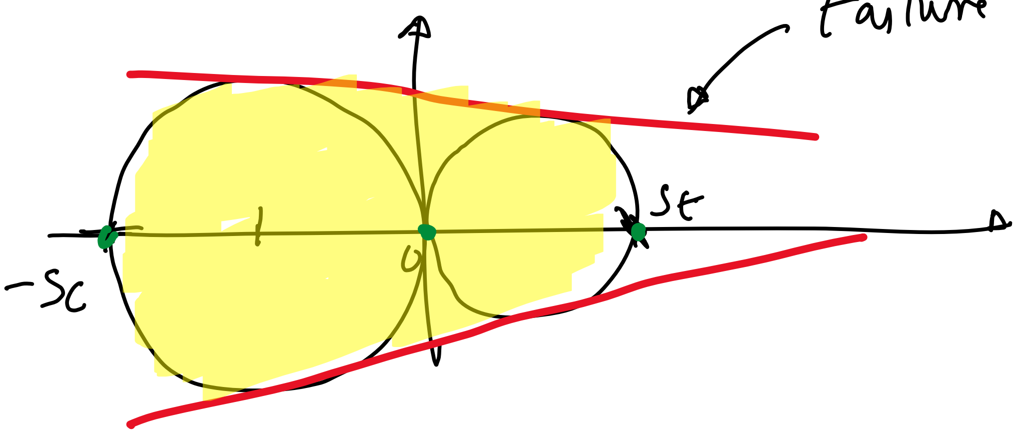
③ Coulomb-Mohr Theory for Ductile Materials

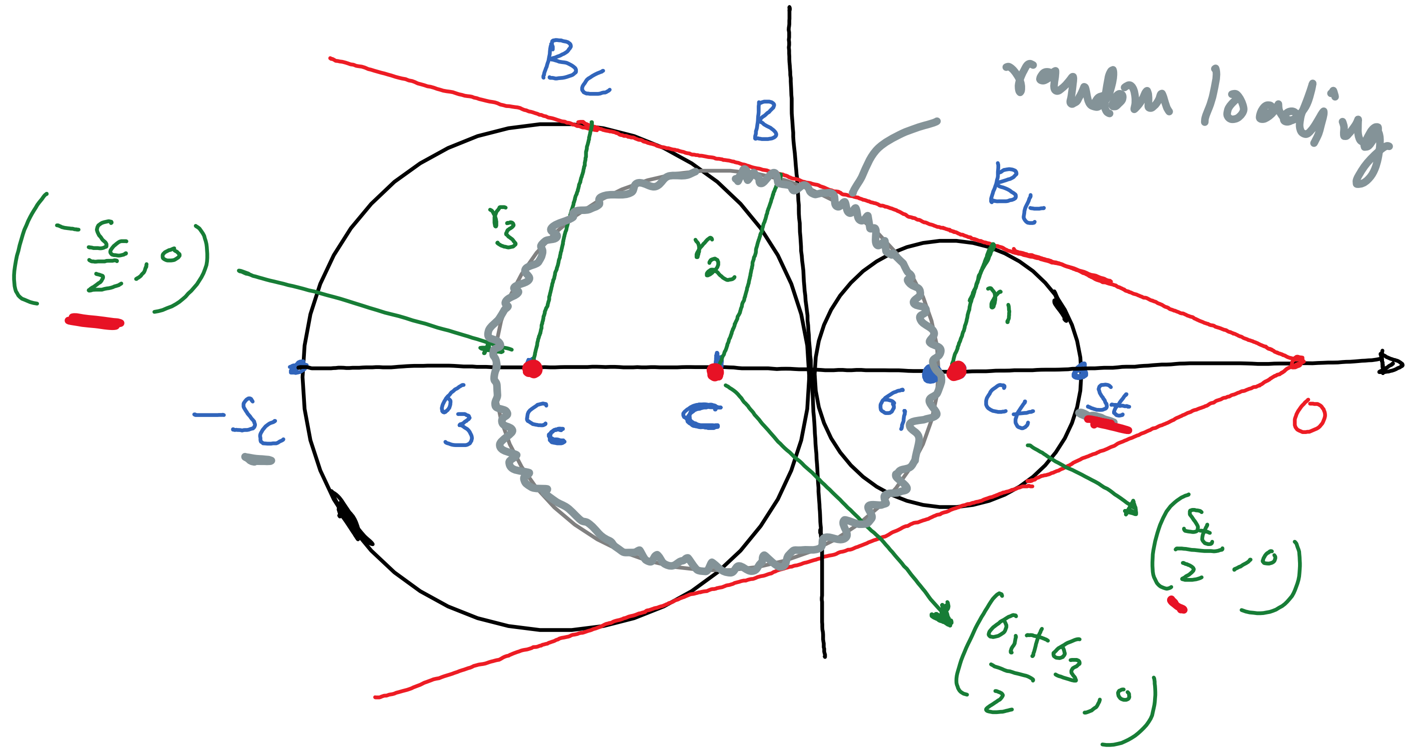
Useful when yield strength in tension is different from that in compress.

