Introduction to Fatigue

- static loading (Failure theories)
  - slow speeds
  - light loads
  - large factor of safety

19th century

- Dynamic loading (Fatigue)
  - fast speeds
  - high loads
  - low factor of safety

industrial revolution

e.g. steam engine, water-wheel, turbines.

Fatigue
- repeated high/low stresses
- failure occurs at much lower strengths (lower than yield strength)
- Failure due to cyclic stresses is Fatigue Failure.

Fatigue-Life methods

The goal is to predict the number of cycles to failure for a specific level of loading

Methods

THEORY DRIVEN

1. Strain-Life method: Uses crack nucleation (growth) to predict failure cyles n

2. Linear elastic fracture mechanics
   Uses crack propagation to predict failure cycles

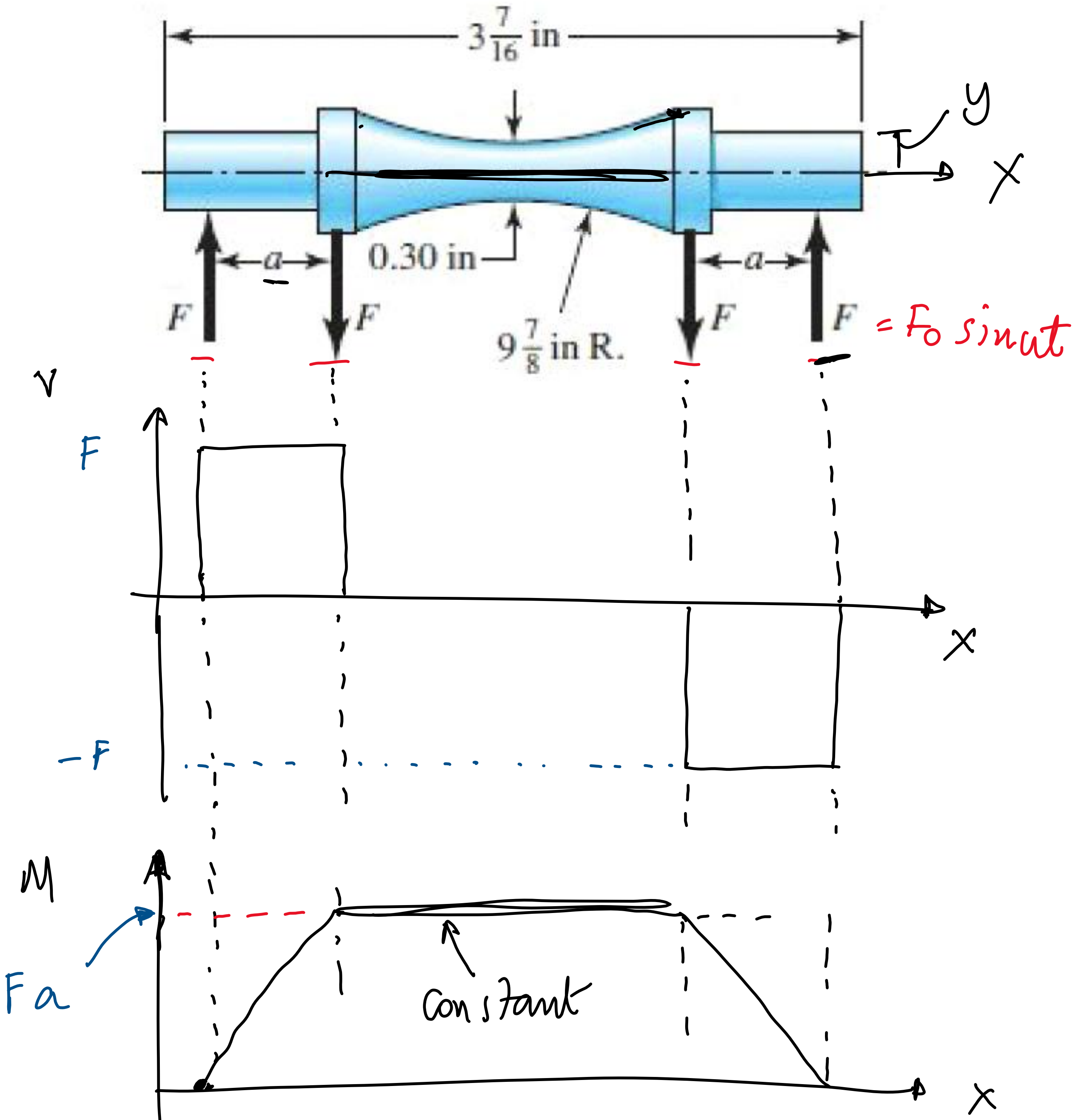
3. Stress-Life method: Uses empirical tests on test specimens to predict failure cycles [DATA DRIVEN)

