Stress concentration / Notch sensitivity

6-10 Stress concentration and Notch Sensitivity

Fatigue stress-concentration factor (K_f)

$$K_f = \frac{\text{Fatigue strength of notched specimen}}{\text{Fatigue strength of notch free specimen}}$$

$$K_f = \frac{\sigma_{max}}{\sigma_0} \quad \& \quad K_{fs} = \frac{\tau_{max}}{\tau_0}$$

Notch sensitivity (q)

$$q = \frac{K_f - 1}{K_t - 1} \quad \& \quad q_s = \frac{K_{fs} - 1}{K_{ts} - 1}$$

K_t – static
table A-15

If $q = 0$ $\quad K_f = 1$ (no sensitivity)
$q = 1$ $\quad K_f = K_t$ (full notch-sensitivity)

How to compute K_f, K_{fs}

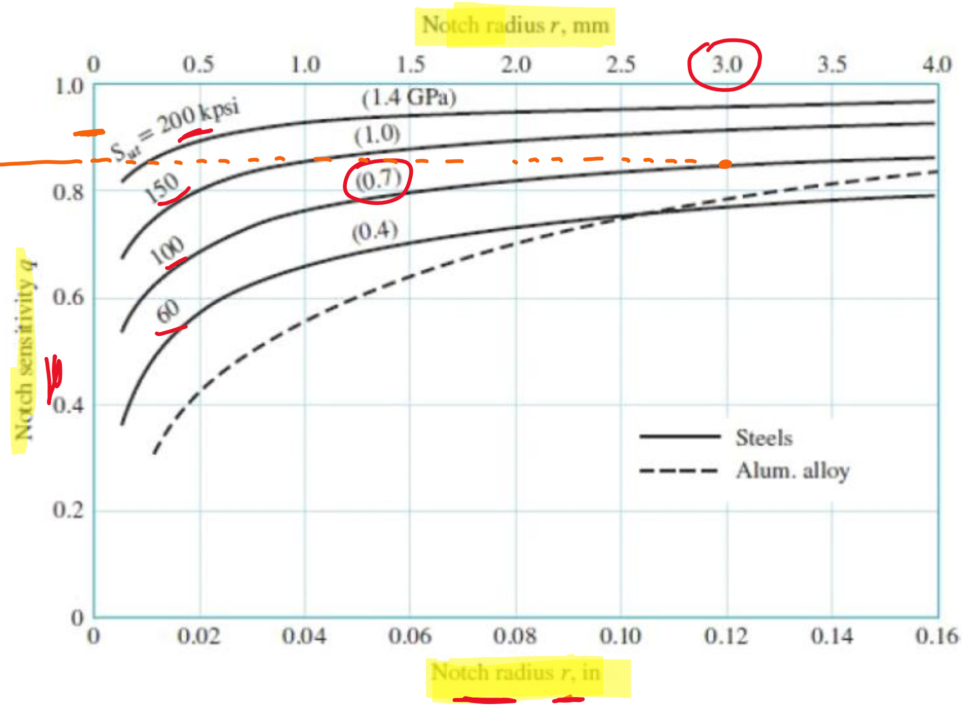
A-15

1) Compute K_t & K_{ts} from geometry (see Chap 5 stress concentration)
2) Compute q, q_s from figure on the next page
3) Compute K_f K_{fs} using the following

$$K_f = 1 + q\,(K_t - 1) \;; \quad K_{fs} = 1 + q_s\,(K_{ts} - 1) \qquad \text{(I)}$$