$$S_t \neq -S_c$$

$$-S_c > S_t$$

Failure envelope.





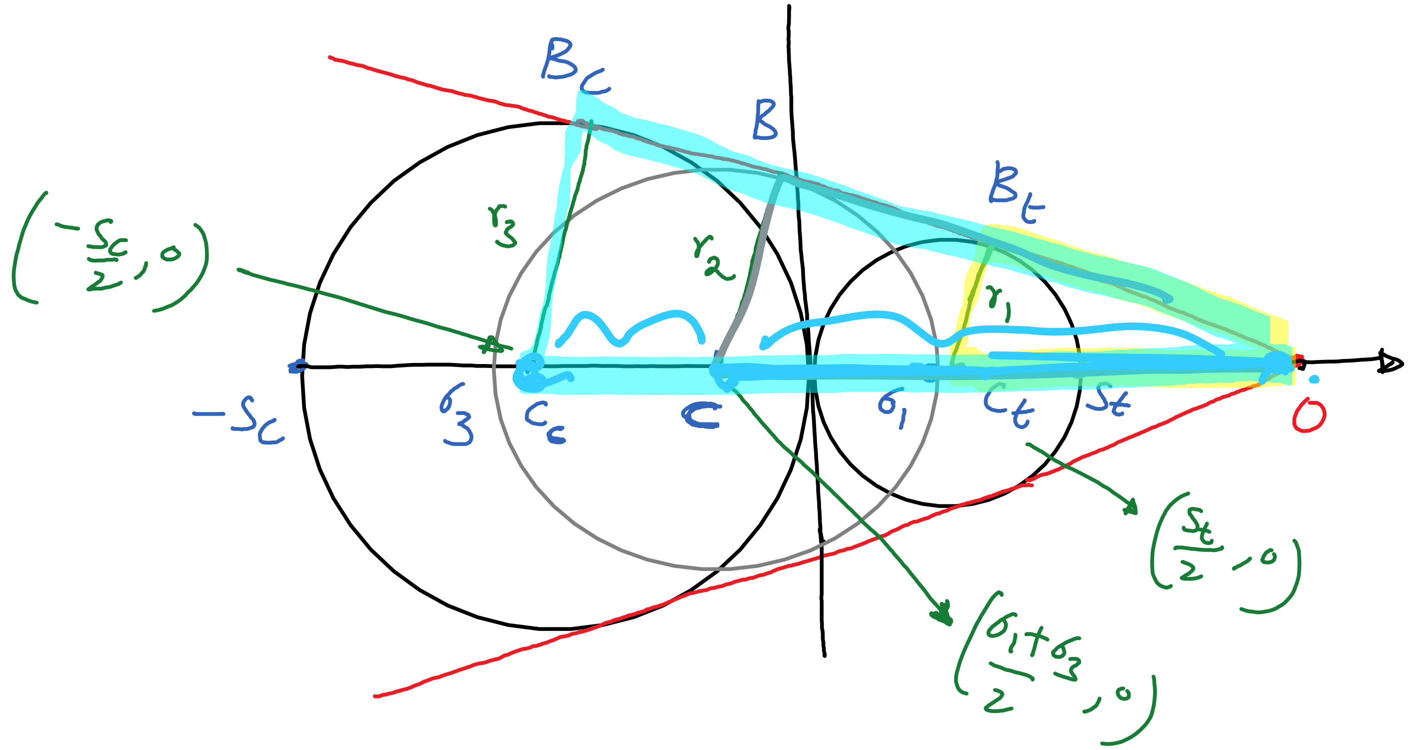
For a given stress condition we order the principal stresses as $\sigma_1 > \sigma_2 > \sigma_3$. The biggest circle has a radius $\frac{\sigma_1 - \sigma_3}{2}$ & is shown above