①, ② — Advanced Solid Mechanics

③ — We will use this approach

Fatigue Test

Test specimen

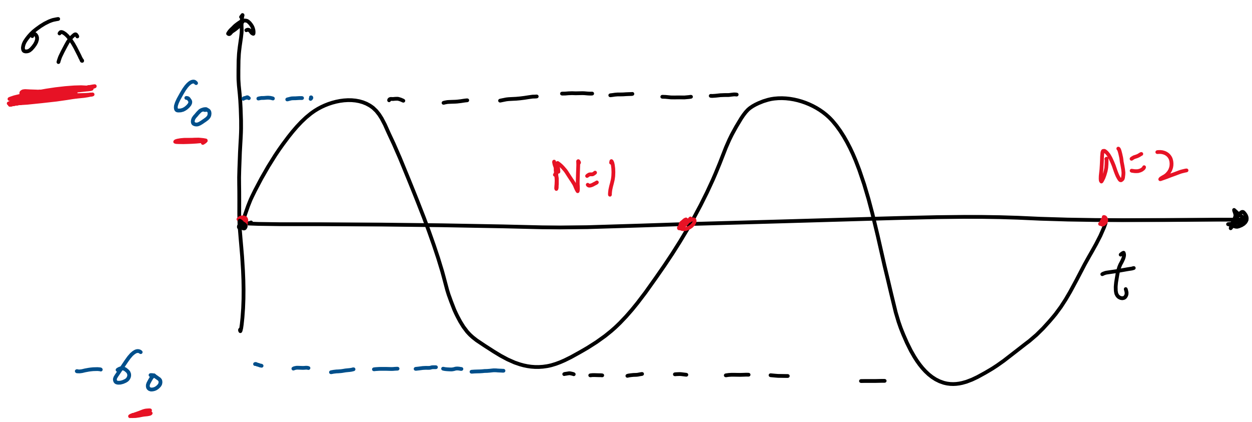


$$\sigma_x = \frac{My}{I} = \frac{Fay}{I}$$

$$\sigma_x = \frac{F a y}{I}$$

