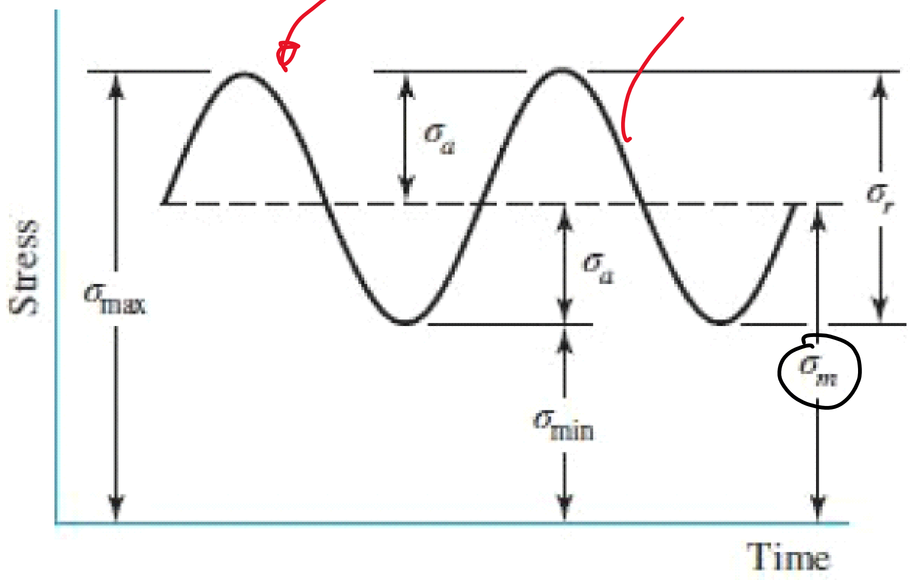
Fluctuating Stress Diagram

frequency, ω $\qquad\qquad$ $\sigma = \sigma_m + \sigma_a \sin(\omega t)$



So far $\sigma_m = 0$; $\sigma_a \neq 0$

→ Here we will look at the effect of σ_m, σ_a

$$\sigma_{max} = \sigma_m + \sigma_a$$

$$\sigma_{min} = \sigma_m - \sigma_a$$

$$\sigma_r = 2\sigma_a$$

Generating SN curve

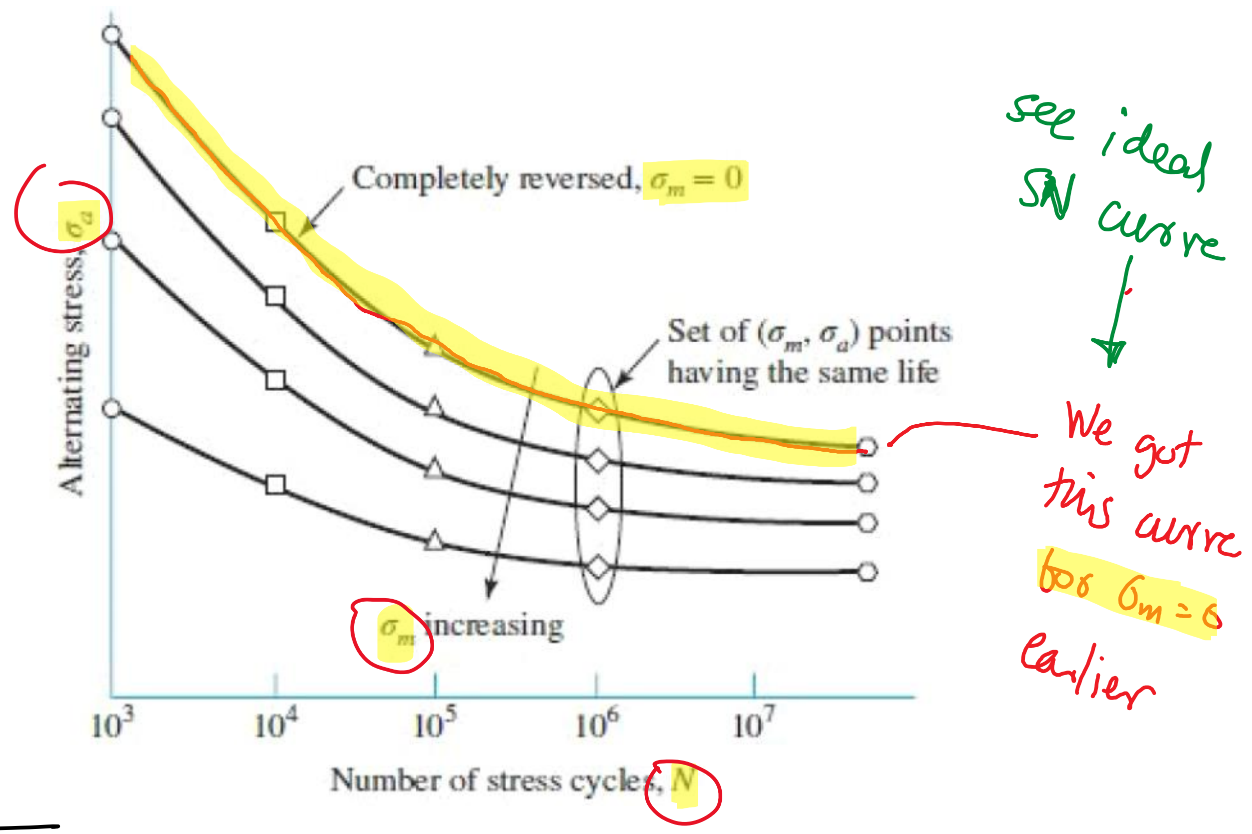
We need to generate a S-N curve. This is done experimentally using the procedure described for reversed loading, but with modification to add the effect of σ_m