Per the figure the centers are

$$C_t \equiv \left(S_t/_2, 0\right) \; ; \; C = \left(\frac{\sigma_1 + \sigma_3}{2}, 0\right) \quad C_c \equiv \left(-\frac{S_c}{2}, 0\right)$$

radius are

$$r_1 = \frac{S_t}{2} \; ; \; r_2 = \frac{\sigma_1 - \sigma_3}{2} \; ; \; r_3 = \frac{S_c}{2}$$



Since triangles OC_tB_t, OCB, OC_cB_c are similar

$$\frac{OC_t}{B_tC_t} = \frac{OC}{BC} = \frac{OC_c}{B_cC_c} = \alpha \text{ (say)}$$

$$OC_t = \alpha B_tC_t \quad \text{— (I)}$$

$$OC = OC_t + CC_t = \alpha BC \quad \text{— (II)}$$

$$OC_c = OC_t + CC_c = \alpha B_cC_c \quad \text{— (III)}$$

(II) − (I): $$CC_t = \alpha (BC - B_tC_t) \quad \text{— (IV)}$$

(III) − (I): $$CC_c = \alpha (B_cC_c - B_tC_t) \quad \text{— (V)}$$

(IV) / (V): $$\frac{CC_t}{CC_c} = \frac{BC - B_tC_t}{B_cC_c - B_tC_t}$$

$$\begin{cases} \dfrac{CC_t}{CC_c} = \dfrac{BC - B_tC_t}{B_cC_c - B_tC_t} \\[2ex] \dfrac{CC_t}{CC_c} = \dfrac{r_2 - r_1}{r_3 - r_1} \end{cases}$$

Per the figure the centers are

$C_t \equiv (S_t/2, 0)$; $C \equiv \left(\dfrac{\sigma_1 + \sigma_3}{2}, 0\right)$ $C_c \equiv \left(-\dfrac{S_c}{2}, 0\right)$

radius are

$r_1 = \dfrac{S_t}{2}$; $r_2 = \dfrac{\sigma_1 - \sigma_3}{2}$; $r_3 = \dfrac{S_c}{2}$

$$\frac{\left[\dfrac{S_t}{2} - \dfrac{\sigma_1 + \sigma_3}{2}\right]}{\left[\dfrac{S_t}{2} - \left(-\dfrac{S_c}{2}\right)\right]} = \frac{\left(\dfrac{\sigma_1 - \sigma_3}{2}\right) - \dfrac{S_t}{2}}{\left(\dfrac{S_c}{2} - \dfrac{S_t}{2}\right)}$$

Simplifying gives

$$\boxed{\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1}$$ — red line

σ_1, σ_3 loading

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{S_y}{2} \Rightarrow \boxed{\frac{\sigma_1}{S_y} - \frac{\sigma_3}{S_y} = 1}$$

$\checkmark \; \frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$ Random loading $\sigma_A, \sigma_B, \sigma_C$

Plane Stress $= 0$

(1) $\underline{\sigma_A} \geq \underline{\sigma_B} \geq 0; \quad \sigma_C = 0$

$\sigma_1 = \sigma_A \qquad \sigma_3 = \sigma_C = 0$

$\frac{\sigma_A}{S_t} - 0 = 1 \Rightarrow \boxed{\sigma_A = S_t}$

Similarly $\underline{\sigma_B} \geq \underline{\sigma_A} \geq 0 \; ; \; \sigma_C = 0 \qquad \boxed{\sigma_B = S_t}$

(i)

