Maximum Normal Stress theory for Brittle materials

- This theory states that failure occurs when one of the principle stresses equals or exceeds the strength.

- Compute principle stresses: $\sigma_A, \sigma_B, \sigma_C$

- Re-arrange in descending order: $\sigma_1 \geq \sigma_2 \geq \sigma_3$

Strength: S_{ut} (tensile strength)
S_{uc} (compressive strength)

$\sigma_1 \geq 0$ Check if $\sigma_1 \geq S_{ut}$ (failure)

$\sigma_3 \leq 0$ check if $\sigma_3 \leq -S_{uc}$ (failure)

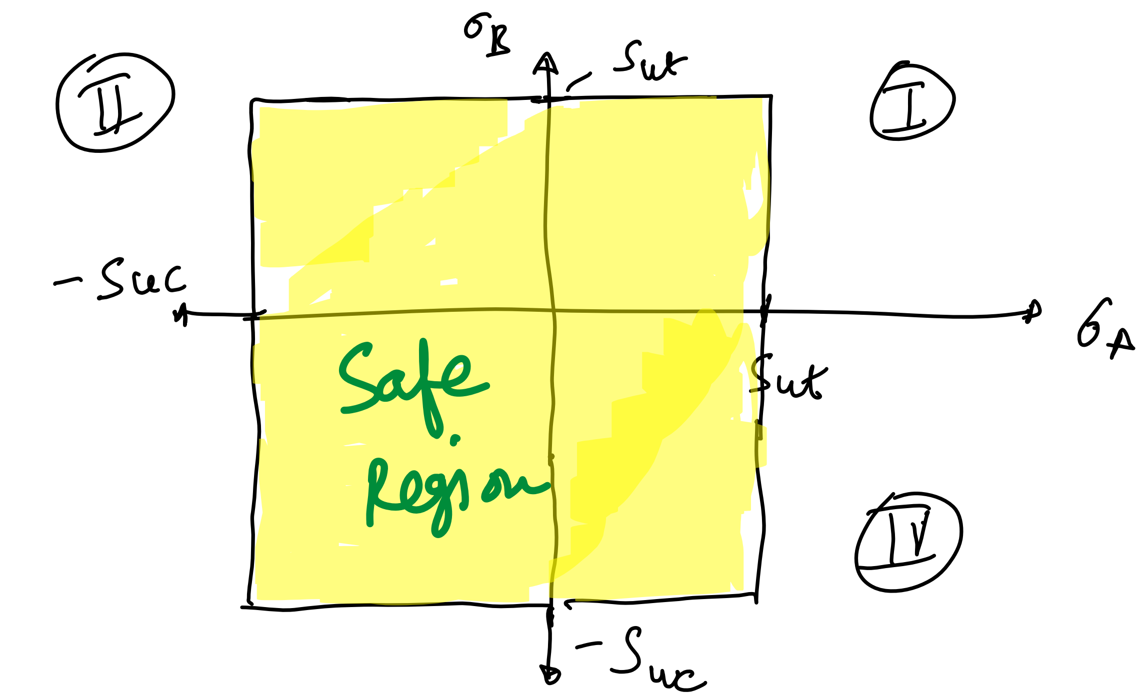
equals or exceeds

For plane stress, σ_A, σ_B, $\sigma_C = 0$

Arrange stresses in descending order

→ $\sigma_A > \sigma_B > 0$; $\sigma_B > \sigma_A > 0$ — I

→ $\sigma_A > 0 > \sigma_B$; $\sigma_B > 0 > \sigma_A$ — II, IV



$0 > \sigma_A > \sigma_B$ or $0 > \sigma_B > \sigma_A$ — III

Equations $\quad \sigma_1 > 0 > \sigma_3$

$$\boxed{\sigma_1 = S_{ut}} \; ; \; \boxed{\sigma_3 = -S_{uc}}$$

Design

$$\boxed{n = \frac{S_{ut}}{\sigma_1}} \; ; \; \boxed{n = -\frac{S_{uc}}{\sigma_3}}$$