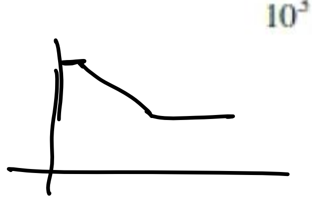
$\sigma_m = 0$

The loading is $\sigma = \sigma_m + \sigma_a \sin(\omega t)$

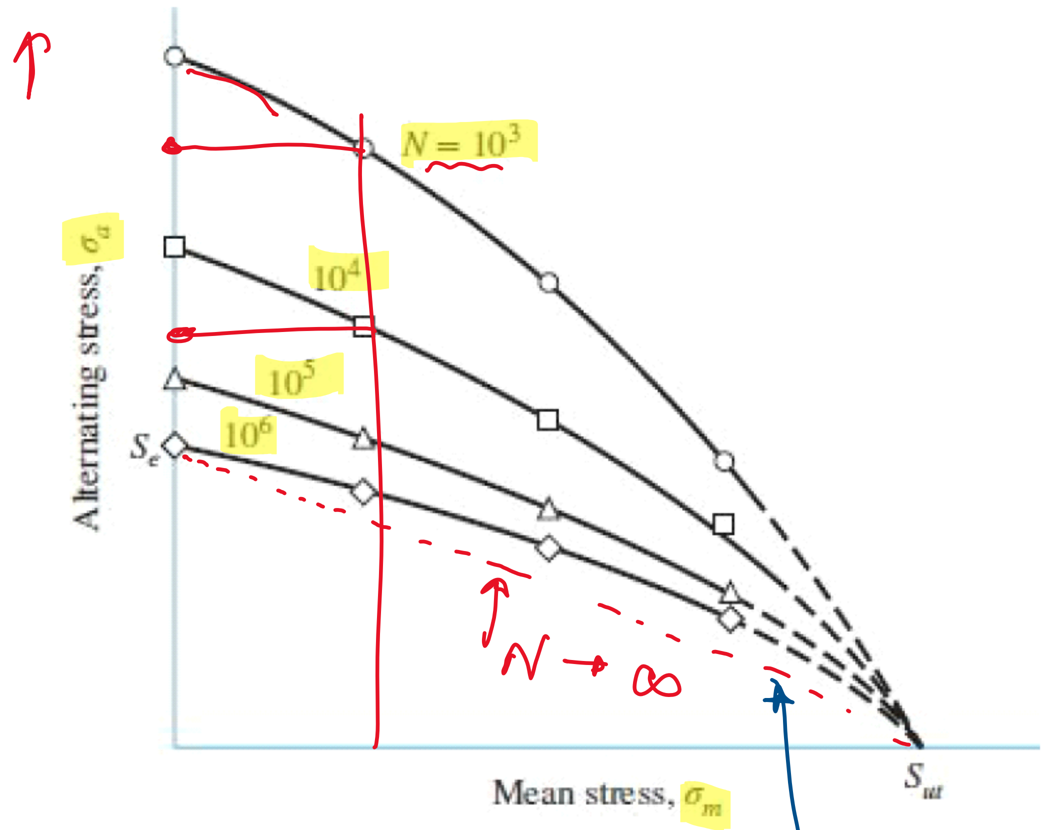
1. Choose σ_m ✓
2. Choose σ_a ✓
3. Apply load $\sigma = \sigma_m + \sigma_a \sin(\omega t)$ ✓ till failure

Repeat for a range of σ_a's & σ_m's.