Compute q, q_s

0.85



$q = f(r)$

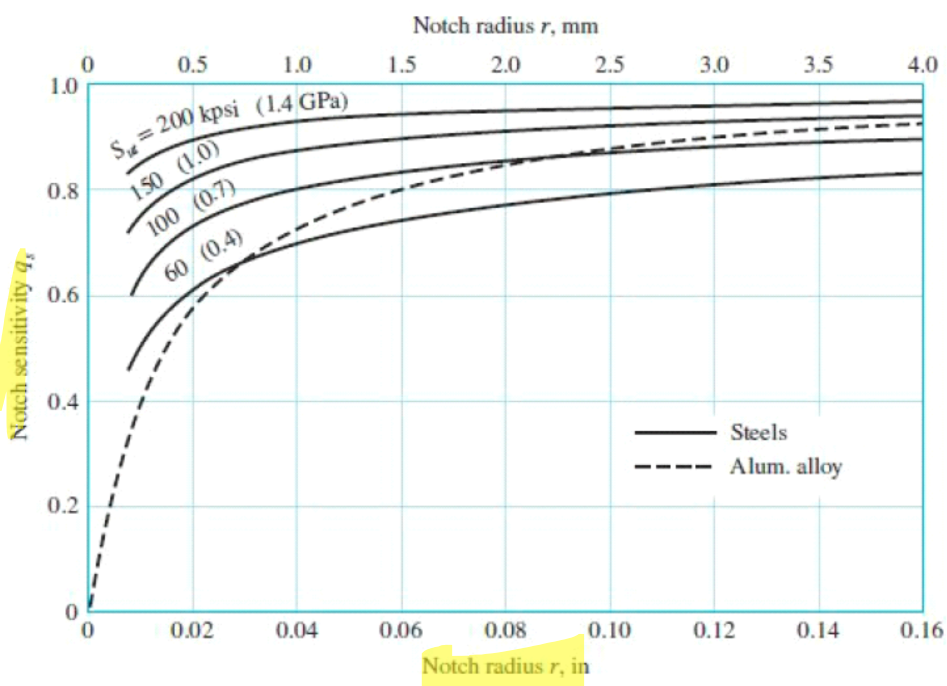
690 MPa

$= 690\ 10^6\ Pa$

$= 0.69\ 10^9\ Pa$

$\cong 0.7$ GPa

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the r = 0.16-in (4-mm) ordinate.



Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_s corresponding to r = 0.16 in (4 mm).

The above figure can be modeled as

$$\boxed{q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}} \quad \text{— (II)}$$

r = notch radius ✓

$\sqrt{a}$ = Neuber constant ✓

(I) and (II) may be combined to give

✓ $$\boxed{K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a}/\sqrt{r}}} \quad \text{— (III)}$$

Neuber constant is experimentally obtained

For steel

(i) Bending and axial loading:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad \text{(IVa)}$$

$$50 \leq S_{ut} \leq 250 \text{ kpsi}$$

$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.6(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3$$

$$340 \leq S_{ut} \leq 1700 \text{ MPa}$$

ips: S_{ut} is in kpsi & $\sqrt{a}$ is in $\text{in}^{1/2}$

SI: S_{ut} is in MPa & $\sqrt{a}$ is in $\text{mm}^{1/2}$

(ii) Torsion:

$$\left.\begin{aligned}
\sqrt{a} &= 0.19 - 2.51(10^{-3})\,S_{ut} + 1.35(10^{-5})\,S_{ut}^{2} - 2.67(10^{-8})\,S_{ut}^{3} \\
&\qquad 50 \le S_{ut} \le 220\ \text{kpsi} \\[1ex]
\sqrt{a} &= 0.958 - 1.83(10^{-3})\,S_{ut} + 1.43(10^{-6})\,S_{ut}^{2} - 4.11(10^{-10})\,S_{ut}^{3} \\
&\qquad 340 \le S_{ut} \le 1500\ \text{MPa}
\end{aligned}\right\}\ \text{IV b}$$

ips: S_{ut} – kpsi ; $\sqrt{a}$ – $\text{in}^{1/2}$

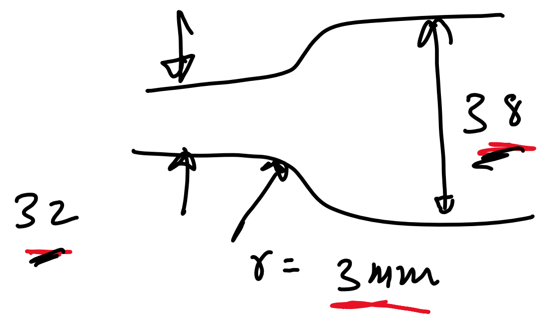
SI: S_{ut} – MPa ; $\sqrt{a}$ – $\text{mm}^{1/2}$

Q1

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure [redacted].
(*b*) Equations [redacted] and [redacted]. III, IV

q vs. r

$\sqrt{a}$

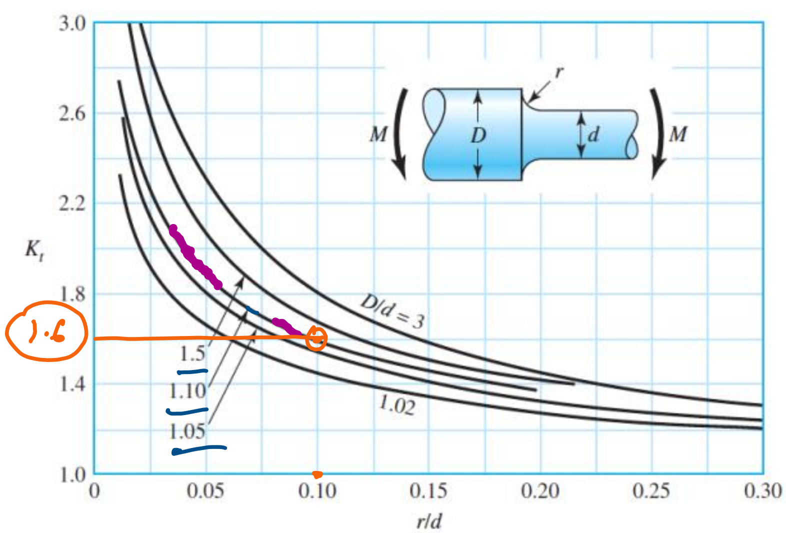


(a) K_t from plot q vs. r

(i) compute K_t (A-15)