(2) $\underline{\sigma_A} \geq 0 \geq \underline{\sigma_B} \qquad \sigma_C = 0$

Is negative

$\sigma_1 = \sigma_A \; ; \; \sigma_3 = \sigma_B$

$$\boxed{\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} = 1}$$

(ii) a

IV th

$\underline{\sigma_B} \geq 0 \geq \underline{\sigma_A} \qquad \sigma_C = 0$

$$\boxed{\frac{\sigma_B}{S_t} - \frac{\sigma_A}{S_c} = 1}$$

(ii) b

③ $0 \geq \sigma_A \geq \sigma_B \qquad \sigma_C = 0$

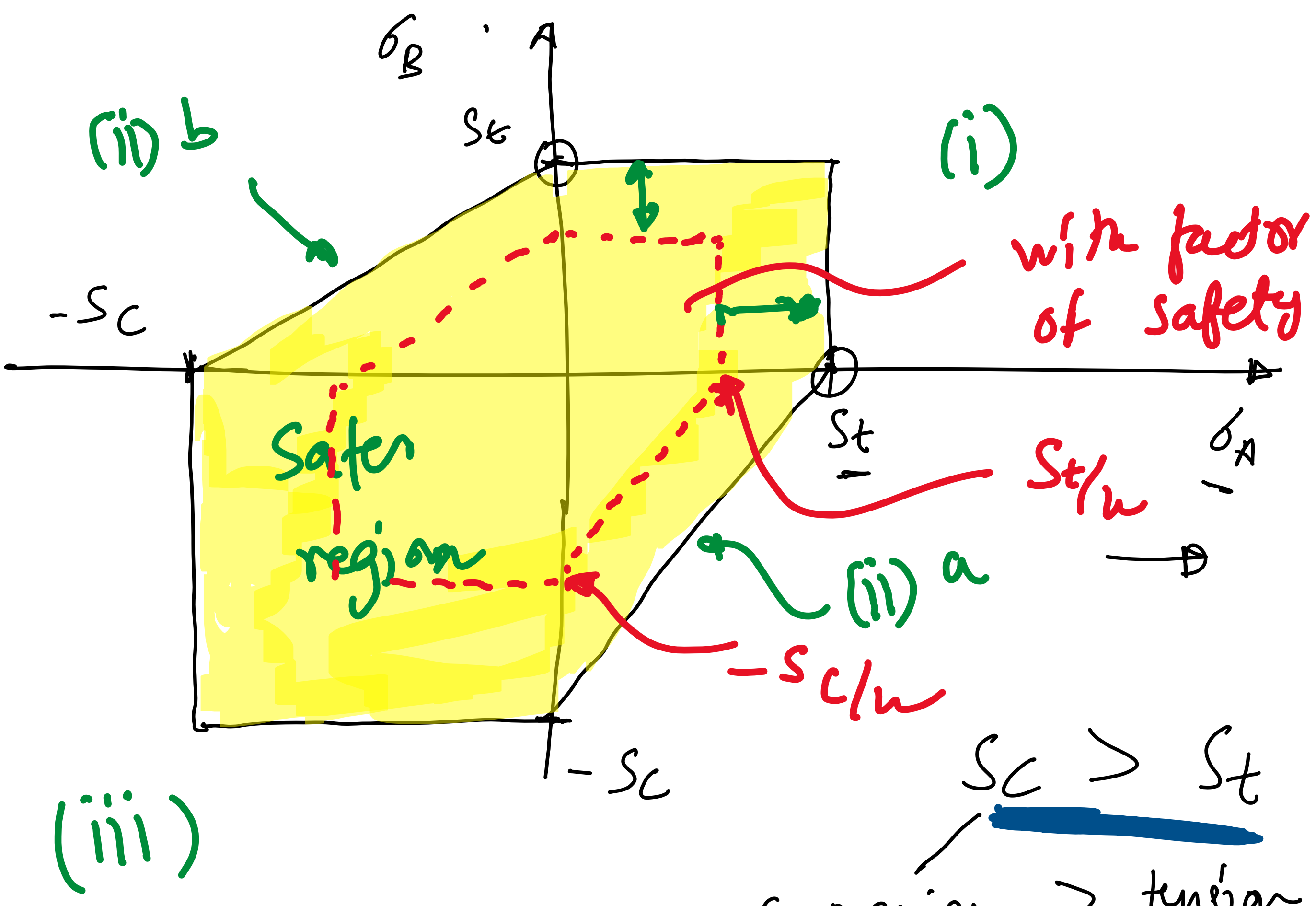
$\sigma_1 = 0$

$\sigma_3 = \sigma_B$

$$\frac{0}{S_t} - \frac{\sigma_B}{S_c} = 1 \qquad \boxed{\sigma_B = -S_c}$$

$$0 \geq \sigma_B \geq \sigma_A \qquad \boxed{\sigma_A = -S_c}$$

(iii)