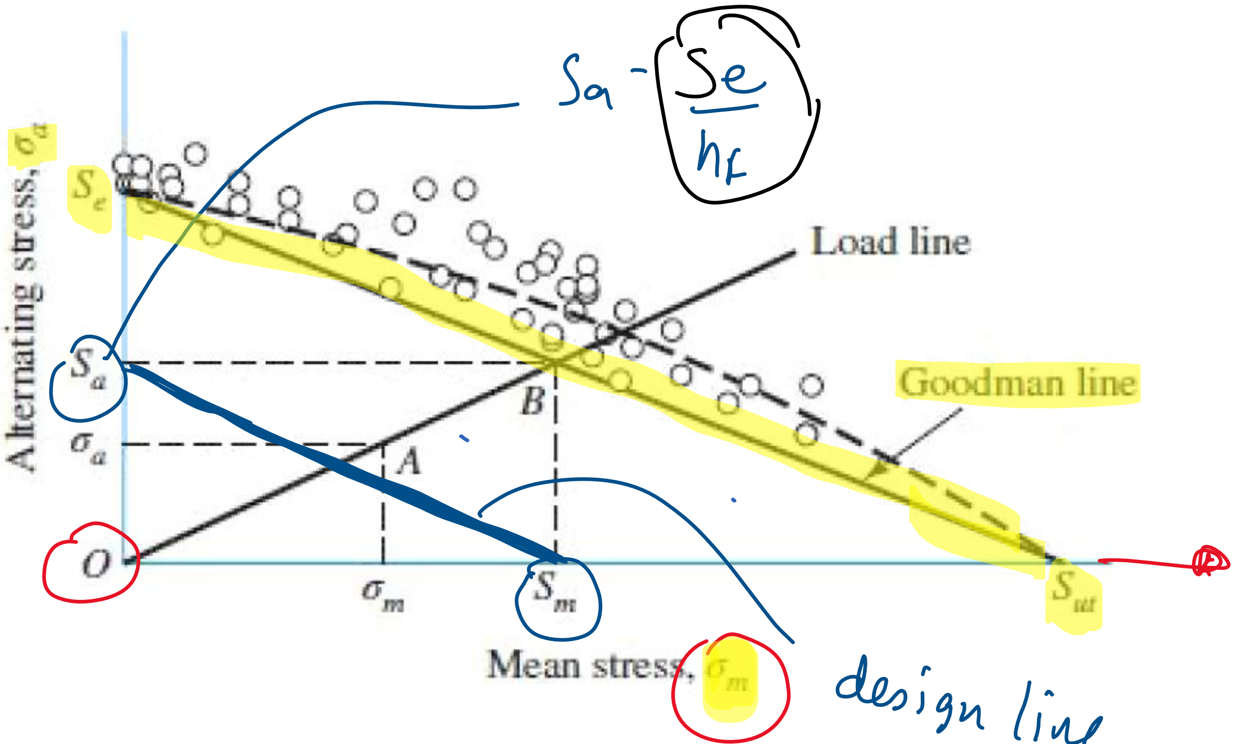


Sigma_a - sigma_m plot (pg 1)



$N \to \infty$ line
is called
1. GOODMAN Line
2. MODIFIED GOODMAN LINE
3. INFINITE LIFE

Sigma_a - sigma_m plot (pg 2)



$$S_m = \frac{S_{ut}}{n_f}$$

Equation for Goodman line

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1}$$

$\sigma_m > 0$

Equation of the blue line (design line)

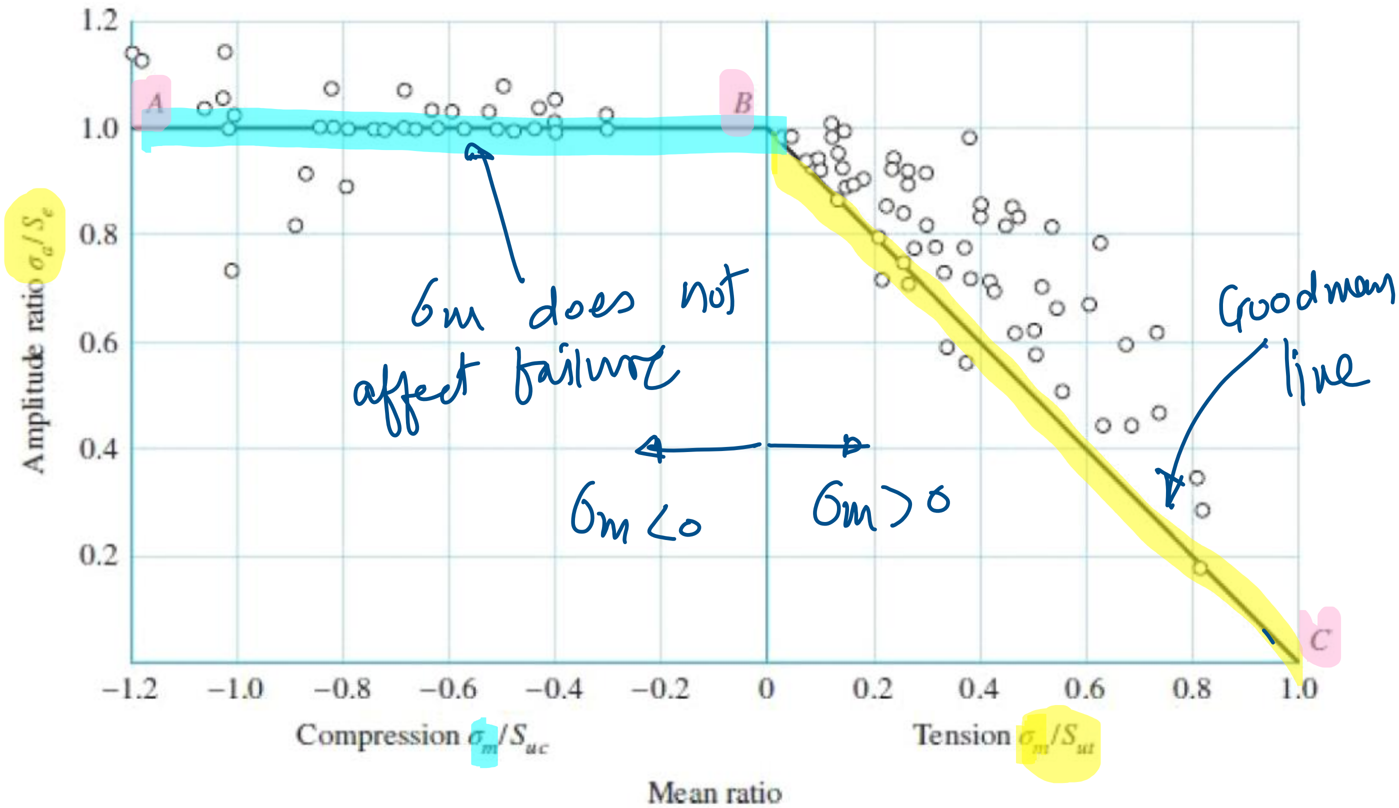
$$\frac{\sigma_a}{S_a} + \frac{\sigma_m}{S_m} = 1$$