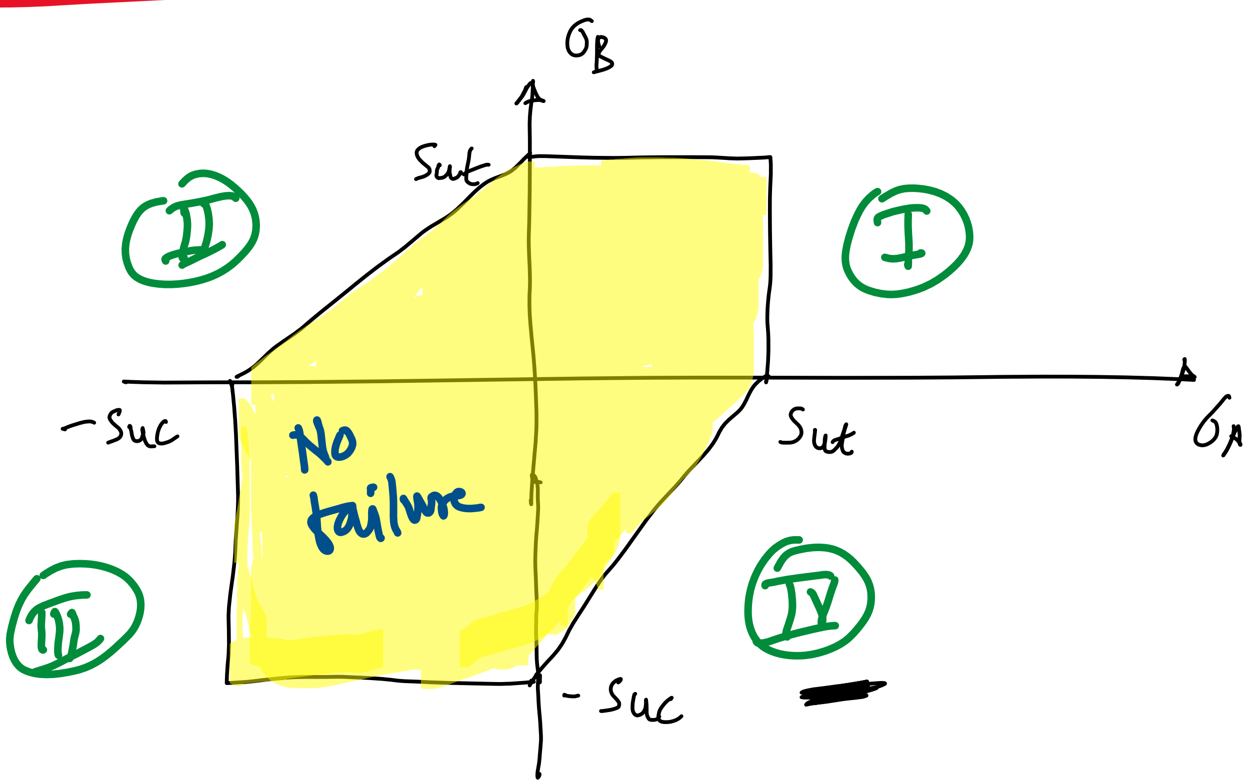
Brittle Coulomb-Mohr Theory

$\sigma_A = S_{ut}$ $\quad$ $\sigma_A \geq \sigma_B \geq 0$ — I

$\sigma_B = -S_{uc}$ $\quad$ $0 \geq \sigma_A \geq \sigma_B$ — III

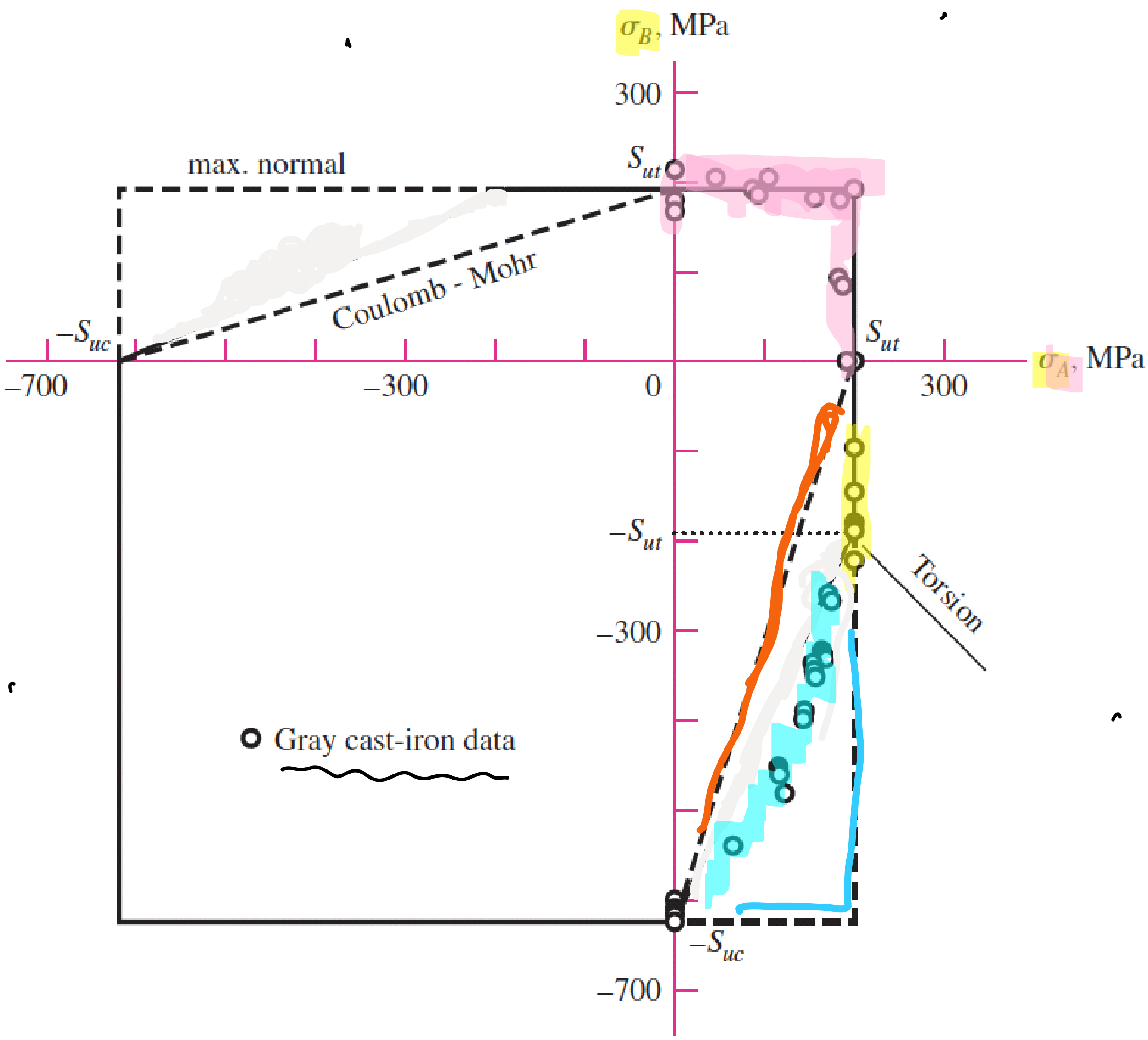
$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = 1 \qquad \sigma_A \geq 0 \geq \sigma_B \quad \text{IV}$$

$$\frac{\sigma_B}{S_{ut}} - \frac{\sigma_A}{S_{uc}} = 1 \qquad \sigma_B \geq 0 \geq \sigma_A \quad \text{— II}$$