$S_c > S_t$

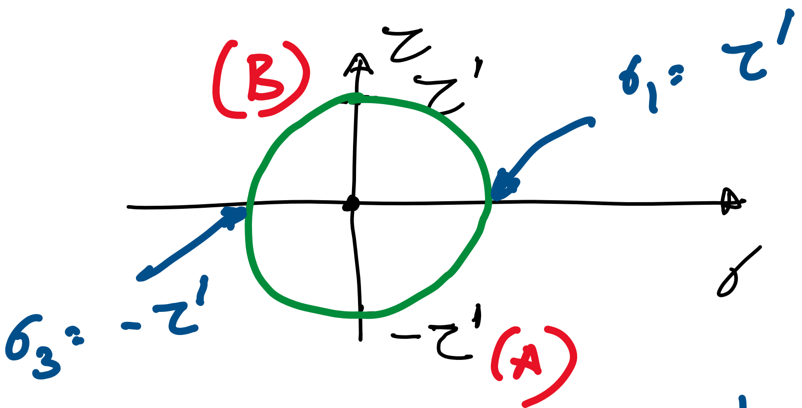
compression strength > tension strength

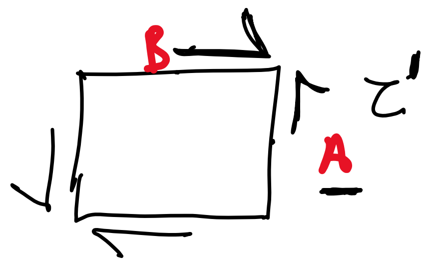
Using DCM for Design (n = factor of safety)

$$\frac{\sigma_1}{\left(S_t/n\right)} - \frac{\sigma_3}{\left(S_c/n\right)} = 1 \qquad \underline{\underline{n > 1}}$$

$$\boxed{\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}}$$

Use DCM for pure shear





DCM: $$\frac{\overset{=\tau'}{\sigma_1}}{S_t} - \frac{\overset{=-\tau'}{\sigma_3}}{S_c} = 1 \Rightarrow \frac{\tau'}{S_t} + \frac{\tau'}{S_c} = 1$$

$$\tau' = \frac{S_t S_c}{S_t + S_c}$$ S_{sy} – yield in shear

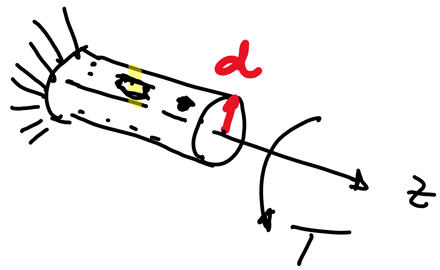
$$\boxed{S_{sy} = \frac{S_t S_c}{S_t + S_c}}$$

DCM Problem

A 25-mm-diameter shaft is statically torqued to 230 N · m. It is made of cast 195-T6 aluminum, with a yield strength in tension of 160 MPa and a yield strength in compression of 170 MPa. It is machined to final diameter. Estimate the factor of safety of the shaft.

