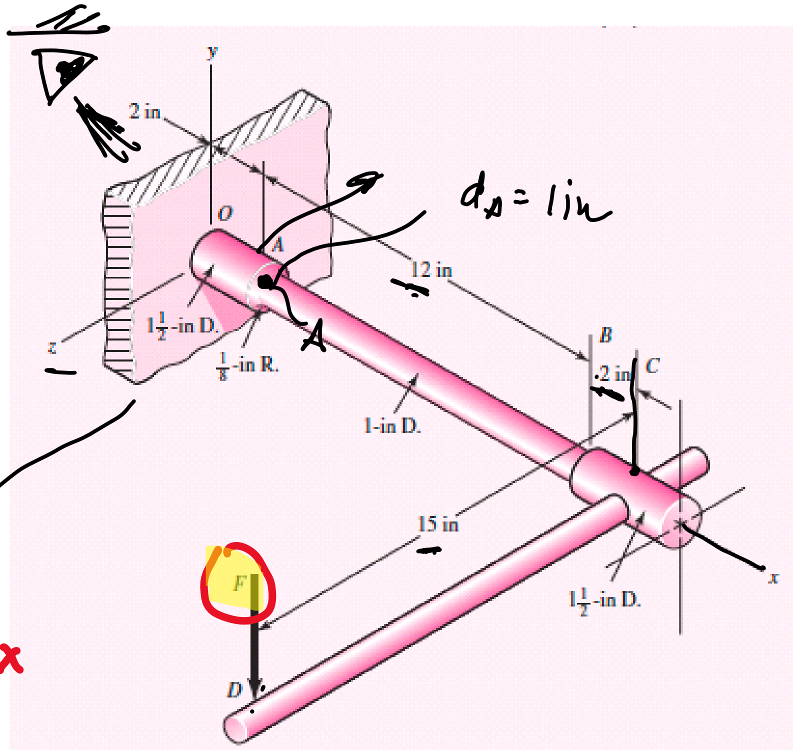
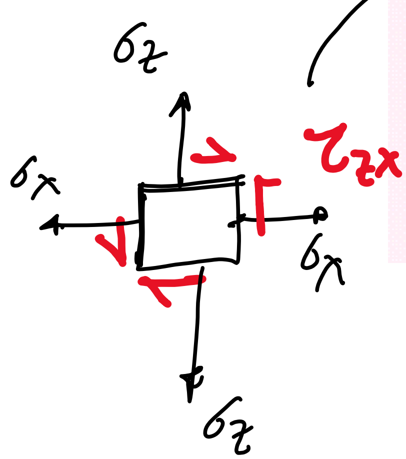
Failure Theories, Q1

A force F is applied at D as shown. Assuming that the material is ductile and the critical point for failure is A. Compute the value of F assuming (a) Maximum shear stress theory, (b) Distortion energy theory
Assume that the yield strength is 81 kpsi



$F = ?$
(a) MSS
(b) DE
$S_y = 81$ kpsi



Steps

FBD/SFD/BMD
Strength of materials

① Compute σ_x, σ_z, τ_{zx} as a function of F.

Mohr's circle

② Compute τ_{max} (MSS)
σ' (von-Mises/DE)