Let $F = F_0 \sin \omega t$

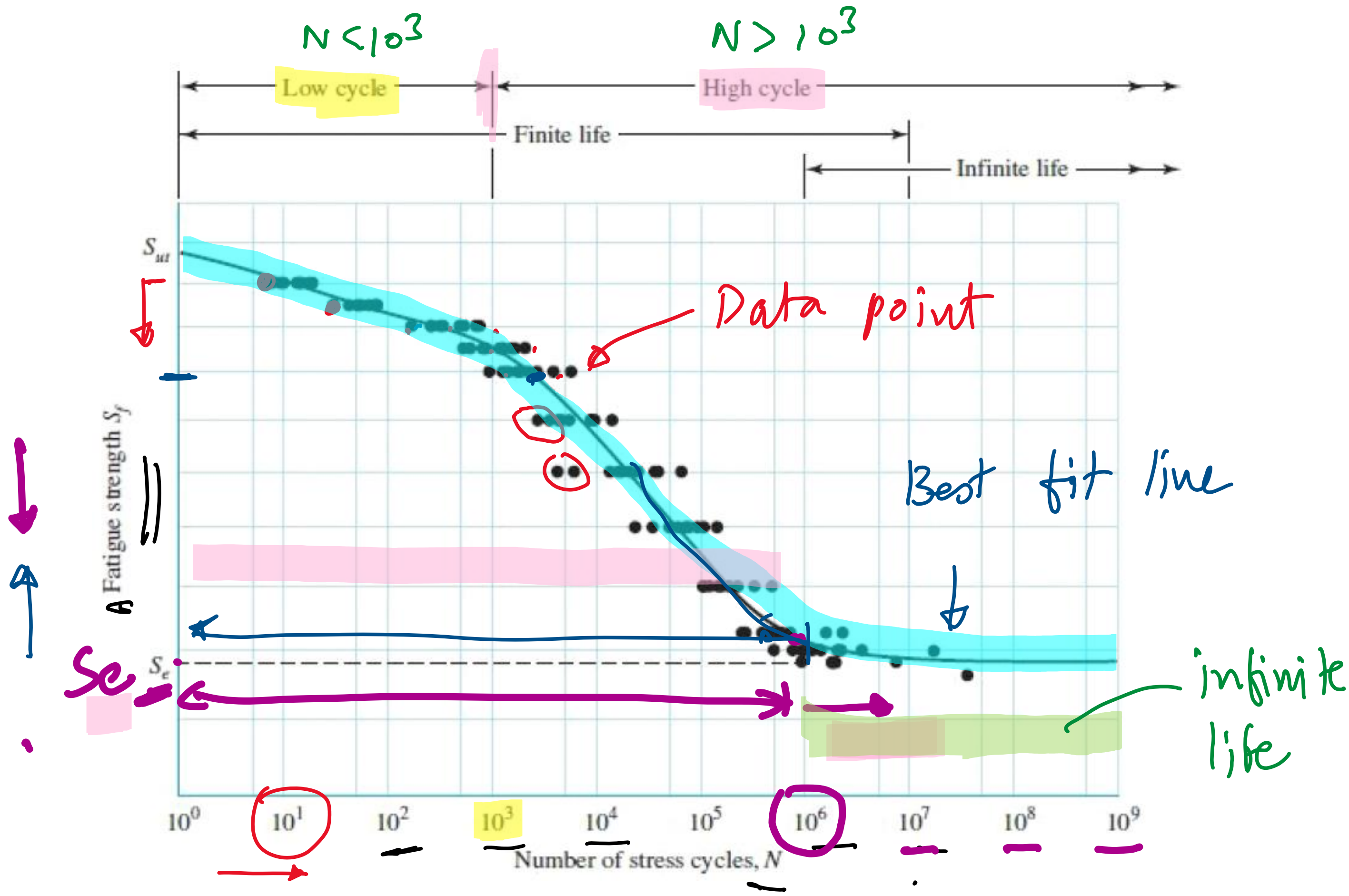
$$\sigma_x = \underbrace{\left(\frac{F_0 a y}{I}\right)}_{= \sigma_0} \sin \omega t = \sigma_0 \sin(\omega t) \; ; \; \sigma_0 = \frac{F_0 a y}{I}$$