$$\frac{\sigma_a}{S_e/n_f} + \frac{\sigma_m}{S_{ut}/n_f} = 1$$

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f}}$$

$\sigma_m > 0$

Sigma_a - sigma_m plot (pg 3)

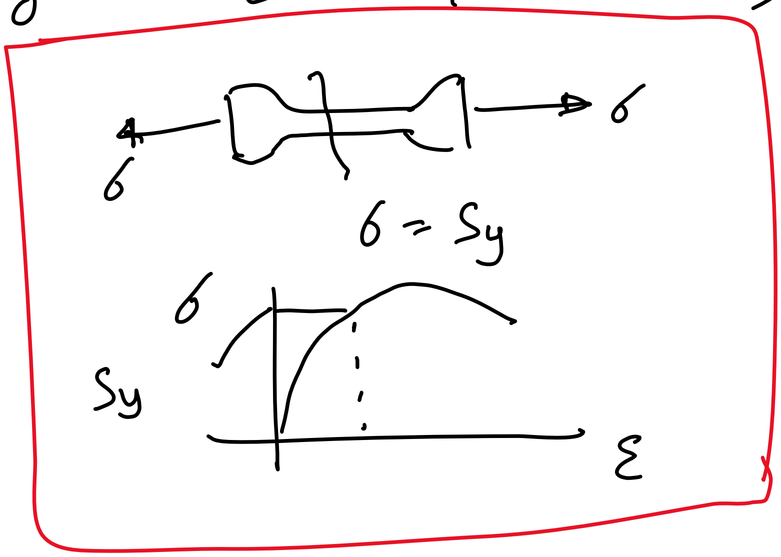


Line BC: $\boxed{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}} = 1}$ $\quad \sigma_m > 0$

Line AB: $\boxed{\dfrac{\sigma_a}{S_e} = 1}$ $\quad \sigma_m < 0$

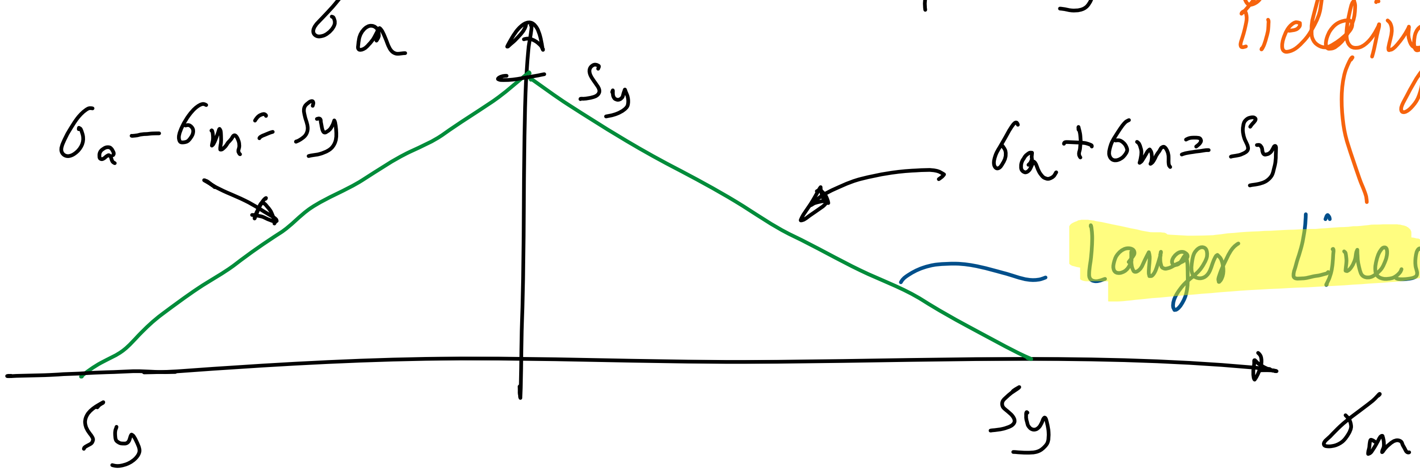
We also need to consider static loading. That is, if the stress is high enough, the material will fail in static loading