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$r/d = 3/32 = 0.09375$
$D/d = 38/32 = 1.1875$ } $K_t = 1.6$

(ii) From figure $r = 3\,mm$

$S_{ut} = 0.7\ GPa$

$$q = \underline{0.85}$$

(iii) $k_f = 1 + q(k_t - 1)$

$= 1 + 0.85(1.6 - 1)$

$= 1.51$

$$\boxed{k_f = 1.51}$$ *

(b)

$$\boxed{\begin{gathered}\sqrt{a} = 1.24 - 2.25(10^{-3})\,S_{ut} + 1.6(10^{-6})\,S_{ut}^2 - 4.11(10^{-10})\,S_{ut}^3 \\ 340 \le S_{ut} \le 1700\ MPa\end{gathered}}$$

Formula IVa

$$\sqrt{a} = 1.24 - 2.25(10^{-3})(690) + (1.6)(10^{-6})(690)^2 - 4.11(10^{-10})\,690^3 \quad mm^{1/2}$$

$$\sqrt{a} = 0.314\ mm^{1/2}$$ ✓

$$\boxed{k_f = 1 + \frac{k_t - 1}{1 + \sqrt{a}/\sqrt{r}}}$$ — Formula III

$k_t = 1.6$ from A-15 (see part a)

$r = 3\,mm$

$\sqrt{a} = 0.314\ mm^{1/2}$

$$k_f = 1 + \frac{1.6 - 1}{1 + \left(0.314/\sqrt{3}\right)} = 1.507 \simeq 1.51$$

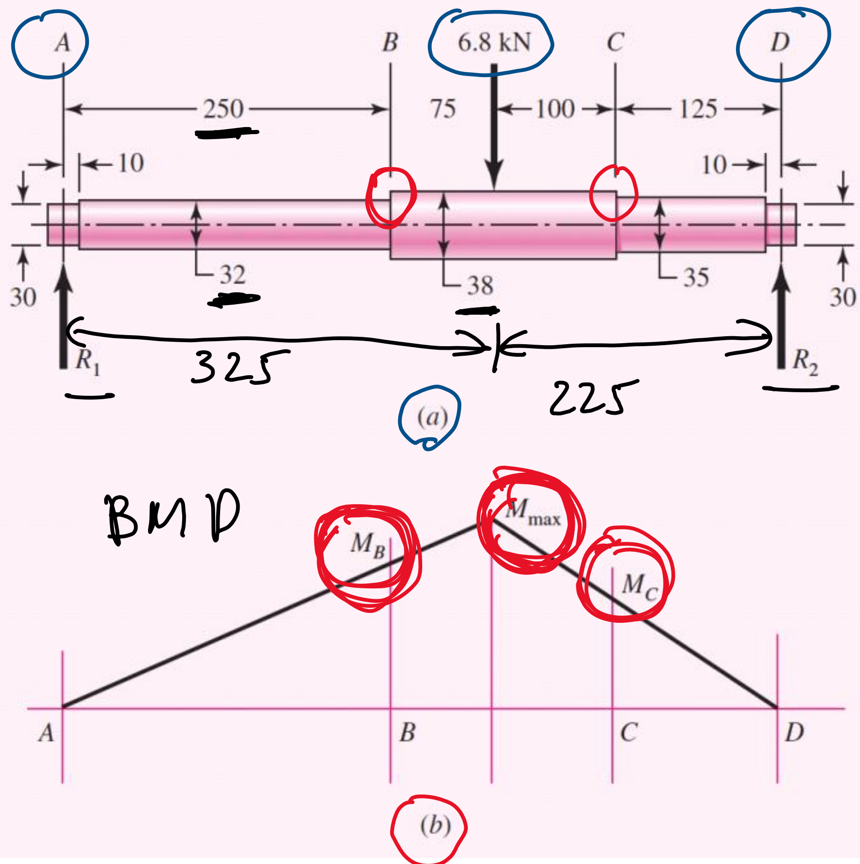
$$\boxed{k_f = 1.51}$$

Q2

Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part. assuming critical point is B.