SN experimental

① Choose σ_0

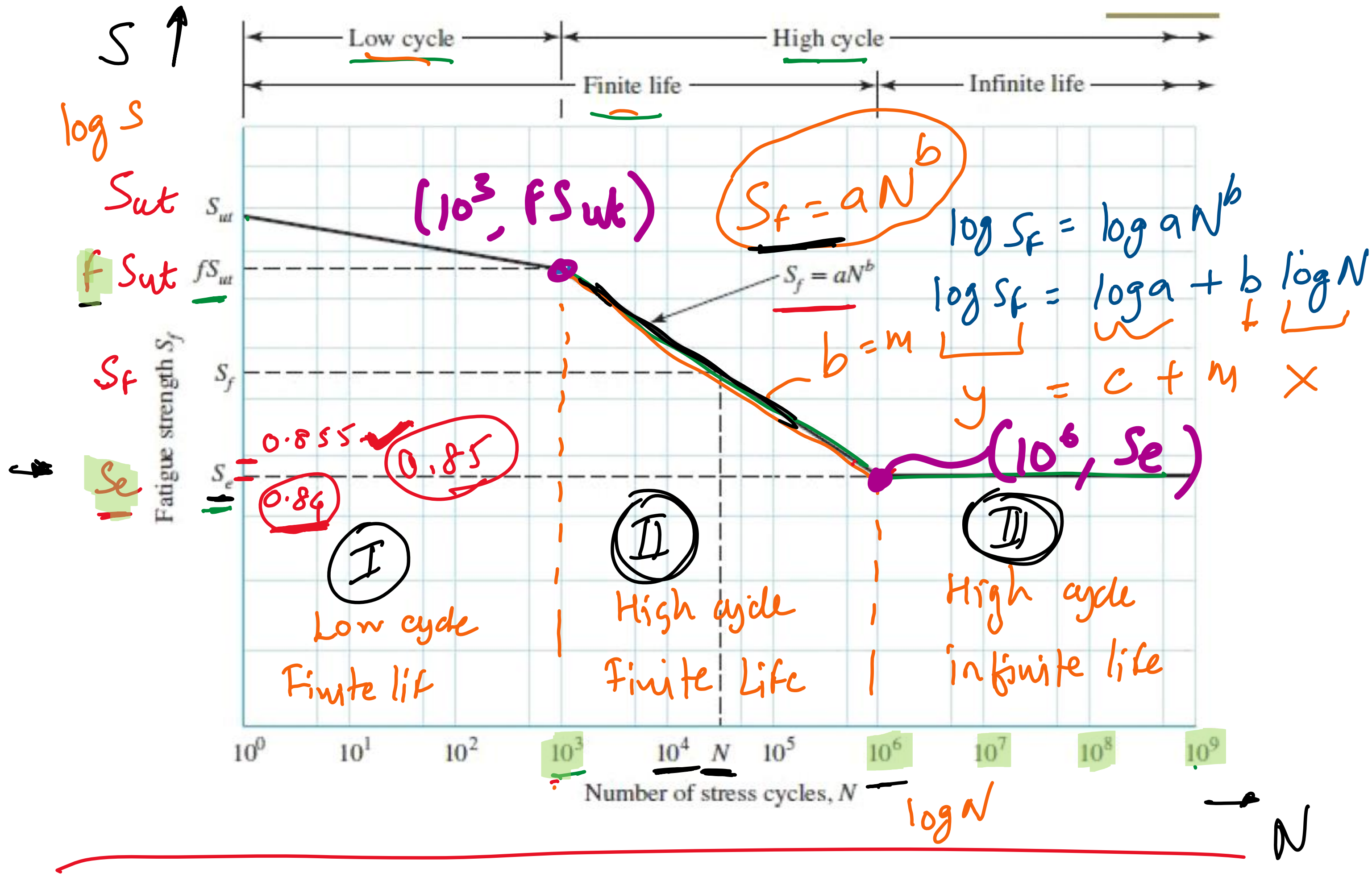
② Keep increasing N till specimen fails



③ Keep repeating ① and ② and plotting on strength vs. log N plot shown above.