$\sigma = S_y$ (see 1st/2nd lecture)

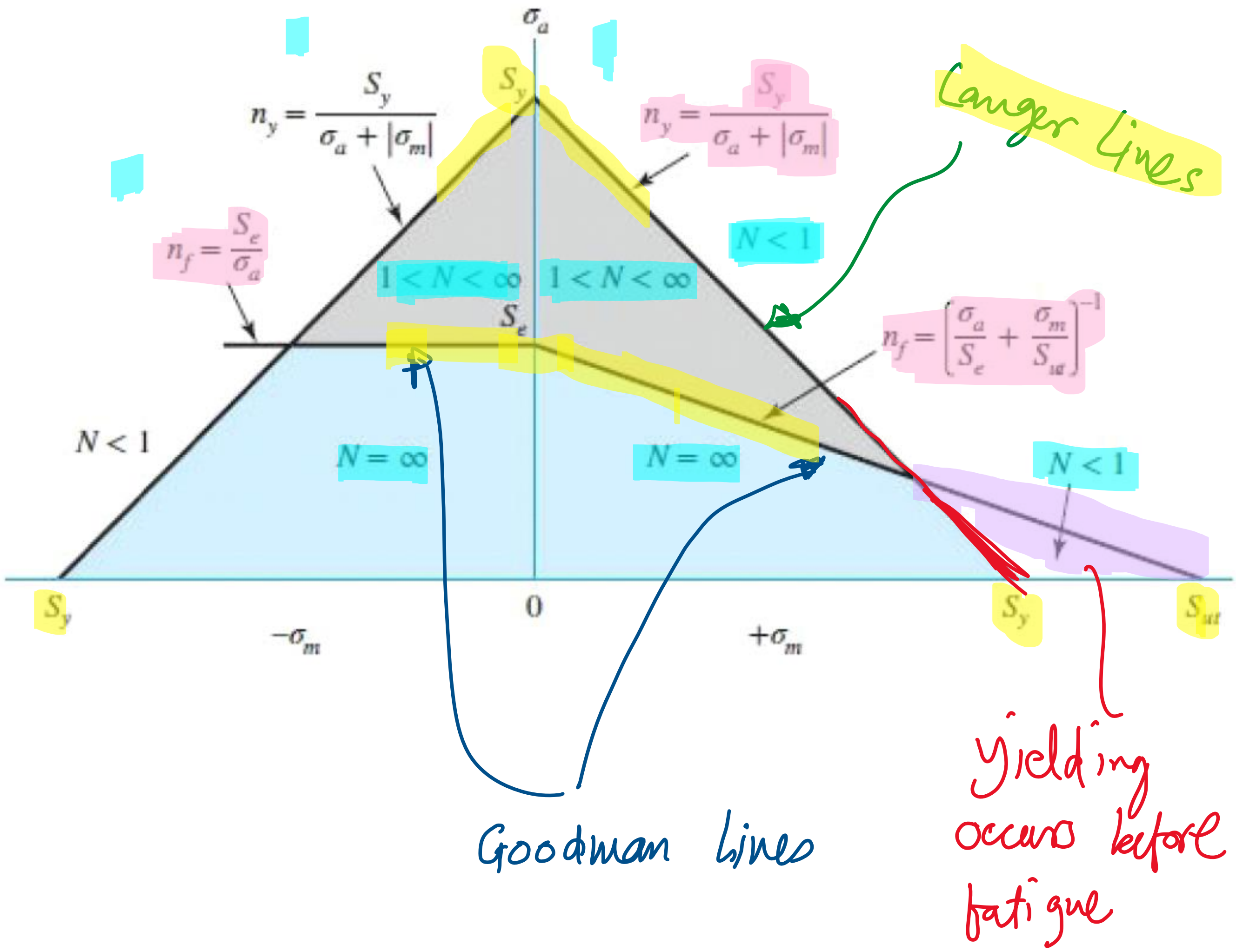


$$\underbrace{\sigma_a + \sigma_m = S_y \quad \text{or} \quad \sigma_a - \sigma_m = S_y}_{\sigma_a + |\sigma_m| = S_y}$$