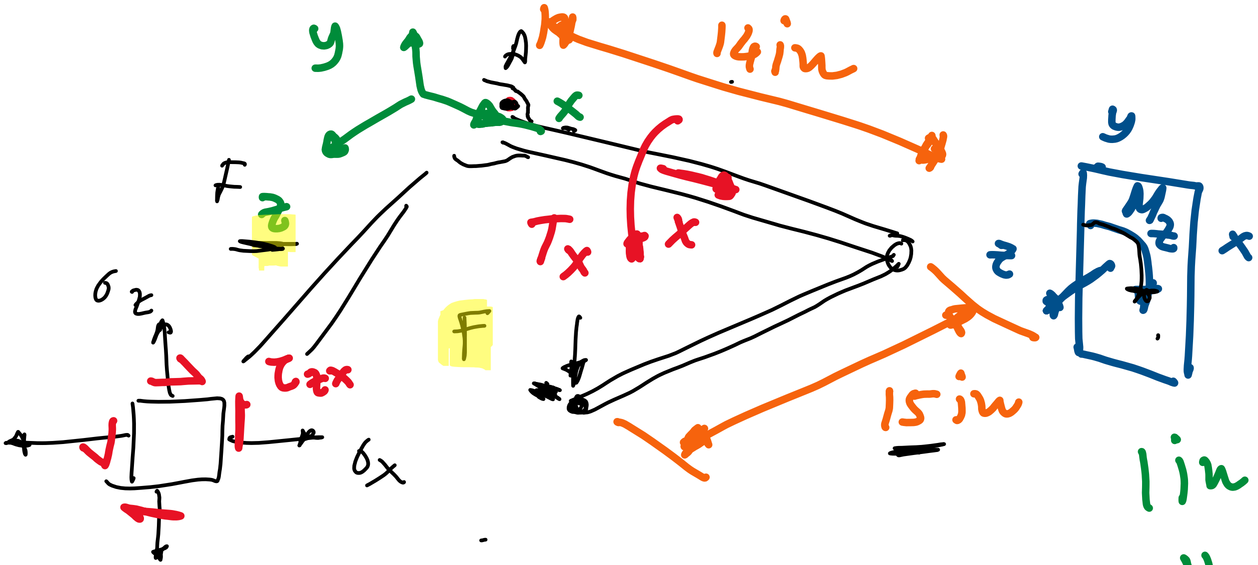
Failure theory

③ $\tau_{max} = \frac{S_y}{2}$ (MSS)
$\sigma' = S_y$ (DE)
Solve for F

theory

$\sigma' = \mathcal{S}_y \, (\mathrm{DE})$

① Compute σ_x, σ_z, τ_{zx} in terms of F



1 in

$$\tau_{zx} = \frac{T_x\, d_A/2}{J_A} = \frac{(15F)(d_A/2)}{\frac{\pi d_A^4}{32}} = \left[\frac{16}{\pi d_A^3}\right][15F]$$

$$\underline{\tau_{zx} = 76.4F}$$

1 in

14F

$$\sigma_x = \frac{M_z\, y}{I} = \frac{M_z\, d_A/2}{\frac{\pi d_A^4}{64}} = \left(\frac{32}{\pi d_A^3}\right)[14F]$$

$$\underline{\sigma_x = 142.6F}$$

$$\sigma_z = 0$$

② Compute τ_{max} (MSS) ; σ' (DE)

✓ $\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2}$

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_z^2 - \sigma_z \sigma_x + 3\tau_{zx}^2}$$

$$\sigma_{A,B} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$\sigma_{A,B} = \frac{142.6F + 0}{2} \pm \sqrt{\left(\frac{142.6F}{2}\right)^2 + (76.4F)^2}$$

$$= 71.3F \pm 104.5F$$

$\sigma_A = 175.8F$; $\sigma_B = -33.2F$; $\underline{\underline{\sigma_C = 0}}$

$$\begin{array}{ccccc} \sigma_A & \geq & \sigma_C & \geq & \sigma_B \\ \| & & & & \| \\ \sigma_1 & & & & \sigma_3 \end{array}$$

$\left.\begin{array}{l} \sigma_1 = 175.8F \\ \sigma_3 = -33.2F \end{array}\right\}$ $\tau_{max} = \dfrac{175.8F - (-33.2)F}{2}$

$\tau_{max} = 104.5F$

$\tau_{max} = 104.5\,F$

$$\sigma_z = 0$$

$$\sigma' = \sqrt{\sigma_x^2 + \cancelto{0}{\sigma_z^2} - \cancelto{0}{\sigma_z}\sigma_x + 3\tau_{zx}^2}$$

$$\sigma' = \sqrt{(142.6\,F)^2 + 3\,(76.4)^2}$$

$$\sigma' = 194.5\,F$$

③ <u>Compute F</u>

MSS : $\tau_{max} = \dfrac{S_y}{2}$

$$104.5\,F = \frac{81\,(10^3)}{2} \Rightarrow \boxed{F_{MSS} = 388\ \text{lbf}}$$