$S_t = 160$ MPa ; $T = 230$ N.m

$S_c = 170$ MPa $d = 25$ mm



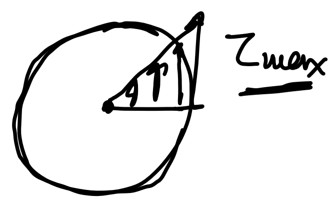
$n = ?$

Assume DCM : $\dfrac{\sigma_1}{S_t} - \dfrac{\sigma_3}{S_c} = \dfrac{1}{n}$?

$$\tau_{max} = \frac{T\rho}{J}$$

ρ — radius

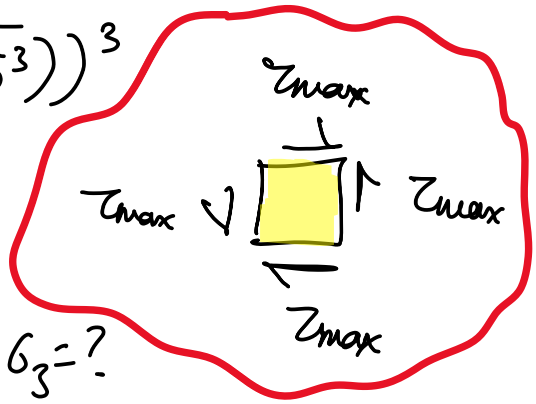
J — polar moment of inertia



$$\tau_{max} = \frac{T(d/2)}{\frac{\pi d^4}{32}}$$ — Table A-15 (?)

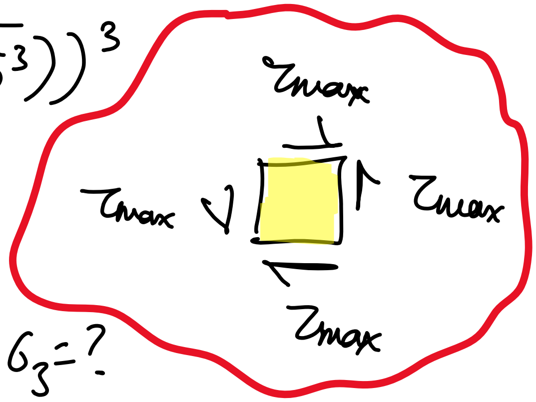
$$\tau_{max} = \frac{16\,T}{\pi d^3} = \frac{16\,(230)}{\pi\,(25(10^{-3}))^3}$$

$$\tau_{max} = 75 \text{ MPa}$$



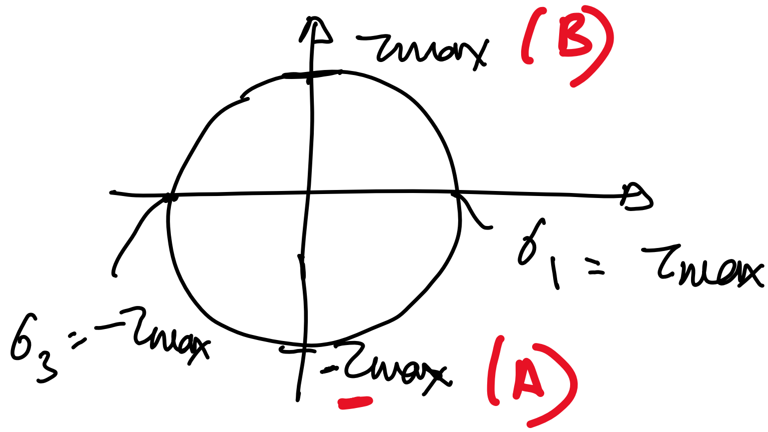
$\sigma_1 = ?$ $\sigma_3 = ?$

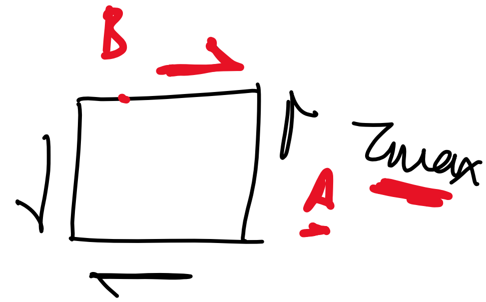
$$\tau_{max} = \frac{16\,T}{\pi d^3} = \frac{16\,(230)}{\pi\,(25(10^{-3}))^3}$$



$$\tau_{max} = 75\ MPa$$

$\sigma_1 = ?$ $\sigma_3 = ?$





$\sigma_2 = 0$

plane stress

$$\sigma_1 = -\sigma_3 = \tau_{max}$$

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

$$\frac{75}{160} - \frac{(-75)}{170} = \frac{1}{n}$$

$$\boxed{n = 1.1}$$

Failure of Ductile Materials