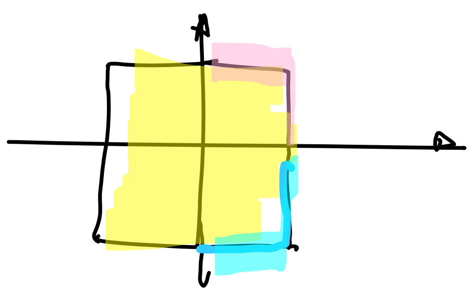


Brittle Materials

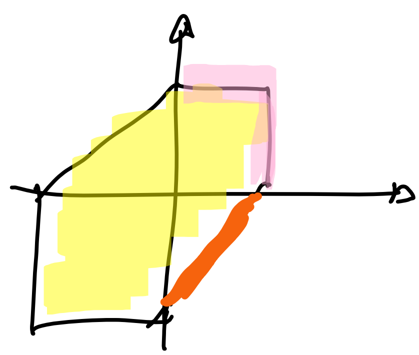
Brittle Coulomb Mohr



Max Normal Stress



Brittle Coulomb-Mohr



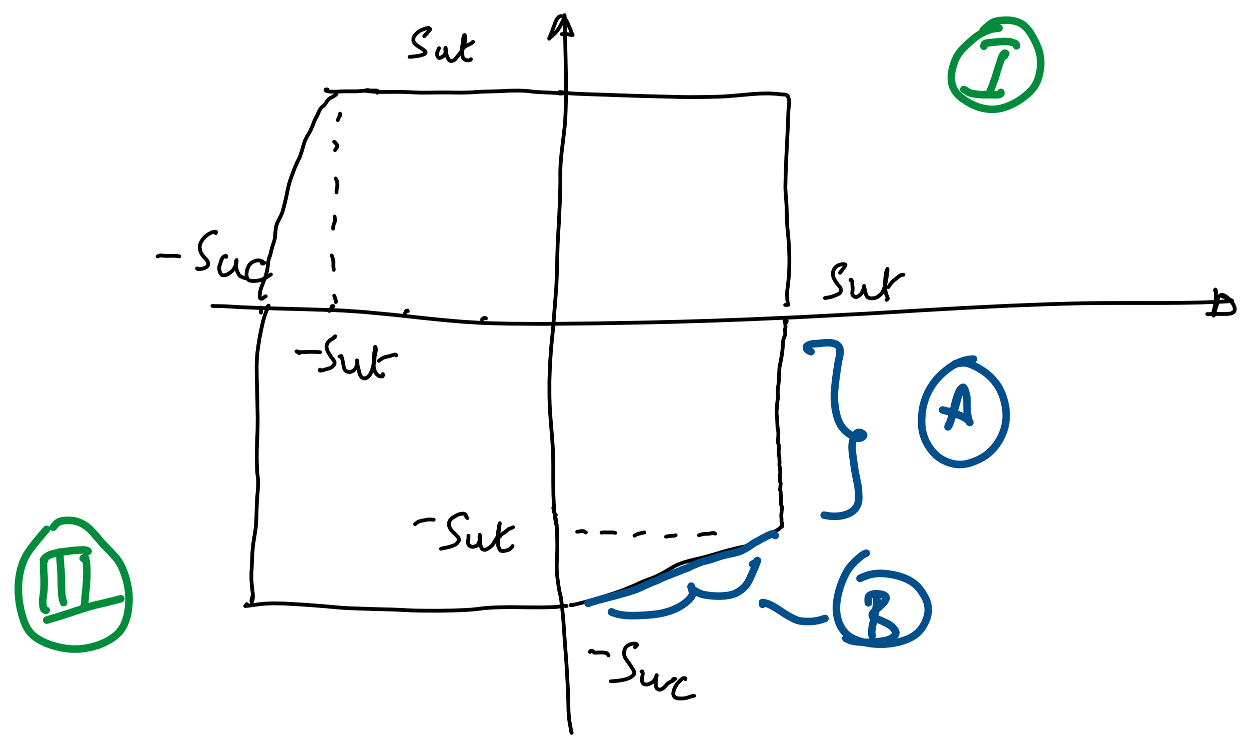
Modified-Mohr Theory

I $\quad \sigma_A = S_{ut} \qquad \qquad \sigma_A \geq \sigma_B \geq 0$

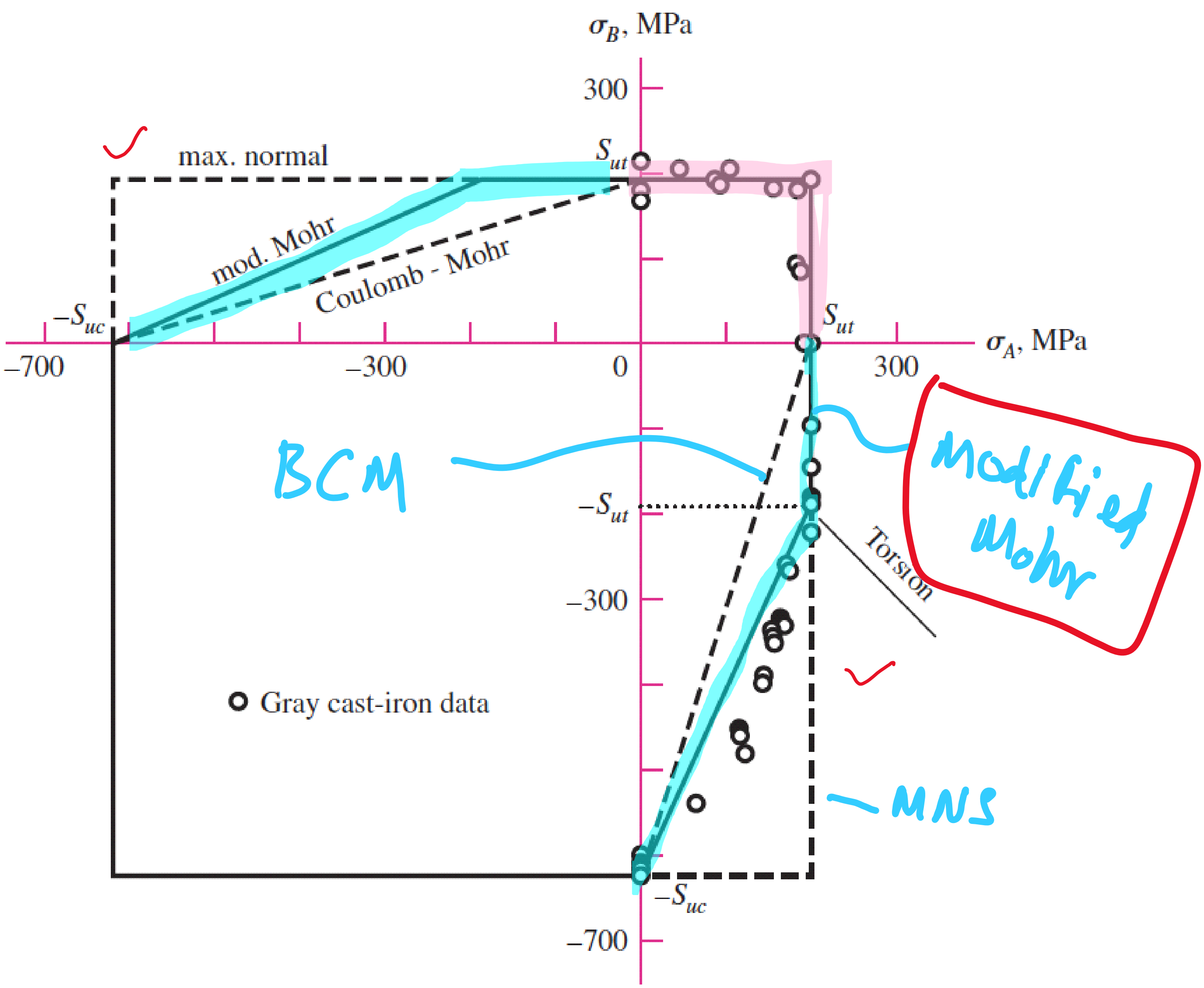
III $\quad \sigma_B = -S_{uc} \qquad \qquad 0 \geq \sigma_A \geq \sigma_B$

A $\quad \sigma_A = S_{ut} \qquad \qquad \sigma_A \geq 0 \geq \sigma_B \; ; \; \left|\dfrac{\sigma_B}{\sigma_A}\right| \leq 1$ ✓

B $\quad \left[\dfrac{S_{uc} - S_{ut}}{S_{uc} S_{ut}}\right]\sigma_A - \dfrac{\sigma_B}{S_{uc}} = 1 \qquad \qquad \sigma_A \geq 0 \geq \sigma_B \; ; \; \left|\dfrac{\sigma_B}{\sigma_A}\right| > 1$