DE : $\sigma' = S_y$

$$194.5\,F = 81\,(10^3) \Rightarrow \boxed{F_{DE} = 416\ \text{lbf}}$$

<u>NOTE:</u>

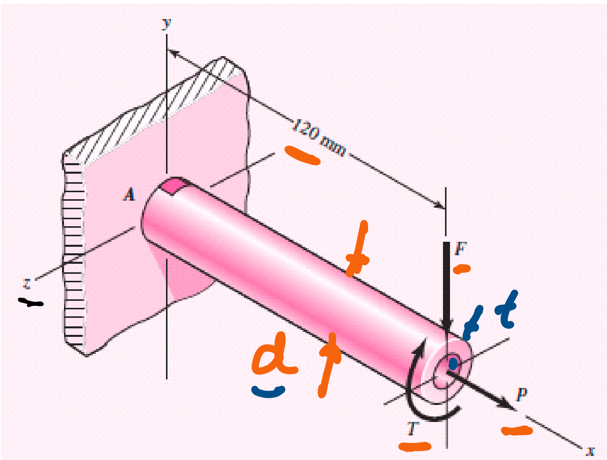
$$\underbrace{F_{DE} > F_{MSS}}_{}$$

MSS is more conservative than DE

Failure Theories, Q2

The shaft shown below is subject to F = 1.75 kN, P = 9 kN, and T = 72 Nm. The material used is Aluminum with a yield strength of 276 Mpa. Using the table shown below and design factor n_d = 4, (a) select a stock size for the shaft, and (b) compute the factor of safety. Assume von Mises stress as the failure criterion

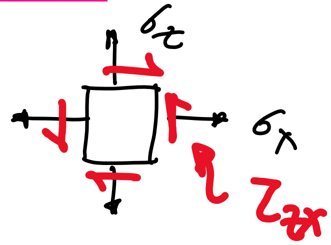
$n_d = 4$, $S_y = 276$ } $n_d = \frac{S_y}{\sigma'}$



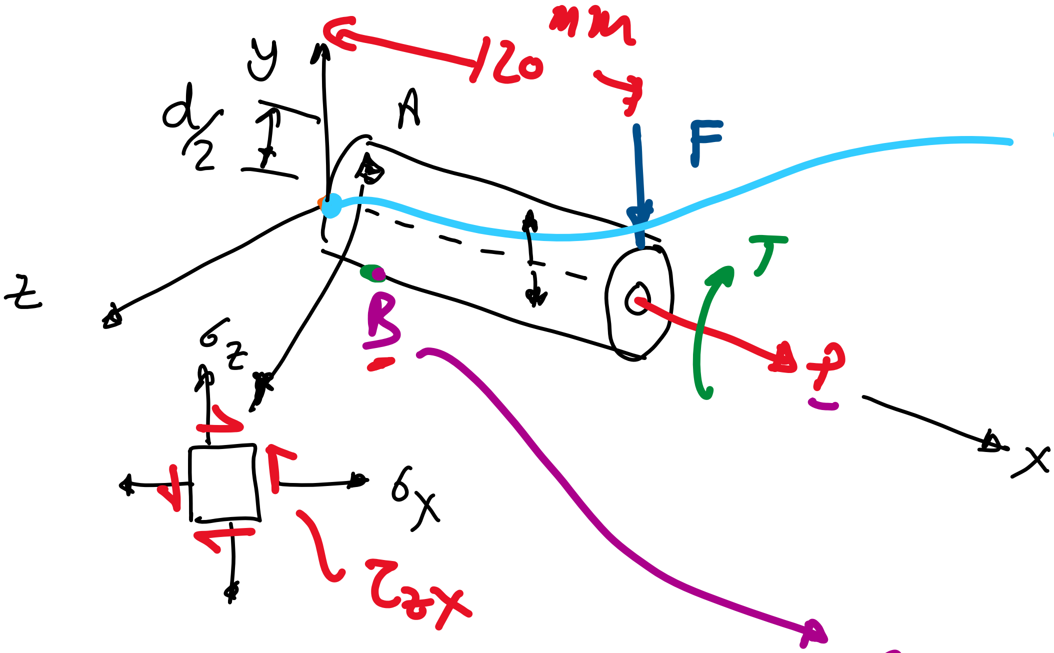
m = unit mass, kg/m
A = area, in^2 (cm^2)
I = second moment of area, in^4 (cm^4)
J = second polar moment of area, in^4 (cm^4)
k = radius of gyration, in (cm)
Z = section modulus, in^3 (cm^3)
d, t = size (OD) and thickness, in (mm)

