Ka

6.09 Endurance Limit Modify Factors

$S_e \neq S_e' \Rightarrow S_e \leq S_e'$

S_e - endurance of material in machines

S_e' - lab tested endurance

$$\boxed{S_e = k_a k_b k_c k_d k_e S_e'}$$

k_a - surface factor
k_b - size factor
k_c - load factor
k_d - temperature factor
k_e - reliability factor

a) Surface factor

Specimen is highly polished, but machine element is not.

$$k_a = a\, S_{ut}^{\,b}$$

S_{ut} is the tensile strength,
a, b constants from the table below

Surface Finish	Factor a: S_{ut}, kpsi	Factor a: S_{ut}, MPa	Exponent b
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

Kb

b) Size factor, k_b

Rotating round specimen

(i) Bending and torsion

$$k_b = \begin{cases} 0.879\, d^{-0.107} & 0.11 < d \leq 2\,\text{in} \\ 0.91\, d^{-0.157} & 2 < d \leq 10\,\text{in} \end{cases} \quad \text{ips}$$

$$k_b = \begin{cases} 1.24\, d^{-0.107} & 2.79 < d \leq 51\,\text{mm} \\ 1.51\, d^{-0.157} & 51 < d \leq 254\,\text{mm} \end{cases} \quad \text{SI}$$

(*)

(ii) Axial

$$k_b = 1$$

$\boxed{\text{Non-rotating}}$ round / non-round specimen.

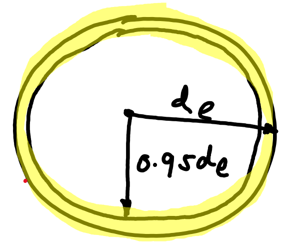
– Critical volume theory. Volume of material experiencing 95% of maximum stress is critical. The goal here is to use critical diameter de obtained by equating:

volume of 95% stress of non-rotating specimen
= volume of 95% stress of rotating round specimen

When volumes are equated lengths drop out and we are left equating areas.

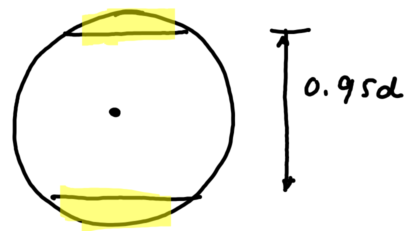
Kb (contd)

For rotating round specimen, the 95% area is



$$A_{0.95\sigma} = \frac{\pi}{4}\left(d_e^2 - 0.95 d_e^2\right) = 0.0766\, d_e^2 \qquad -(1)$$

For non-rotating round specimen

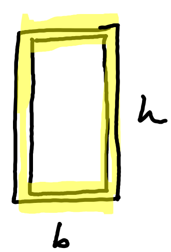


$$A_{0.95\sigma} = 0.1046\, d^2 \qquad -(2)$$

Equating (1) to (2)

$$\boxed{d_e = 0.370\, d}$$

For rectangular section



$$A_{0.95} = 0.05\, bh \qquad -(3)$$

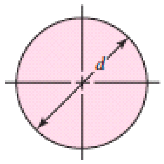
Equating (1) to (3)

$$\boxed{d_e = 0.808\sqrt{hb}}$$

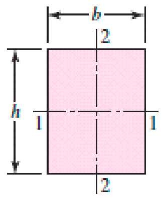
This d_e is used in place of d in equations (*)

Kc

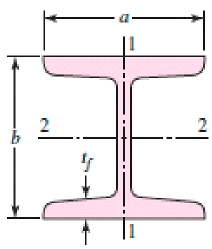
k_b



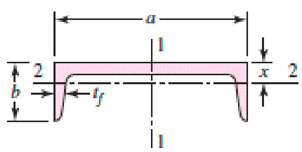
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

(c) Loading factor (k_c)

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

Kd

(d) Temperature factor (k_d)

$$\frac{S_T}{S_{RT}} = 0.98 + 3.5(10^{-4})\,T_F - 6.3(10^{-7})\,T_F^2$$

$$\frac{S_T}{S_{RT}} = 0.99 + 5.9(10^{-4})\,T_C - 2.1(10^{-6})\,T_C^2$$

(~)

T_F - Fahrenheit;

T_C - Celsius

S_T - ultimate strength at operating temperature

S_{RT} - ultimate strength at room temperature

field — lab expt.