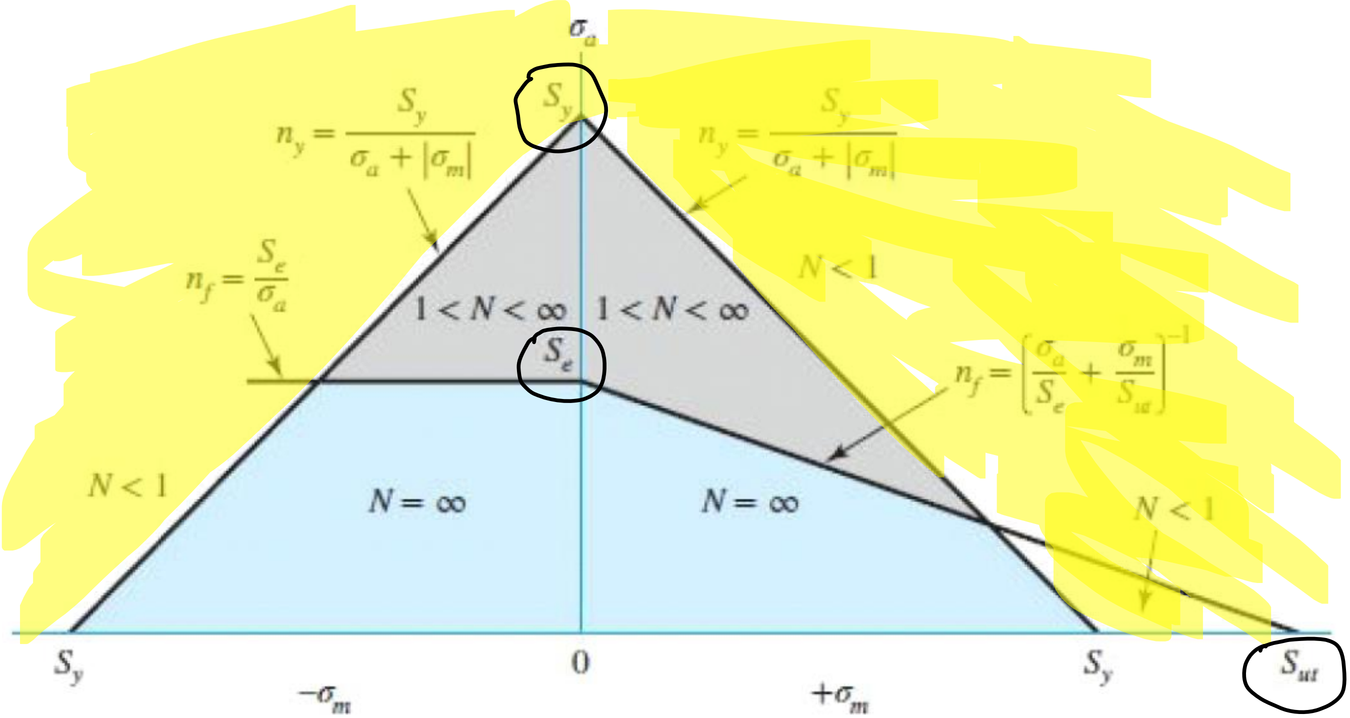
First cycle Yielding



Design line (a)



Design line (b)



Given σ_a, σ_m, S_e, S_{ut}, S_y predict if the material will fail at

(i) $N < 1$ (first cycle yielding)

(ii) $1 < N < \infty$ (finite life)

(iii) $N \to \infty$ (infinite life

2 methods:

(i) Graphical: Create the plot above using S_e, S_{ut}, S_y, σ_a, σ_m

(ii) Numerical

(a) $n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1}$ $\sigma_m > 0$

$= \frac{S_e}{\sigma_a}$ $\sigma_m < 0$

$n_y = \frac{S_y}{\sigma_a + |\sigma_m|}$

(b) If $n_f > 1$ Infinite life ($N \to \infty$) (Gray)

If $n_f < 1$ but $n_y > 1$ finite life ($1 < N < \infty$) (Blue region)

If $n_f < 1$ & $n_y < 1$ First-cycle yield (Yellow Region)

Q1

A steel bar undergoes cyclic loading with nominal stress at the notch location given as Sigma_max = 40 kpsi, sigma_min = 20 kpsi, stress concentration factor Kf = 1.2. The material ultimate strength is S_ut = 100 kpsi, yield strength is S_y = 85 kpsi, and fully corrected endurance limit is S_e = 40 kpsi