Size, mm	m	A	I	k	Z	J
12 × 2	0.490	0.628	0.082	0.361	0.136	0.163
16 × 2	0.687	0.879	0.220	0.500	0.275	0.440
16 × 3	0.956	1.225	0.273	0.472	0.341	0.545
20 × 4	1.569	2.010	0.684	0.583	0.684	1.367
25 × 4	2.060	2.638	1.508	0.756	1.206	3.015
25 × 5	2.452	3.140	1.669	0.729	1.336	3.338
30 × 4	2.550	3.266	2.827	0.930	1.885	5.652
30 × 5	3.065	3.925	3.192	0.901	2.128	6.381
42 × 4	3.727	4.773	8.717	1.351	4.151	17.430
42 × 5	4.536	5.809	10.130	1.320	4.825	20.255
50 × 4	4.512	5.778	15.409	1.632	6.164	30.810
50 × 5	5.517	7.065	18.118	1.601	7.247	36.226

d x t

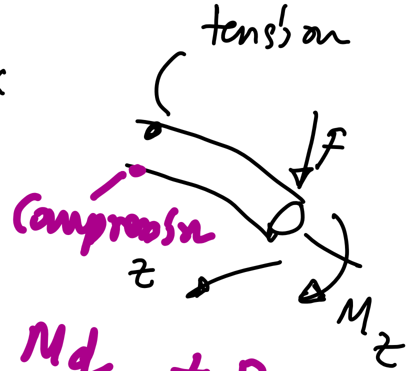


1. compute stresses $\sigma_x, \sigma_z, \tau_{zx}$
2. $\sigma' = \sqrt{\sigma_x^2 + \sigma_z^2 - \sigma_z\sigma_x + 3\tau_{zx}^2}$
3. $n_d = S_y/\sigma'$ and table to choose stock size.



$\sigma_x = \frac{P}{A} + 0$

neutral axis



$\sigma_x = -\frac{M d/2}{I} + \frac{P}{A}$

$F(0.12) = 1.75(10^3)(0.12)$

$\sigma_x = +\frac{M d/2}{\underbrace{I}_{\frac{My}{I}}} + \frac{P}{A}$

$\frac{P}{A} \rightarrow 9(10^3)$

$\sigma_z = 0$

$\tau_{zx} = \frac{T d/2}{\underbrace{J}_{T\rho/J}}$

$T \rightarrow 72\,Nm$

(2) $\sigma' = \sqrt{\sigma_x^2 + \cancelto{0}{\sigma_z^2} - \cancelto{0}{\sigma_z}\,\sigma_x + 3\tau_{xz}^2}$

$$\sigma' = \sqrt{\left(\frac{P}{A} + \frac{0.12F\,d/2}{I}\right)^2 + 3\left(\frac{T\,d/2}{J}\right)^2} \quad \text{— I}$$

(3) $n_d = \dfrac{S_y}{\sigma'} \Rightarrow 4 = \dfrac{276}{\sigma'}$

$\sigma' = 69$ MPa — II

EXAM 2 material upto here.

Table

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d x t	A	I	J	sigma'
12 x 2	0.628	0.082	0.164	1740.751938 x 10^6
16 x 2	0.879	2.220	4.440	179.476881 x 10^6
16 x 3	1.225	0.273	0.546	712.676020 x 10^6
20 x 4	2.010	0.684	1.368	363.413031 x 10^6
25 x 4	2.638	1.508	3.016	214.508339 x 10^6
25 x 5	3.140	1.669	3.338	191.716983 x 10^6
30 x 4	3.266	2.827	5.654	142.865852 x 10^6
30 x 5	3.925	3.192	6.384	125.094305 x 10^6
42 x 4	4.773	8.717	17.434	71.052901 x 10^6
42 x 5	5.809	10.130	20.260	60.426032 x 10^6
50 x 4	5.778	15.409	30.818	50.667537 x 10^6
50 x 5	7.065	18.118	36.236	42.593595 x 10^6

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Pa

I

$< 69(10^6)$ Pa

II

(a) Stock size: 42x5 (d x t)

(b) $n = \frac{S_y}{\sigma'} = \frac{276}{60.4} = 4.56$

$\boxed{n = 4.56}$