ASME-Elliptic: 0.81 Least conservative

$$n_f \propto \frac{1}{\sigma} \qquad \sigma = \frac{My}{I} = \frac{M\,d/2}{\pi d^4/64} = \frac{32M}{\pi d^3}$$

$$\sigma \propto \frac{1}{d}$$

$$n_f \propto d$$