Figure 6–22

(*a*) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel. (*b*) Bending-moment diagram.



S_{ut} = 690 MPa

$S_f = a N^b$

$a = \frac{(f S_{ut})^2}{S_e}$

$b = \frac{1}{3} \log_{10} \left(S_e / f S_{ut} \right)$

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1. Compute f, Se → a, b
2. S_f from the BMD — critical pt. is B
3. Use $S_f = a N^b$

① Compute S_e

$$\underline{S_e'} = \begin{cases} 0.5\, S_{ut} \checkmark & S_{ut} \le 1400\,\text{MPa} \\ 700\ \text{MPa} & S_{ut} > \underline{1400}\,\text{MPa} \end{cases} \quad \| \ SI$$

Since $S_{ut} = 690$ MPa

$$S_e' = 0.5\, S_{ut} = 0.5(690)$$
$$= \underline{345}\ \text{MPa}$$

$$S_e = \underline{k_a}\, k_b\, k_c\, k_d\, k_e\, S_e'$$

Cold drawn : $a = 3.4$; $b = -0.317$

$$k_a = a\, S_{ut}^{\,b}$$
$$= 3.4\,(690)^{-0.317}$$
$$= 0.74$$

$$k_b = \begin{cases} 1.24\, d^{-0.107} & 2.79 < \underline{d} \le 51\ \text{mm} \\ 1.51\, d^{-0.157} & 51 < d \le 254\ \text{mm} \end{cases} \quad \| \ SI$$

Rotating / Bending

$$k_b = 1.24\,(32)^{-0.107}$$
$$= 0.86$$

$k_c = 1$ (Bending)

$$S_e = k_a k_b k_c S_e'$$

$$S_e = (0.74)(0.86)(1)(345)$$

$$S_e = 220 \text{ MPa}$$

$$\boxed{f = 1.06 - 4.1(10^{-6}) S_{ut} + 1.5(10^{-7}) S_{ut} \qquad 500 \text{ MPa} \le S_{ut} \le 1400 \text{ MPa} \quad \text{SI}}$$

$$f = 1.06 - 4.1(10^{-6})(690) + (1.5)(10^{-7})(690)$$

$$f \simeq 0.85$$

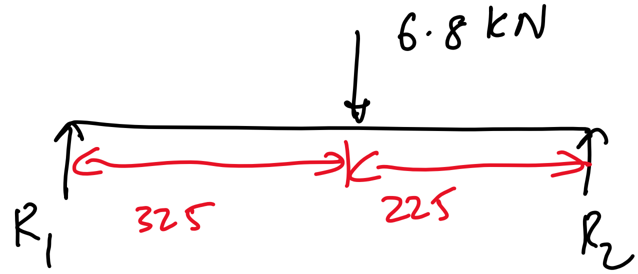
$$a = \frac{(f S_{ut})^2}{S_e} \qquad b = \frac{1}{3}\log_{10}\frac{S_e}{f S_{ut}}$$

$$a = \frac{[(0.85)(690)]^2}{220} \qquad b = \frac{1}{3}\log_{10}\left[\frac{220}{(0.85)(690)}\right]$$

$$a = 1564 \text{ MPa} \qquad b = -0.1419$$

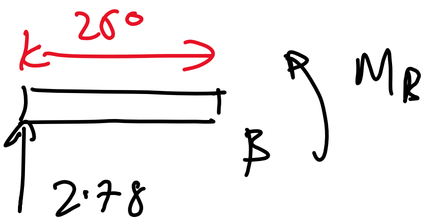
② $S_f = \sigma_B = \left(\dfrac{M_B\, y}{I}\right) K_f$

Compute M_b



Moment at R_2 : $-R_1\,(0.325 + 0.225) + 6.8\,(0.225) = 0$

$\Rightarrow \; R_1 = 2.78 \text{ kN}$



$- 2.78 \times 10^3 (0.25) + M_B = 0 \quad \Rightarrow \quad M_B = 695 \text{ Nm}$

$K_f = 1.51$ (see Q1, same geometry)

$S_f = \sigma_b = \left(\dfrac{M_B\,(d/2)}{\pi d^4/64}\right) K_f = \dfrac{32\, K_f\, M_B}{\pi d^3}$

$S_f = \sigma_B = \dfrac{(32)(1.51)(695)}{\pi\,(0.32)^3} = 326.16$

$\boxed{S_f = 326.16}$

(3) $S_f = a N^b$

$$326.16 = (1564)(N)^{-0.1419}$$

$$N = \left(\frac{326.16}{1564}\right)^{-\frac{1}{0.1419}} = 62775$$

$$\boxed{N \simeq 63\,(10)^3 \text{ cycles}}$$