(a) factor of safety assuming infinite life ✓ n_f

(b) factor of safety assuming yielding ✓ n_y

(c) actual region based on given stresses ← $N<1,\ 1<N<\infty,\ N\to\infty$

$\sigma_{max} = 40$ kpsi ; $\sigma_{min} = 20$ kpsi ; $K_f = 1.2$

$S_{ut} = 100$ kpsi ; $S_y = 85$ kpsi, $S_e = 40$ kpsi

nominal

$$\sigma_m^{o} = \frac{\sigma_{max}+\sigma_{min}}{2} = \frac{40+20}{2} = 30 \text{ kpsi}$$

$$\sigma_a^{o} = \left|\frac{\sigma_{max}-\sigma_{min}}{2}\right| = \frac{40-20}{2} = 10 \text{ kpsi}$$

$$\sigma_m = K_f \sigma_m^{o} = (1.2)(30) = 36 \text{ kpsi}$$

$$\sigma_a = K_f \sigma_a^{o} = (1.2)(10) = 12 \text{ kpsi}$$

(a) $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{12}{40} + \frac{36}{100}\right)^{-1}$$

$$\boxed{n_f = 1.52}$$

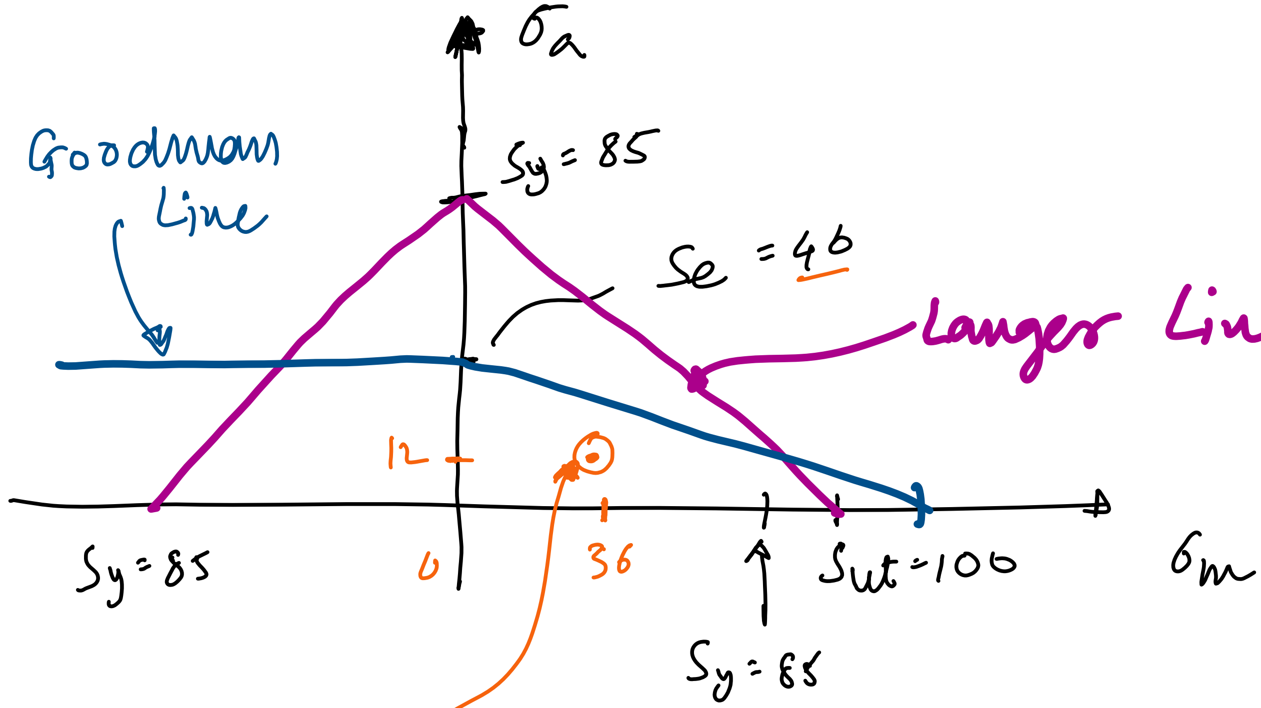
(b) $$n_y = \frac{S_y}{|\sigma_m| + \sigma_a} = \frac{85}{|36| + 12} \Rightarrow \boxed{n_y = 1.8}$$

(c) Numerical

Since $n_f > 1$ Infinite Life $N \to \infty$

Graphical

$S_{ut} = 100$ kpsi; $S_y = 85$ kpsi, $S_e = 40$ kpsi



$\sigma_m = 36$ kpsi
$\sigma_a = 12$ kpsi

Region $N \to \infty$

Q2

A steel bar undergoes cyclic loading with nominal stress at the notch location given as Sigma_max = 60 kpsi, sigma_min = -20 kpsi, stress concentration factor Kf = 1.2. The material ultimate strength is S_ut = 100 kpsi, yield strength is S_y = 85 kpsi, and fully corrected endurance limit is S_e = 40 kpsi

(a) factor of safety assuming infinite life n_f

(b) factor of safety assuming yielding n_y

(c) actual region based on given stresses ~ N ?