(i) S_e' is unknown

(a) Estimate S_T/S_{RT} from (~)

(b) Estimate $(S_{ut})_T$ using $(S_{ut})_T = \left(\frac{S_T}{S_{RT}}\right)(S_{ut})_{RT}$

Room temp

(c) Estime S_e using

$$S_e' = \begin{cases} 0.5\,(S_{ut})_T & (S_{ut})_T \le 200\text{ kpsi} \\ 100\text{ kpsi} & (S_{ut})_T > 200\text{ kpsi} \end{cases} \quad \text{ips}$$

$$S_e' = \begin{cases} 0.5\,(S_{ut})_T & (S_{ut})_T \le 1400\text{ MPa} \\ 700\text{ MPa} & (S_{ut})_T > 1400\text{ MPa} \end{cases} \quad \text{SI}$$

(d) and $k_d = 1$ $\quad S_e = S_e'$

Ke

(ii) S_e' is known

(a) Estimate S_T/S_{RT} from ~

(b) Estimate $k_d = S_T/S_{RT}$

(c) Estimate $S_e = k_d S_e'$

(e) Reliability factor (k_e)

$$k_e = 1 - 0.08\, z_a$$

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

$$S_e = k_e S_e'$$

Q1

A steel specimen has a minimum $\overbrace{\text{ultimate strength}}^{S_{ut}}$ of 520 MPa has a machine surface. Estimate the factor Ka

From the table, Row 2

$a = 3.04$

$b = -0.217$

$S_{ut} = 520$ MPa

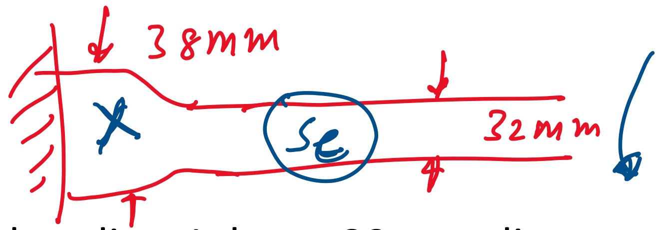
$K_a = a\, S_{ut}^{\,b}$

$K_a = (3.04)(520)^{-0.217}$

$\boxed{K_a = 0.78}$

$S_e = K_a\, S_e'$

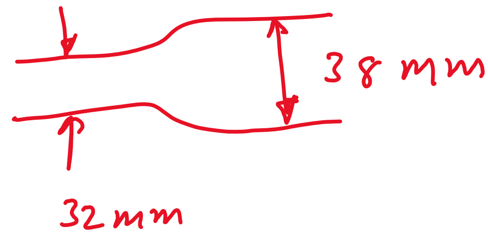
Q2



A steel shaft is loaded in bending. It has a 32 mm diameter and a filleted shoulder of 38 mm diameter. Estimate the size factor kb for

(a) rotating mode

(b) nonrotating mode



(a) rotating mode

$$\boxed{k_b = \begin{cases} 1.24\, d^{-0.107} & 2.79 < d \le 51 \text{ mm} \\ 1.51\, d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \quad \text{SI}}$$

$d = 32\,\text{mm}$

$k_b = (1.24)(32)^{-0.107}$

$\boxed{k_b = 0.855}$

$S_e = 0.855$

~~$d = 38\,\text{mm}$~~

~~$k_b = (1.24)(38)^{-0.107}$~~

~~$\boxed{k_b = 0.84}$~~

$S_e = 0.84$

$S_e = k_b S_e'$

0.84

NOTES

(b) non-rotating mode

$d_e = 0.37d = 0.37(32) \Rightarrow d_e = 11.84\,\text{mm}$

$k_b = (1.24)\, d_e^{-0.107} = (1.24)(11.84)^{-0.107}$

$K_b = (1.24)\,\theta_e = (1.24)(11.84)$

$$\boxed{K_b = 0.95}$$

Q3

A 1035 steel has a tensile strength of 80 kpsi and is to be used for a part that operates at a temperature of 750 F. Estimate the endurance limit at the operating temperature if

$S_e = ?$

(a) only the tensile strength at room temperature is known

(b) at the room-temperature of 70F, the endurance limit for the material is found by test to be Se' = 39 kpsi

(a) $(S_{ut})_{RT} = (S_{ut})_{70F} = 80$ kpsi RT = room temp

$(S_e)_T = (S_e)_{750F} = ?$ T = operating temp

S_e' is unknown

(a)
$$\boxed{\frac{S_T}{S_{RT}} = 0.98 + 3.5(10^{-4})\,T_F - 6.3\,(10^{-7})\,T_F^2}$$