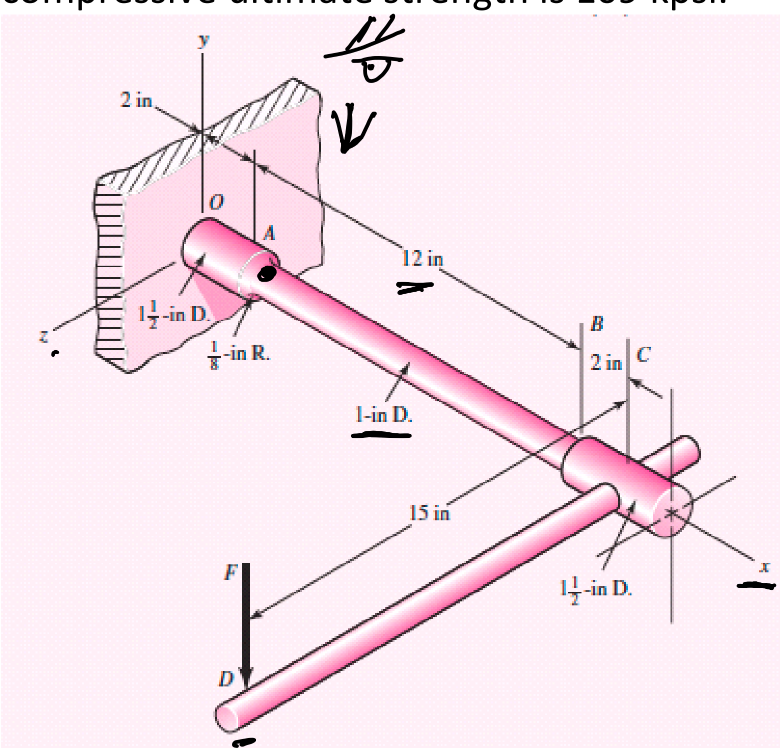


Brittle Materials



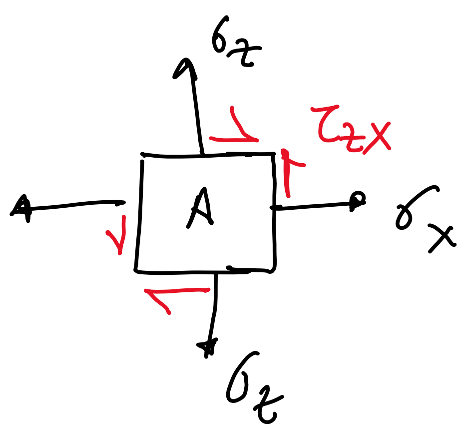
A force F is applied at D as shown. Assuming that the material is brittle and the critical point for failure is A. Compute the value of F assuming (a) Coulomb-Mohr Theory and (b) Modified Mohr Theory

Assume that the tensile ultimate strength is 31 kpsi and compressive ultimate strength is 109 kpsi.