$\sigma_{max} = 60$ kpsi ; $\sigma_{min} = -20$ kpsi ; $K_f = 1.2$

$S_{ut} = 100$ kpsi, $S_y = 85$ kpsi, $S_e = 40$ kpsi

$$\sigma_m^o = \frac{1}{2}(\sigma_{max} + \sigma_{min}) = \frac{60-20}{2} = 20 \text{ kpsi}$$

$$\sigma_a^o = \frac{1}{2}\left|(\sigma_{max} - \sigma_{min})\right| = \left|\frac{60-(-20)}{2}\right| = 40 \text{ kpsi}$$

$$\sigma_m = K_f \sigma_m^o = (1.2)(20) = 24 \text{ kpsi}$$

$$\sigma_a = K_f \sigma_a^o = (1.2)(40) = 48 \text{ kpsi}$$

(a) $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{48}{40} + \frac{24}{100}\right)^{-1}$$

Since $\sigma_m > 0$

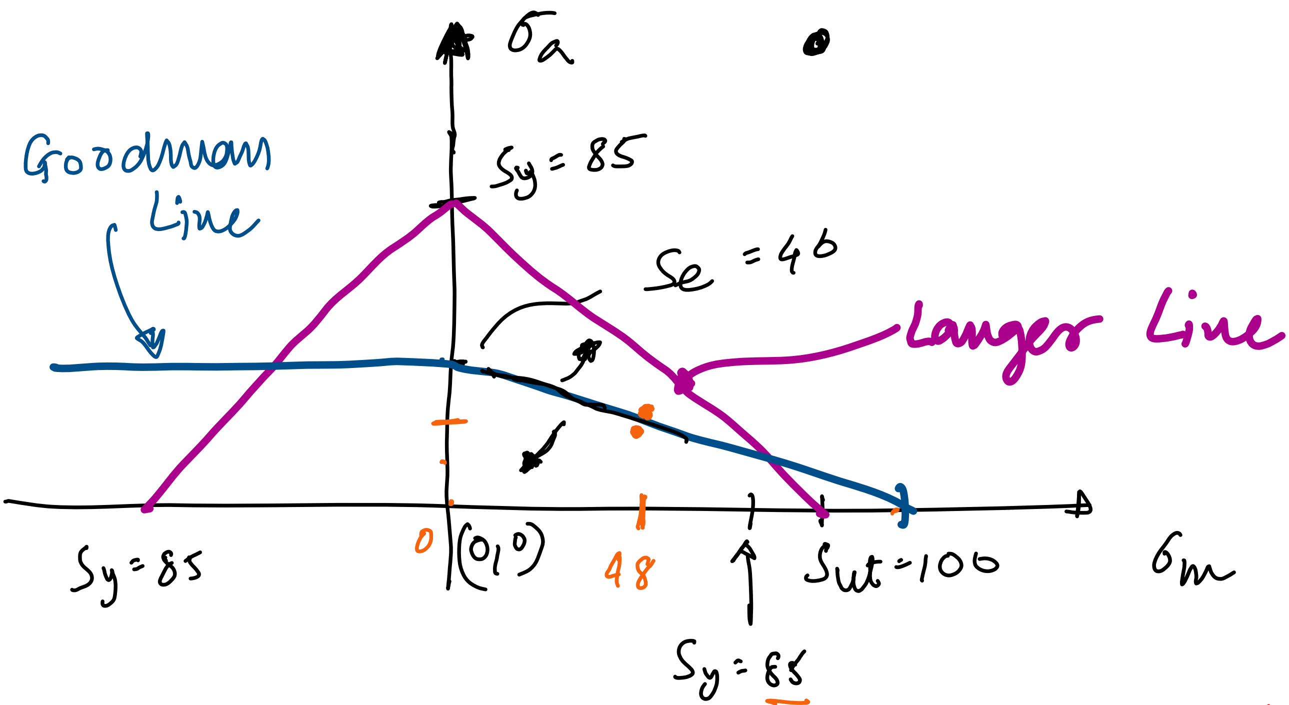
$$\boxed{n_f = 0.69}$$

(b) $$n_y = \frac{S_y}{\sigma_a + |\sigma_m|} = \frac{85}{48+24} \Rightarrow \boxed{n_y = 1.2}$$

(c) Numerical

Since $n_f < 1$ and $n_y > 1$ $\boxed{1 < N < \infty \text{ Finite Life}}$

Graphically



$\sigma_a = 24$; $\sigma_m = 48$

(a) < 1 $N \to \infty$

$\boxed{(a) > 1 \quad 1 < N < \infty}$

Goodman: $\underbrace{\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e}}_{(a)} = 1$

$$\frac{48}{100} + \frac{24}{40} = 1.08 > 1$$

$\boxed{1 < N < \infty \quad \text{Finite Life}}$

9

100 40

$1 < N < \infty$ Finite Life

Q3

A steel bar undergoes cyclic loading with nominal stress at the notch location given as Sigma_max = -20 kpsi, sigma_min = -40 kpsi, stress concentration factor Kf = 1.2. The material ultimate strength is S_ut = 100 kpsi, yield strength is S_y = 85 kpsi, and fully corrected endurance limit is S_e = 40 kpsi
(a) factor of safety assuming infinite life
(b) factor of safety assuming yielding
(c) actual region based on given stresses

Try yourself

(a) $n_f = 3.3$
(b) $n_y = 1.8$
(c) Infinite Life