From kd data

$$\frac{S_T}{S_{RT}} = 0.98 + 3.5\,(10^{-4})(750) - 6.3\,(10^{-7})\,(750)^2$$

$$\frac{S_T}{S_{RT}} = \frac{S_{750}}{S_{70}} = 0.89$$

(b) $(S_{ut})_T = \left(\frac{S_T}{S_{RT}}\right)(S_{ut})_{RT}$

$(S_{ut})_{750} = (0.89)(80) \Rightarrow (S_{ut})_{750} = 71.2$ kpsi

(c) $$\boxed{S_e' = \begin{cases} 0.5\,(S_{ut})_T & (S_{ut})_T \le 200\,\text{kpsi} \\ 100\,\text{kpsi} & (S_{ut})_T > 200\,\text{kpsi} \end{cases}}$$

$$(S_{ut})_{750} = 71.2\ \text{kpsi} \le 200\,\text{kpsi}$$

$$(S_e')_T = 0.5\,(S_{ut})_T = 0.5\,(S_{ut})_{750} = 0.5\,(71.2)$$

$$(S_e')_{750} = 35.6\ \text{kpsi}$$

(d) $k_d = 1$ $$\boxed{S_e = S_e' = 35.6\,\text{kpsi}}$$

(ii) $(S_e')_{RT} = (S_e')_{70} = \underline{39}$ kpsi

$\underbrace{S_e' \text{ is known}}$

(a) $S_T/S_{RT} = S_{750}/S_{70} = 0.89$ [see part (i)]

(b) $k_d = S_T/S_{RT} = 0.89$

(c) $(S_e)_{750} = k_d\,(S_e')_{70} = 0.89\,(39) \doteq 35$ kpsi

$$\boxed{(S_e)_{750} = 35\ \text{kpsi}}$$

Q4

A 1080 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70,000 cycles to failure in an operating environment of 650 F. Estimate the endurance limit and fatigue strength at 70,000 cycles. Assume reliability for the endurance limit estimate to be 99% and ultimate tensile strength (S_ut) at 70 F to be 112 kpsi.

$d = 1\,in$; $N = 70{,}000$; $RT = 70\,F$; $T = 650\,F$

$R_e = 99\%$. $(S_{ut})_{RT} = (S_{ut})_{70} = 112\ kpsi$;

(i) $S_e = ?$ (ii) $S_f = a N^b$

$S_e = k_a k_b k_c k_d k_e S_e'$ || $a = \frac{(f S_{ut})^2}{S_e}$; $b = \frac{1}{3} \log\left(\frac{S_e}{f S_{ut}}\right)$

(i) $k_a = a (S_{ut})^b = 11\,(?)^b$

$(S_{ut})_T = (S_{ut})_{650}$

(i) $\boxed{\dfrac{S_T}{S_{RT}} = 0.98 + 3.5(10^{-4})\,T_F - 6.3(10^{-7})\,T_F^{\,2}}$ from k_d

(a) $$\frac{S_T}{S_{RT}} = 0.98 + 3.5(10^{-4})(650) - 6.3(10^{-7})(650)^2$$

$$\frac{S_{650}}{S_{70}} = 0.94$$

(b) $$(S_{ut})_T = \left(\frac{S_T}{S_{RT}}\right)(S_{ut})_{RT}$$

$$(S_{ut})_{650} = (0.94)(112) = 105 \text{ kpsi} \quad \text{I}$$

(c) $$\boxed{S_e' = \begin{cases} 0.5\,(S_{ut})_T & (S_{ut})_T \le 200 \text{ kpsi} \\ 100 \text{ kpsi} & (S_{ut})_T > 200 \text{ kpsi} \end{cases}}$$ || ips

Copied from k_a

$$S_e' = (0.5)(S_{ut})_{650} = (0.5)(105)$$

$$S_e' = 52.5 \text{ kpsi}$$

(d) $k_d = 1$ $\qquad$ $S_e = S_e' = 52.5$ kpsi

(i) $k_a = a(\underline{S_{ut}})^b = 11(?)^b$

$(S_{ut})_T = (S_{ut})_{650}$

$= 11(105)^{-0.65}$

$k_a = 0.534$

$k_b = 1$ (axial)

$k_c = 0.85$ (axial)

$k_d = 1$

$k_e = 0.814$ (99% reliability)

$S_e = (k_a)(k_b)(k_c)(k_d)(k_e)\, S_e'$

$= (0.534)(1)(0.85)(1)(0.814)(52.5)$

at $T = 650F$

$$\boxed{S_e = 19.39}$$

(ii) $a = \frac{(f\, S_{ut})^2}{S_e}$ $\qquad b = \frac{1}{3} \log\left(\frac{S_e}{f\, S_{ut}}\right)$

$(S_{ut})_T = (S_{ut})_{650} = \underline{105}$ kpsi

At $S_{ut} = 105$ kpsi $\quad f = 0.84$ (see Lec 17)

$$a = \frac{[(0.84)(105)]^2}{19.39} = 401.2$$

$$b = \frac{1}{3} \log\left(\frac{19.39}{0.84(105)}\right) = -0.22$$

$$S_f = a N^b = (401.2)(70{,}000)^{-0.22}$$

$$\boxed{S_f = 34.47 \text{ kpsi}}$$