S-N curve; Wöhler curve; stress-life curve

Idealized SN curve



I $\quad 0 \leq N \leq 10^3$

$S_{ut} \leq S \leq f S_{ut}$

$f = 0.8 - 0.9$
(emperical)

II $\quad 10^3 \leq N \leq 10^6$

$f S_{ut} \leq S_{ut} \leq S_e$

S_e – endurance limit

III $\quad N \geq 10^6$

$S \leq S_e$

① S_e is found empirically

For steel

$$S_e' = \begin{cases} 0.5\, S_{ut} & S_{ut} \le 200\,\text{kpsi} \\ 100\,\text{kpsi} & S_{ut} > 200\,\text{kpsi} \end{cases}$$ ips

$$S_e' = \begin{cases} 0.5\, S_{ut} & S_{ut} \le 1400\,\text{MPa} \\ 700\,\text{MPa} & S_{ut} > 1400\,\text{MPa} \end{cases}$$ SI

S_e' – endurance strength of the lab specimen

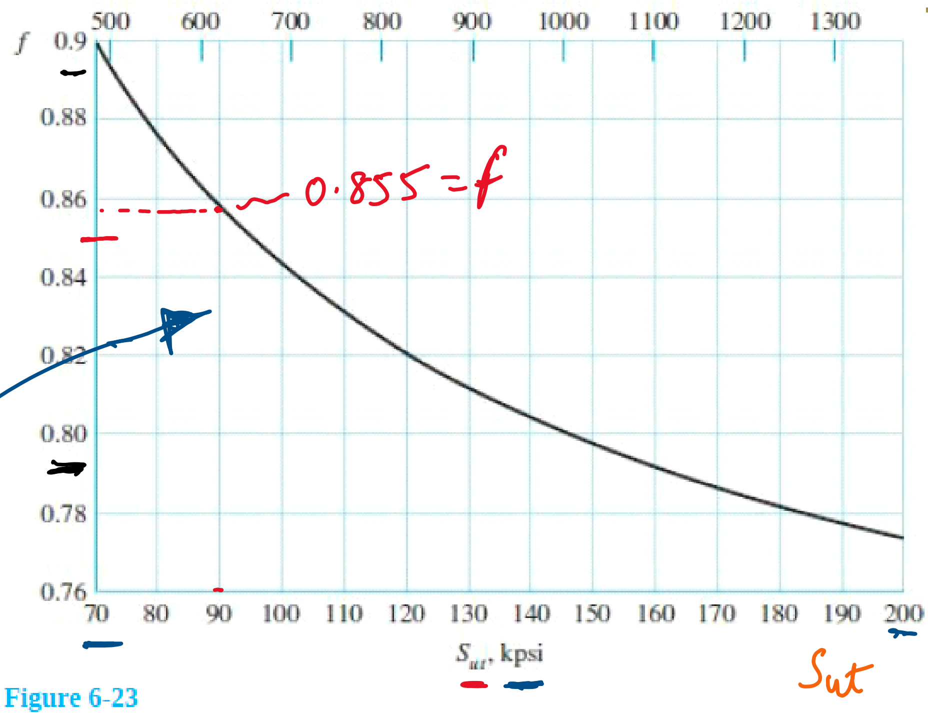
S_e – endurance strength of actual material in the field.

$S_e \neq S_e'$ due to imperfections in actual material in the field.

F vs Sut

f

F



(i)

Figure 6-23
Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for steels, with $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.

$$f = 1.06 - 2.8(10^{-3})\,S_{ut} + 6.9(10^{-6})\,S_{ut}^2$$

(ii)

$$70\text{ kpsi} \le S_{ut} \le 200\text{ kpsi}$$

ips

$$f = 1.06 - 4.1(10^{-6})\,S_{ut} + 1.5(10^{-7})\,S_{ut}^2$$

$$500\text{ MPa} \le S_{ut} \le 1400\text{ MPa}$$

SI

$$f = 0.8 \text{ or } 0.9$$

(i) graph (ii) formula (iii) $f = 0.8$ or 0.9

Compute the constant a, b in $S_f = a N^b$

We know that the curve passes through $(10^3, f S_{ut})$ and $(10^6, S_e)$

$$\rightarrow \quad (f S_{ut}) = a (10^3)^b \quad \text{— I}$$

$$\rightarrow \quad (S_e) = a (10^6)^b \quad \text{— II}$$

Solving for a, b using I and II

$$\frac{\text{I}}{\text{II}} \qquad \frac{f S_{ut}}{S_e} = \frac{10^{3b}}{10^{6b}} = 10^{-3b}$$