$S_{ut} = 31$ kpsi

$S_{uc} = 109$ kpsi

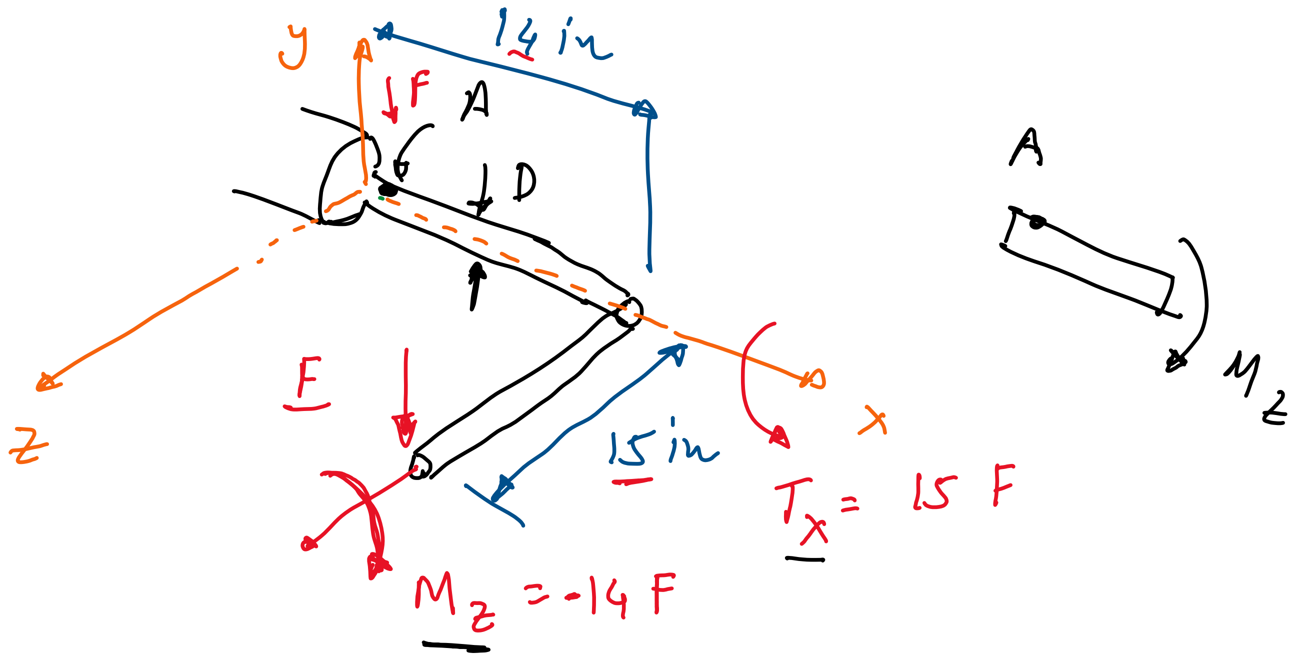


① Compute σ_z, σ_x, τ_{zx} in terms of F

② Compute the principle stress at A
σ_A, σ_B, $\sigma_C = 0$

③ Apply BCM and MM to compute F.

①



$$\tau_{zx} = \frac{T\rho}{J} = \frac{T_x\, D/2}{\pi D^4/32} = \frac{16}{\underbrace{\pi D^3}_{=1\,in}}(15F) = 76.4F$$

$$\sigma_x = +\frac{My}{I} = \frac{M_z\, D/2}{\pi D^4/64} = \frac{32}{\underbrace{\pi D^3}_{=in}}(14F) = 142.6F$$

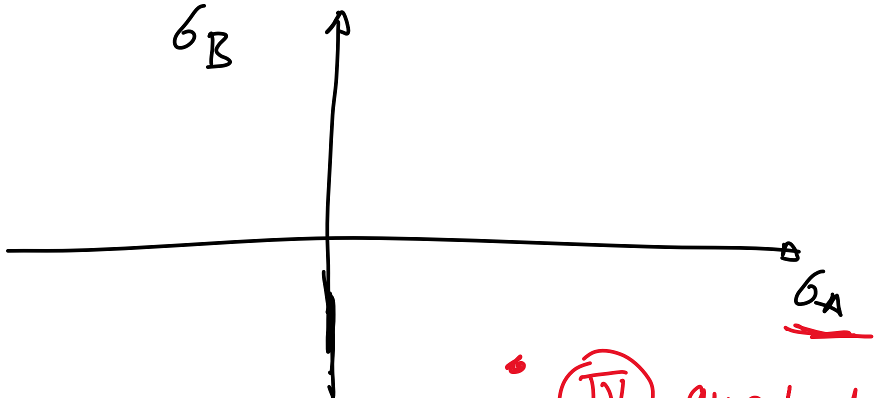
$$\sigma_z = 0$$

② $$\sigma_{A,B} = \frac{\overset{142.6F}{\sigma_x} + \overset{0}{\sigma_z}}{2} \pm \sqrt{\left(\frac{\overset{142.6F}{\sigma_x} - \overset{0}{\sigma_z}}{2}\right)^2 + \overset{76.4F}{\tau_{xz}^2}}$$

$\sigma_A = \underline{\underline{175.8F}}$; $\sigma_B = \underline{\underline{-33.2}}\,F$

$\sigma_C = 0$ {Third principle stress is 0 since this is a plane stress problem}

(3)



$\sigma_A = \underline{175.8}\ F$ $\qquad$ (IV) quadrant

$\sigma_B = \underline{-33.2}\ F$ $\qquad$ $\sigma_A > 0$

$\sigma_B < 0$

(i) BCM – Brittle Coulomb Mohr Theory

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = 1 \Rightarrow \underbrace{\frac{175.8F}{31(10^3)} - \frac{(-33.2F)}{109(10^3)} = 1}$$

Solve $\boxed{F_{BCM} = 167\ lbf}$

(ii) Modified-Mohr theory

$$\left|\frac{\sigma_B}{\sigma_A}\right| = \left|\frac{-33.2F}{175.8F}\right| < 1$$

$$\sigma_A = S_{ut} \Rightarrow \underbrace{175.8F = 31(10^3)}$$

Solve $\boxed{F_{MM} = 176\ lbf}$

$F_{MM} > F_{BCM}$ $\qquad$ BCM is more conservative