$$\frac{S_e}{f S_{ut}} = 10^{3b}$$

Taking $\log_{10}$

$$\log_{10}\left(\frac{S_e}{f S_{ut}}\right) = \log_{10}\left(10^{3b}\right) = 3b$$

$$\boxed{b = \frac{1}{3} \log_{10}\left(\frac{S_e}{f S_{ut}}\right)}$$

$$S_e = a\,(10)^{6b} \qquad \underline{b} = \frac{1}{3}\log_{10}\frac{S_e}{f S_{ut}}$$

$$\log_{10} S_e = \log_{10} a + 6b\,\underbrace{\log_{10} 10}_{1}$$

$$\log_{10} S_e = \log_{10} a + \underline{2}\,\log_{10}\frac{S_e}{f S_{ut}}$$

$$\log_{10} a = \log_{10} S_e - \log_{10}\left(\frac{S_e}{f S_{ut}}\right)^2$$

$$\log_{10} a = \log_{10}\left[\frac{S_e}{\left(\frac{S_e}{f S_{ut}}\right)^2}\right] = \log_{10}\left(\frac{f^2 S_{ut}^2}{S_e}\right)$$

$$\boxed{a = \frac{f^2 S_{ut}^2}{S_e}}$$

$$\boxed{S_f = a N^b \;;\; a = \frac{f^2 S_{ut}^2}{S_e} \;;\; b = \frac{1}{3}\log_{10}\left(\frac{S_e}{f S_{ut}}\right)}$$

In region II of the SN curve (idealized) given $S_f = \sigma$, then we can compute the cycles to failure (N) as follows

$$S_f = a N^b$$

$$\sigma = a N^b$$

$$\Rightarrow \quad \boxed{N = \left(\frac{\sigma}{a}\right)^{1/b}}$$

a, b are computed using formulae given earlier.

Q1

$S_e = ?$

Given a 1050 HR steel, *estimate* $= N$ ✓

III (*a*) the rotating-beam endurance limit at 10^6 cycles.

II (*b*) the endurance strength of a polished rotating-beam specimen corresponding to 10^4 cycles to failure. $= N$

S_f

I (*c*) the expected life of a polished rotating-beam specimen under a completely reversed stress of 55 kpsi. $\sigma = 55$ kpsi ; $N = ?$

II $S_f = aN^b$

1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi) S_{ut}	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 $\boxed{(90)}$	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

(a) $S_{ut} = 90$ kpsi for 1050 HR steel.

Since $S_{ut} \leq 200$ kpsi

$$S_e = 0.5\, S_{ut}$$
$$S_e = 0.5\,(90)$$

$$\boxed{S_e = 45 \text{ kpsi}}$$

(b) $N = 10^4$; $S_f = ?$

$$S_f = a\, N^b$$

$f = 0.855$ from f vs. S_{ut} curve

$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$a = \frac{[(0.855)(90)]^2}{45} \Rightarrow a = 131.58$$

$$b = \frac{1}{3}\log_{10}\left(\frac{S_e}{f\,S_{ut}}\right)$$

$$b = \frac{1}{3}\log_{10}\left(\frac{45}{(0.855)(90)}\right) \Rightarrow b = -0.0776$$

$$S_f = a N^b$$

$$S_f = (131.58)(10^4)^{-0.0776}$$

$$\boxed{S_f = 64.39 \text{ kpsi}}$$

(c) $$S_f = \sigma = a N^b$$

$$55 = (131.58)\, N^{(-0.0776)}$$

$$\rightarrow \quad N = \left[\left(\frac{55}{131.58}\right)\right]^{\frac{1}{-0.0776}}$$

$$\rightarrow \quad N = 7.62\,(10^4)$$

$$\boxed{N \approx 76\,000 \text{ cycles}}$$